

# THE ORTHOGONAL METHOD FOR THE DESIGN OF CONFORMAL ARRAYS IN THE PRESENCE OF MUTUAL COUPLING

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## ABSTRACT

The Orthogonal Method (OM) for the synthesis of conformal arrays is presented. The arrays are positioned on smooth or singular convex conducting surfaces (cylinders, paraboloids, elliptic cones) and contain either microstrip antenna elements or slots. Coupling effects between the elements are taken into account. Several examples of the design under constraints on SLL, on scanning capabilities and on specific patterns are included.

## INTRODUCTION

A general procedure that can be employed to design conformal arrays on smooth or singular conducting convex surfaces is presented. It makes use of the proper, for each host geometry, analysis procedure and suggests the Orthogonal Method (OM), [1], for the synthesis of the array. The present study includes arrays conformed on single curved surfaces (Cylinder), double curved surfaces (Paraboloid) and also surfaces having tip singularities (Elliptic cone). For the analysis of the host platform, analytic as well as asymptotic methods are employed, [2]. The cylinder and the elliptic cone are analyzed using, numerically stable and accurate, Green's function approaches. The respective problem of the paraboloid surface is treated by a caustic corrected UTD analysis. Arrays of patch and/or slot elements are synthesized. In the design examples, constraints on the side lobe level, on scanning capabilities and on specific (i.e. cosecant) pattern shaping are included. The method addresses design problems both at the deep lit/lit region of the radiating structure as well as at the transition region. In the last case (transition region) successful employment of the OM together with linear constraint methods is demonstrated.

The addressed design problems prove the effectiveness of the OM to synthesize arrays, conformal on convex smooth or singular surfaces, with strict radiation pattern requirements.

## THE SYNTHESIS PROCEDURE

A conformal array with  $N$  radiators is a linear system. The field of the array is a vector in a vector space consisting of all the possible fields that this system can produce. After defining a norm, one can use whatever set of  $N$  linearly independent fields to define this space. When there is no coupling the total field is the summation of the fields produced by each one of the radiating elements. The electric field of an  $N$  element array is given by:

$$\vec{E}(\theta, \phi) = \sum_{n=1}^N W_n \cdot \vec{F}_n(\theta, \phi) = [W]^T \cdot [\vec{F}] \quad (1)$$

where  $[W]$ ,  $[\vec{F}]$  are  $N \times 1$  column vectors. The  $n$ -th element  $W_n$  of  $[W]$  and the  $n$ -th element  $\vec{F}_n(\theta, \phi)$  of  $[\vec{F}]$  are the excitation and the electric field of the respective radiating element. For a desired electric field  $\vec{E}(\theta, \phi)$  given in (1) the corresponding  $[W]$  has to be derived. Following the OM one constructs an orthonormal basis  $[\hat{F}]$

$$[\hat{F}] = [C] \cdot [\vec{F}] \quad (2)$$

by using the Gram-Schmidt procedure. In the new basis the prescribed field is now written:

$$\vec{E}(\theta, \phi) = [B]^T \cdot [\hat{F}] \quad (3)$$

Combining (1) and (2) we have:

$$[W]^T = [B]^T \cdot [C] \quad (4)$$

If coupling is to be taken into account then  $[\vec{F}]$  in (1) has to be substituted by  $[\vec{F}_c]$ . The elements of the  $[\vec{F}_c]$

consist of the fields produced when each radiator is excited in the presence of the rest  $N-1$  ones. Coupling is presented by a linear transformation in the working vector space and the new basis,  $[\vec{F}_c]$

$$[\vec{F}_c] = [X] \cdot [\vec{F}] \quad (5)$$

For (4) and (5) the excitation  $[W_c]^t$  becomes:

$$[W_c]^t = [B]^t \cdot [C] \cdot [X]^{-1} \Rightarrow [W_c]^t = [W]^t \cdot [X]^{-1} \quad (6)$$

In (6)  $[B]$  is a column vector coming from of the projections of the prescribed field on the orthonormal basis,  $[C]$  is the matrix produced by the Gram-Schmidt procedure and  $[X]$  represents the matrix containing the mutual coupling between radiators. After the norm is defined, the above method is a global optimizer.

## SYNTHESIZED ARRAY EXAMPLES

In this section, synthesis examples are given for single and double curved surfaces as well as surfaces having tip singularities. The examples are accompanied by brief description of the analysis method that is used in each different case.

