The Hybrid Finite Element - Boundary Integral - Multilevel Fast Multipole - Uniform Geometrical Theory of Diffraction Method

Thomas F. Eibert[(1)] and Andreas Tzoulis[(2)]
[(1)] *FGAN-FHR, Neuenahrer Str. 20, 53343 Wachtberg, Germany, Email: eibert@fgan.de.*
[(2)] *As [(1)] above, but Email: tzoulis@fgan.de.*

ABSTRACT

The Multilevel Fast Multipole Method (MLFMM) is hybridized with the ray-optical Uniform Geometrical Theory of Diffraction (UTD) and the methodology is combined with the hybrid Finite Element Boundary Integral Technique (FEBI). Thus, a very general hybrid FEBI-MLFMM-UTD technique results that can be applied to a wide variety of radiation and scattering problems. The MLFMM acceleration of the BI provides for the necessary flexibility to treat medium size objects within the numerically exact part of the hybrid technique and the asymptotic UTD must only be applied to large objects. The formulation of the method is discussed and numerical results are presented.

INTRODUCTION

Boundary Integral (BI) solutions by the Method of Moments (MoM) [1, 2] deliver usually most accurate and reliable results, if a given problem can be approached by such a formulation. Since conventional MoM solutions were impractical for somewhat larger geometries (compared to wavelength), the BI was hybridized with asymptotic high-frequency methods such as the Uniform Geometrical Theory of Diffraction (UTD) or Physical Optics (PO) many years ago [3, 4]. On the other hand, the BI was hybridized with local methods such as the Finite Element Method (FE) in order to handle inhomogeneous volumetric objects, for which conventional BI was inefficient as well [2]. Later, both hybrid methods were hybridized again resulting in hybrid FEBI-UTD techniques [5]. The resulting approaches were very versatile, but the FE and BI portions of the overall problem were still restricted to relatively small size, due to the bad numerical complexity of the BI. The complexity problems of stand-alone BI and hybrid FEBI methods could considerably be relieved by the development of fast integral methods, especially the Multilevel Fast Multipole Method (MLFMM). However, the MLFMM is not readily applicable within the BI-UTD hybridization.
In this contribution, the various hybrid methods and the MLFMM are brought together and this results in a very flexible hybrid FEBI-MLFMM-UTD method, where the electromagnetic model can be adapted to the requirements of various electromagnetic antenna and scattering problems. The hybrid method is especially attractive, if the UTD is restricted to few very large and simple scattering objects and this can be achieved due to the efficiency of BI-MLFMM. As in conventional hybrid BI-UTD methods, the UTD is employed to modify the Green's function used in the BI and the explicitly computed near-coupling matrix elements are thus modified. The combination of MLFMM and UTD is performed in the translation procedure on the various levels of the MLFMM, using a far-field representation of the appropriate translation operator to obtain the high-frequency incident fields at the critical points of the UTD. Due to the hierarchical multilevel structure of MLFMM, it can still be achieved that the UTD objects are very close to the FEBI-objects, such that only individual BI-basis functions must be far away from the critical points on the UTD-objects.

FORMULATION

FEBI-UTD Formulation for CFIE

Arbitrarily mixed conducting/dielectric objects treated within the FEBI part of the hybrid technique and electrically large conducting objects treated by UTD are assumed to be in the same homogeneos environment such as free space, see Fig. 1. The electric field intensity within the inhomogeneos volumes of the FEBI objects are modelled by the Finite Element Method (FEM) and are expanded in terms of edge element basis functions $\boldsymbol{\alpha}_n$ on tetrahedral volume meshes [2]. The fields in the exterior homogeneos space regions are expressed via the BI over the boundaries of the FEBI objects and are expanded in terms of RWG basis functions $\boldsymbol{\beta}_n$, in order to discretize the BI with a Galerkin type MoM [1]. The coupling of the fields in the various regions is given by the field continuity conditions at the boundaries of the FEBI objects and on the basis of Huygens' equivalence principle. UTD objects are taken into account by modifying the Green's functions and the incident fields within the BI with additionally received high-frequency contributions [5]. Here, up to 3rd order reflections

and diffractions on flat structures are considered.
Thus, the central building block of the hybrid technique is the BI according to Huygens' equivalence principle, where we work with the Combined Field Integral equation (CFIE) according to