### 1. Single curved surfaces – Coated Conducting Circular Cylinder

#### 1a. Analysis Method

The rigorous analysis of the problem of a layered-grounded circular cylinder with patch antenna radiators on it entails first the derivation of the Green's function in a form of radially propagating waves. The patch radiators are replaced by surface electric current densities which are to be derived. The MoM for the currents derivation and the OM for the synthesis problem are applied. The radiation field is provided by the inverse Fourier Transform.

#### 1b. Array examples

An array consisting of rectangular microstrips having dimensions  $9.3\text{mm} \times 5.5\text{mm}$  is studied (see Fig. 1). The patches are excited with a probe being at distance of  $2.5\text{mm}$  from their smaller side. The operating frequency is  $10\text{GHz}$  and the cylinder has a radius of  $0.5\text{m}$ . The dielectric substrate of the array has a thickness of  $1.57\text{mm}$  and  $\epsilon_r=2.2$ . The array has 24 elements at an interelement angle  $2^\circ$  which corresponds to a distance of  $1.163$  wavelengths. Due to this distance only the mutual coupling between adjacent elements has a considerable value. A pattern of  $7^{\text{th}}$  degree Chebyshev polynomial with a half – power beamwidth of  $\sim 6^\circ$  and  $\text{SLL} \leq -30\text{dB}$  on the horizontal plane is desired and the OM is employed to provide the element excitations. Fig.2 shows  $E_\theta$  field and the current excitation for the antenna for both (coupling and noncoupling) cases. The resulting pattern, for the noncoupling case, has  $\text{HPBW}=5.05^\circ$  and  $\text{SLL}=-30.1\text{db}$ . By incorporating the coupling into the Orthogonal Method the obtained results, for the previously computed array, are:  $\text{HPBW}=5.06^\circ$  and  $\text{SLL}= -29.8\text{db}$ . In the case at hand the coupling produces practically the same results which was something that was expected due to the large ( $1.1635\lambda_0$ ) interelement distance.

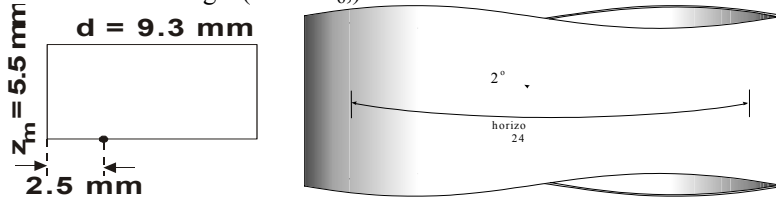


Fig.1. The element radiator and the cylindrical antenna.

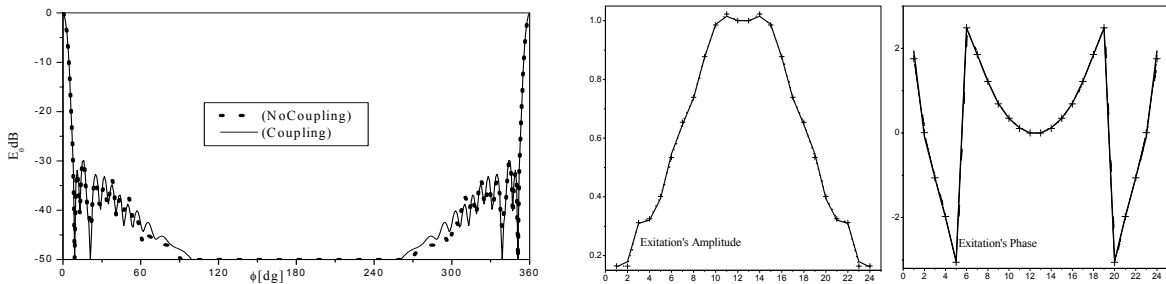


Fig.2.  $E_\theta$  field and current excitation of the cylindrical conformal array.

For the beamsteering of such an array one can use the sliding excitation method. An array with  $M$  ( $M > 24$ ) elements from which only 24 consecutive ones are excited at a time could solve the beamsteering problem providing the desired coverage angle/sector.

## 2. Double curved surfaces – Conducting Paraboloid

### 2a. Analysis Method

The host platform contains an array of slots and is analyzed by the Uniform Theory of Diffraction (UTD). The convex double curved surface inserts caustic problems in the analysis which can be properly treated [3]. The UTD-OM is a promising scheme that can be applied in the conformal array design, involving various conducting geometries. In fact, all the conducting platforms falling into the BOR (Body of Revolution) class can be addressed.

### 2b. Array examples

A slot array is under study (see Fig. 3a)

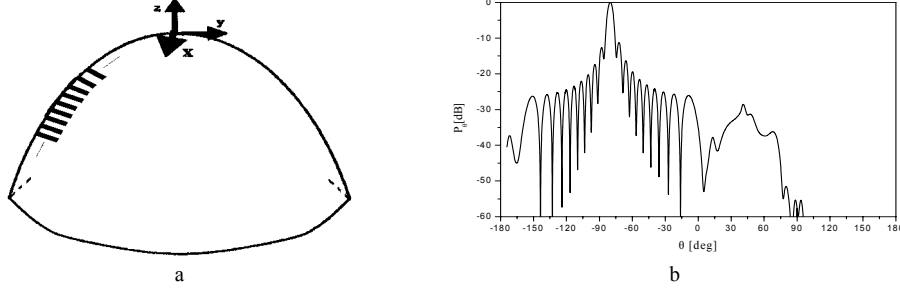


Fig. 3 a) A slot array with 21 circumferential slot, b) Far field pattern for uniform excitation.

It is formed by 21 circumferential slots. The slots are rectangular with thin widths and a length of half-wavelength ( $\lambda/2 \times \lambda/5$ ). The field on each slot can be well – approximated by a simple cosine distribution, i.e., the so – called “one mode approximation”. The distance between slots is  $\lambda/4$  measured on the paraboloid surface. In the case of uniform excitation the resulting far field pattern is given in Fig. 3b. The far field of the array in the zy plane is given by

$$F(\theta, \phi) = \sum_{n=1}^{21} F_n(\theta, \phi) \cdot E_n^+ \quad (7)$$

where  $F_n(\theta, \phi)$  is the far field produced by the array when only the n-th slot exists and  $E_n^+$  is the excitation amplitude of the dominant mode incident to this slot from its feeding waveguide.

For the array to meet strict requirements (low SLL, small HPBW, scanning capability), non-uniform excitation is required. The non-uniform excitation ( $E_n^+ \neq 1$ ) is provided by the Orthogonal Method (OM). In our example the desired pattern is a Chebyshev polynomial  $T_{20}$  with a SLL -25dB. The HPBW is  $\sim 8^\circ$  at the broadside. Employing OM, a wide area-scanning sector is formed ( $-40^\circ - -120^\circ$ ), (Fig. 4a). It is noticed that the appropriate excitations (see Fig. 4c) classify the solution as stable against construction or feeding imperfections.

In the previous results the mutual coupling was not taken into account. The proper design of conformal arrays requires an accurate estimation of this factor. The coupling information is included in the scattering matrix. To cope with mutual coupling the OM is enforced in (7) by substituting  $F_m(\theta, \phi)$  with  $\tilde{F}_m(\theta, \phi)$ .  $\tilde{F}_m(\theta, \phi)$  is the array’s far-field when only the m-th slot is excited

$$\tilde{F}_m(\theta, \phi) = \sum_{n=1}^{21} F_n(\theta, \phi) (S_{nm} + I_{nm}) \quad (8)$$

$S_{nm}$  is the nm element of the scattering matrix and  $I_{nm}$  is the nm element of the identity matrix. Notice the equivalence between the [X] matrix of (5) and the S+I matrix of (8). OM produces the same far-field patterns but the excitations due to the coupling inclusion have changed

The inclusion of the mutual coupling without recalculation of the excitations with the OM gives patterns of inferior compliance with the requirements, e.g. SLL > -25dB, (see Fig. 4b)

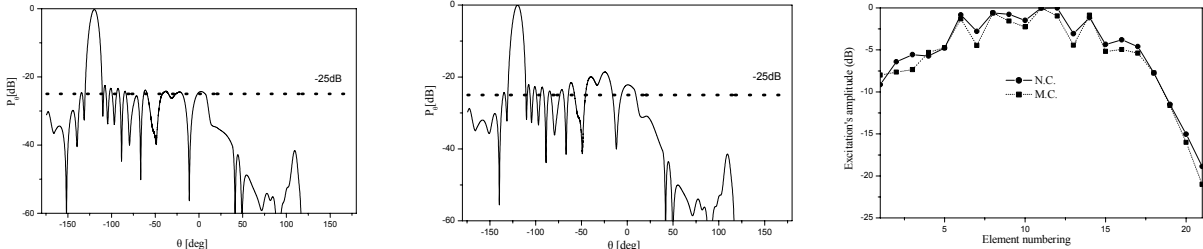


Fig. 4. a) Chebyshev pattern with maximum at  $\theta = -120^\circ$ , b) Pattern degradation due to lack of coupling information ( $\theta = -120^\circ$ ), c) Excitation in dBs for no coupling (N.C.) and mutual coupling (M.C.) cases ( $\theta = -120^\circ$ ).