$$Z(1-\alpha)MFIE - \alpha EFIE = 0\,. \tag{1}$$

α is the combination parameter with values from 0 to 1 and Z the wave impedance of the considered solution space. The Electrical Field Integral Equation (EFIE) is given by

$$\hat{n}\times\left[\hat{n}\times\left(\iint\limits_{A}\left[\bar{\mathbf{G}}^{E}_{J,tot}(\mathbf{r},\mathbf{r}')\cdot\mathbf{J}_A(\mathbf{r}')+\bar{\mathbf{G}}^{E}_{M,tot}(\mathbf{r},\mathbf{r}')\cdot\mathbf{M}_A(\mathbf{r}')\right]da'+\mathbf{E}^{inc}_{tot}(\mathbf{r})\right)+\frac{1}{2}\mathbf{M}_A(\mathbf{r})\right]=0, \tag{2}$$

where $\bar{\mathbf{G}}^{E}_{J,tot}(\mathbf{r},\mathbf{r}')$ and $\bar{\mathbf{G}}^{E}_{M,tot}(\mathbf{r},\mathbf{r}')$ are the total Green's functions of the electric field due to electric and magnetic surface currents, respectively, and $\mathbf{E}^{inc}_{tot}(\mathbf{r})$ is the total incident electric field at the observation point $\mathbf{r}$. The Magnetic Field Integral Equation (MFIE) can be expressed in the same way by duality. The total Green's functions of the hybrid problem are the superposition of the Green's functions of the direct coupling of the currents $\bar{\mathbf{G}}^{E/H}_{J/M}(\mathbf{r},\mathbf{r}')$ and of

$$\bar{\mathbf{G}}^{E/H}_{J/M,UTD}(\mathbf{r},\mathbf{r}')=\sum_{s}A_{R_s}\,\bar{\mathbf{R}}^{E/H}_{s}\cdot\,\bar{\mathbf{G}}^{E/H}_{J/M}(\mathbf{r}_{R_s},\mathbf{r}')+\sum_{v}A_{D_v}\,\bar{\mathbf{D}}^{E/H}_{v}\cdot\,\bar{\mathbf{G}}^{E/H}_{J/M}(\mathbf{r}_{D_v},\mathbf{r}')+\cdots\,, \tag{3}$$

which are the Green's functions of the high-frequency ray-based fields due to UTD objects. A_{R_s} and A_{D_v} are the divergence and phase factors for reflection and diffraction, respectively, and $\bar{\mathbf{R}}^{E/H}_{s}$ and $\bar{\mathbf{D}}^{E/H}_{v}$ the dyadic reflection and diffraction coefficients for electric or magnetic field, defined by basic GO and UTD concepts [7]. Direct coupling is ignored in the case of shadowing due to UTD objects. The incident electric and magnetic fields at the observation point $\mathbf{r}$ are similarly modified by high-frequency contributions due to UTD objects.

MLFMM-UTD Formulation

The matrix-vector product computations of the BI contribution within an iterative linear equation system solver are accelerated by MLFMM. The correspondig CFIE matrix elements due to electric currents can be written in FMM representation as [6]

$$\begin{aligned} Z^{CFIE}_{mn,J} = \; & - \; j\frac{\omega\mu}{4\pi}\alpha\oiint\tilde{\boldsymbol{\beta}}^{*}_{m}(\hat{k})\cdot T_L(\hat{k}\cdot\hat{r}_{m'n'})\left(\bar{\mathbf{I}}-\hat{k}\hat{k}\right)\cdot\,\tilde{\boldsymbol{\beta}}_{n}(\hat{k})\,d\hat{k}^2 \\ & + \; jZ\frac{k}{4\pi}(1-\alpha)\oiint\left(\hat{k}\times\,\tilde{\boldsymbol{\alpha}}^{*}_{m}(\hat{k})\right)\cdot T_L(\hat{k}\cdot\hat{r}_{m'n'})\,\tilde{\boldsymbol{\beta}}_{n}(\hat{k})\,d\hat{k}^2\,, \end{aligned} \tag{4}$$

where $\tilde{\boldsymbol{\beta}}_{n,m}(\hat{k})$, $\tilde{\boldsymbol{\alpha}}_{n,m}(\hat{k})$ are the $\hat{k}$-space representations of the basis functions and $T_L(\hat{k}\cdot\hat{r})$ is the translation operator. Also, the * denotes complex conjugation. The CFIE matrix elements for magnetic currents are obtained by duality.

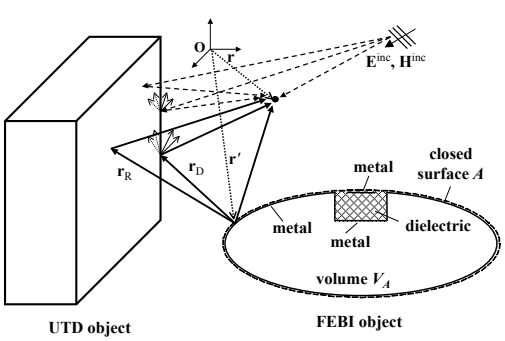

Fig. 1: Hybrid FEBI-UTD concept.

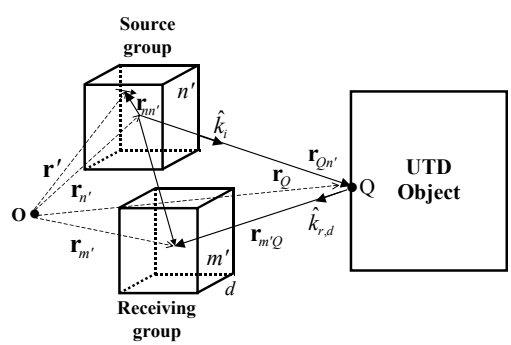

Fig. 2: Hybrid MLFMM-UTD concept.

UTD contributions in the matrix-vector product MLFMM computations are taken into account as illustrated in Fig. 2 by extending the translation operators on the various MLFMM levels. It is assumed, that the ray path from the source group to the UTD object $\mathbf{r}_{Qn'}$ is much greater than the dimensions of the group itself, so that only one $\hat{k}$ direction is required to represent the radiated group field. Assuming far-field conditions, the scalar Green's function from the source current to the local point on the UTD object is expressed with the far-field MLFMM approximation

$$G(\mathbf{r}_Q, \mathbf{r}') = \frac{e^{-jk|\mathbf{r}_Q - \mathbf{r}'|}}{|\mathbf{r}_Q - \mathbf{r}'|} = e^{j\mathbf{k}_i \cdot \mathbf{r}_{nn'}} T_L^{FF}\,, \tag{5}$$

where $\mathbf{k}_i = k\hat{k}_i$, $\hat{k}_i = \hat{r}_{Qn'}$ is the direction of ray incidence and $T_L^{FF} = \frac{e^{-jkr_{Qn'}}}{r_{Qn'}}$ is the corresponding far-field translation operator [8]. The received high-frequency fields are taken into account only for the reflection $\hat{k}_r$ or diffraction $\hat{k}_d$ directions from the critical point on the UTD object to the receiver groups. The high-frequency contributions of the EFIE matrix elements for electric currents in the hybrid MLFMM-UTD approach can be written as

$$\begin{aligned} Z_{mn,J,UTD}^{EFIE} = & - j\frac{\omega\mu}{4\pi}\,\tilde{\boldsymbol{\beta}}_m^*(\hat{k}_r) \cdot \sum_s A_{R_s}\,\bar{\mathbf{R}}_s^E T_L^{FF} \cdot \left(\bar{\mathbf{I}} - \hat{k}_r\hat{k}_i\right) \cdot \tilde{\boldsymbol{\beta}}_n(\hat{k}_i) \\ & - j\frac{\omega\mu}{4\pi}\,\tilde{\boldsymbol{\beta}}_m^*(\hat{k}_d) \cdot \sum_v A_{D_v}\,\bar{\mathbf{D}}_v^E T_L^{FF} \cdot \left(\bar{\mathbf{I}} - \hat{k}_d\hat{k}_i\right) \cdot \tilde{\boldsymbol{\beta}}_n(\hat{k}_i) + \cdots\,, \end{aligned} \tag{6}$$

where $\mathbf{k}_r = k\hat{k}_r$ and $\mathbf{k}_d = k\hat{k}_d$ are the propagation directions of the reflected and diffracted rays, respectively.