### 3. Surfaces having tip singularities – Conducting Elliptic Cone

#### 3a. Analysis Method

In this case the dyadic Green's function for the design of slot arrays conformed on a perfectly conducting elliptic cone is used. The Green's function of an elliptic cone can be expressed in terms of non-periodic and periodic Lamé functions. The latter are derived from the solution of the wave equation in the sphero-conal co-ordinate system, where the elliptic cone is one of the co-ordinate surfaces, [2].

#### 3b. Array examples

The Orthogonal Method (OM), for two different slot arrays will be studied. One has to do with circumferential and the other with radial slots. Slots are rectangular with dimensions  $\lambda/2 \times \lambda/5$ . The array of circumferential slots (Fig. 5a) consists of 21 slots positioned on a cone with  $k^2 = 0.6$ ,  $\theta_c = 160^\circ$  at distances:  $k \cdot r_n = (2 - 0.3 + n \cdot 0.3) \cdot 2\pi$ ,  $n = 1, \dots, 21$ . The position of the first slot is  $2\lambda$ , while the last is located at  $8\lambda$  from the tip. Fig. 5d shows a quasi-radial array consisting of 20 slots positioned on a  $k^2 = 0.6$ ,  $\theta_c = 160^\circ$  elliptic cone at distances:  $k \cdot r_n = (2 - 0.5 + n \cdot 0.5) \cdot 2\pi$ ,  $n = 1, \dots, 20$ . The first slot is  $2\lambda$ , while the last is  $12\lambda$  far from the tip. A  $T_{20}$  and a  $T_{19}$  Chebyshev array with SLL of  $-30\text{dB}$  on the xz plane are desired for the two cases respectively. For both the circumferential and the radial slot arrays the main lobe can be directed  $\pm 35^\circ$  from the broadside direction. (Fig. 5b & 5e). The minimum relative excitation values, for all acceptable designs ( $\phi \sim \pm 35^\circ$  from broadside), become  $\sim 0.3$ . This classifies the solution as stable.

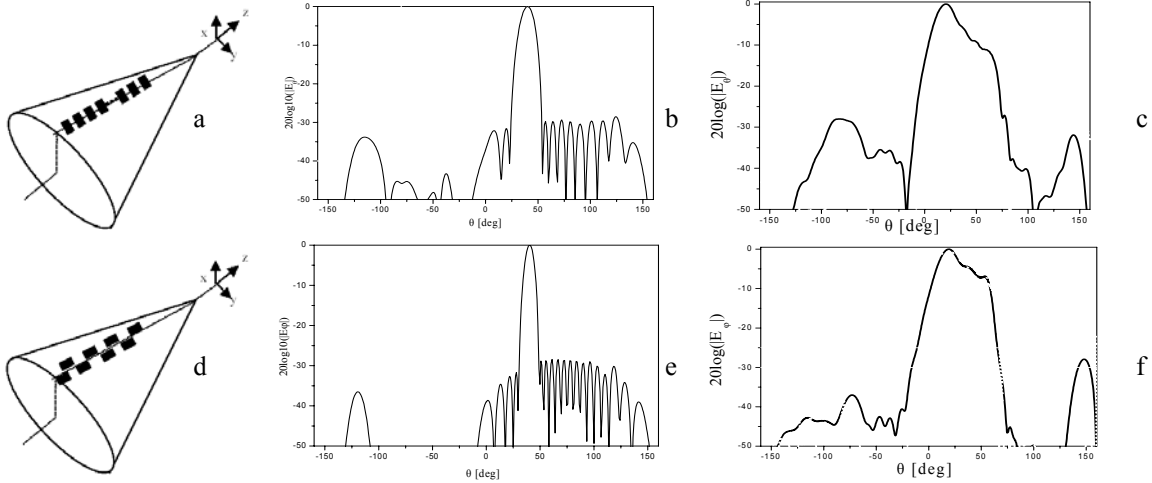


Fig. 5. a) The circumferential slot array, d) The radial slot array, b&e) respective scanning patterns, c&f) respective cosecant patterns.

Figs. 5c & 5f present results derived to produce cosecant patterns. This design demonstrates the successful employment of the OM together with linear constraint and pattern smoothing methods.

### CONCLUSION

The work at hand presents an advanced application of the Orthogonal Method. OM is used for the synthesis of patch/slot arrays on various non-planar platforms. The OM implementation verifies several useful features that can be summarized in the following list:

- The OM can be applied for any type of elements.
- Provides the desired pattern.
- Makes use of simple algorithms.
- Takes coupling into account.
- Produces stable solutions.

### REFERENCES

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