In general, ray directions of incidence $\hat{k}_i$, reflection $\hat{k}_r$ or diffraction $\hat{k}_d$ do not match with any of the sampling points used for numerical integration in the $\hat{k}$-space. Therefore, the required direction of incidence must be interpolated from the neighboring sampling points. Similarly, after reflection or diffraction, the appropriate ray direction must be anterpolated to the neighboring sampling points.

Care must be excercised in evaluating the UTD contributions within the MLFMM, since on coarser MLFMM levels it is more difficult to achieve the necessary far-field conditions required to justify UTD. Therefore, the MLFMM procedure is modified by extending the translation range around the source groups for coarser MLFMM levels. As a consequence, the UTD contributions are typically evaluated using fewer MLFMM levels than used for the direct contributions, which are still evaluated using the smallest possible translation ranges around the source groups. Also, improved far-field representations of the group fields can be achieved by performing the far-field expansion around the gravity centers of all basis functions in the source group instead of the usually used midpoints.

APPLICATIONS

First, the very general scattering configuration shown in Fig. 3 is considered, where full combination of the hybrid FEBI-MLFMM-UTD method is clearly demonstrated. The dimensions of the flat metallic plates (UTD objects) are $28.5\lambda_0 \times 60\lambda_0$, where λ_0 is the free space wavelength. The lossy dielectric structure with $\epsilon_r = 2.5 - j0.005$ is mounted in a metallic cavity. The mixed metallic-dielectric body has a square cross section in the xy-plane and the metallic torus is rotationally symmetric around the z-axis. All objects are placed symmetrically with respect to both the xz- and the yz-plane. The problem is excited by a y-polarized plane wave travelling in -z-direction with $|E_0| = 100$ V/m and $f_0 = 30$ GHz. In the hybrid FEBI-UTD computation the metallic torus and the mixed metallic-dielectric structure were discretized with tetrahedral elements within the dielectric volume and triangles on the surfaces, resulting in a model with 113049 unknowns. The electrically large plates were treated by UTD. The results are compared to the full FEBI solution, where all objects were discretized with 1715195 unknowns. In Fig. 4, the total electric near-field in direction of propagation is given for both computations.

The second example is a dielectric rod antenna in front of a large metallic screen with a quadratic aperture as depicted in Fig. 5. The dielectric rod is excited via a transition from a circular metallic hollow waveguide filled with the same dielectric and the hollow waveguide is excited by a metallic post connected to the center conductor of a coaxial feed. Fig. 6 shows computed radiation pattern results in the H-plane obtained with full FEBI-MLFMM and FEBI-MLFMM-UTD computations. The influence of the metallic screen is clearly seen when compared to the configuration without screen. FEBI-MLFMM-UTD and FEBI-MLFMM results show excellent agreement, where modelling the metallic screen with the rectangular aperture by UTD leads to considerable savings of computer resources.

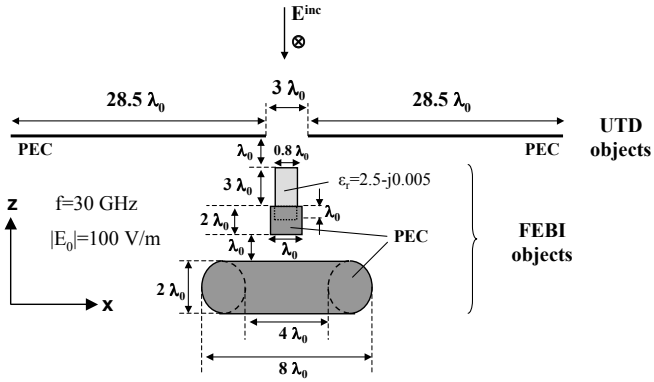

Fig. 3: Scattering problem of two FEBI and two UTD objects.

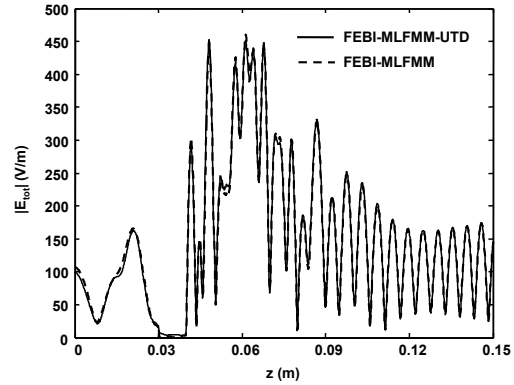

Fig. 4: Electric field in direction of propagation for scattering problem in Fig. 3.

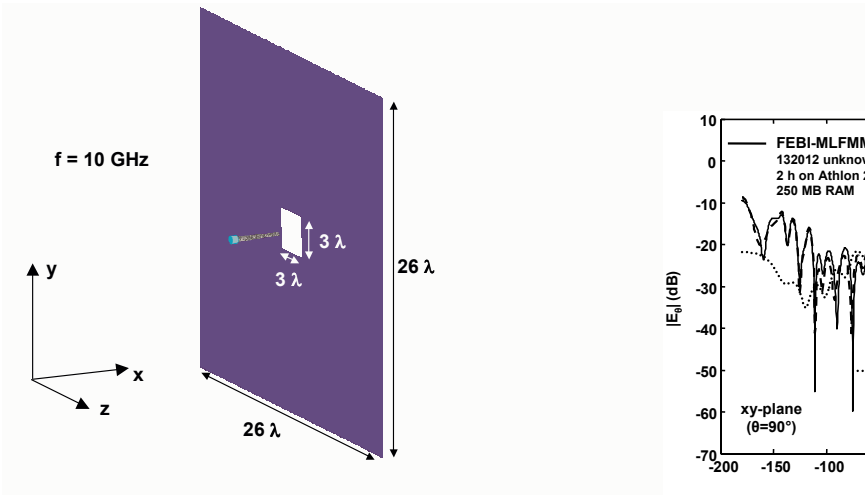

Fig. 5: Dielectric rod antenna in front of metallic screen with rectangular aperture.

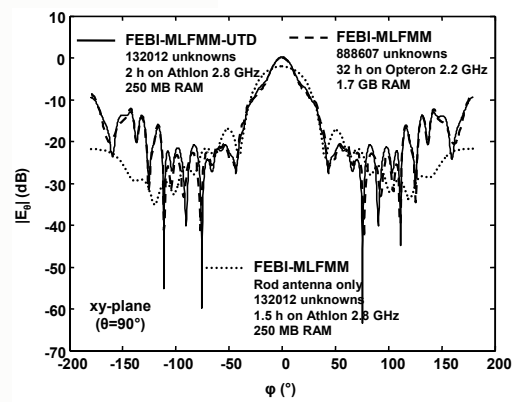

Fig. 6: H-plane radiation patterns and computer resources for configuration in Fig. 5.

References

[1] S.M. Rao, D.R. Wilton, and A.W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Trans. Antennas Propagat.,* vol. 30, no. 3, pp. 409-418, May 1982.

[2] J.-M. Jin, *The Finite Element Method in Electromagnetics,* 2nd Ed., John Wiley & Sons, Inc., New York, 2002.

[3] G.A. Thiele, "Overview of selected hybrid methods in radiating system analysis," *Proc. IEEE,* vol. 80, no. 1, pp. 66-78, Jan. 1992.

[4] U. Jakobus and F.M. Landstorfer, "Improved PO-MoM hybrid formulation for scattering from three-dimensional perfectly conducting bodies of arbitrary shape," *IEEE Trans. Antennas Propagat.,* vol. 43, pp. 162-169, Feb. 1995.

[5] M. Alaydrus, V. Hansen, and T.F. Eibert, "Hybrid^2: Combining the three-dimensional hybrid finite element - boundary integral technique for planar multilayered media with the uniform geometrical theory of diffraction," *IEEE Trans. Antennas Propagat.,* vol. 50, no. 1, pp. 67-74, Jan. 2002.

[6] W.C. Chew, J.-M. Jin, E. Michielssen, *Fast and Efficient Algorithms in Computational Electromagnetics,* Boston: Artech House, 2001.

[7] R.G. Kouyoumjian and P.R. Pathak, "A uniform geometrical theory of diffraction for an edge in a perfectly conducting surface," *Proc. IEEE,* vol. 62, no. 11, pp. 1448-1461, Nov. 1974.

[8] W.C. Chew, T.J. Cui and J.M. Song, "A FAFFA-MLFMA algorithm for electromagnetic scattering," *IEEE Trans. Antennas Propagat.,* vol. 50, no. 11, pp. 1641-1649, Nov. 2002.