A NEW AND EFFECTIVE MODEL FOR 3-D FORWARD AND INVERSE SCATTERING PROBLEMS IN LOSSY SCENARIOS

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ABSTRACT

The demanding computational requirements of 3D scattering problems push towards the development of suitable solution models. Since the recently introduced CS-EB model has shown to be particularly effective for solving 2D scalar problems in presence of losses, in this work we have pursued its exploitation in the 3D vectorial case. Owing to the dyadic nature of the involved operators, a suitable reformulation is required. Derivation of the new model and its application to solution of forward and inverse scattering problems are discussed. Finally, its performances are assessed through numerical examples.

INTRODUCTION

Despite their potential performances, exploitation of microwave diagnostics techniques is nowadays still limited by several problems. For instance, when moving from the widely studied 2D geometry to the more realistic 3D one, many approaches proposed in the literature become hardly pursuable (or even unfeasible) due to increased complexity of the scenario at hand, as well as to the growing computational requirements. In order to possibly overcome these limitations, development of effective and efficient models and methods is therefore in order.
In this communication, we introduce a new model for solving 3D scattering problems in lossy environments (i.e. when losses are present in the embedding medium and/or in the scatterers), as for instance happens in biomedical imaging or subsurface sensing.
The starting point is given by the Contrast Source – Extended Born (CE-EB) model, recently introduced in the 2D scalar case [1]. With reference to lossy scenarios, in [1] it has been shown that the CS-EB model allows a reduction of computational costs in forward problems and an increased robustness against false solutions occurrence in inverse ones. Moreover, its effectiveness has also been proved in [2] for the lossless case.
Accordingly, it seems convenient to adopt the CS-EB also in the demanding 3D vectorial case. However, due to the dyadic nature of the operators at hand, such a task is far from being straightforward and a suitable reformulation of the CS-EB is required.

THE CONTRAST SOURCE EXTENDED BORN SCALAR MODEL (CS-EB-s)

Let us consider the geometry shown in Fig.1, in which a system of scatterers of equivalent permittivity $\varepsilon_x(\underline{r})$ is enclosed in a volume V embedded in a lossy homogeneous medium of equivalent permittivity ε_b and let $\chi(\underline{r})=\varepsilon_x(\underline{r})/\varepsilon_b$-1 be the contrast function. When the scatterers are illuminated by a known incident field $\underline{E}_{inc}$, an electrical current $\underline{J}$, the *contrast source*, related to the electrical field by the constitutive relation $\underline{J}(\underline{r})=\chi(\underline{r})\underline{E}(\underline{r})$, is induced in V. By omitting the time factor $\exp(j\omega t)$, the scattering problem can be described by the following coupled integral equations [3]:

$$\underline{J}(\underline{r}) = \chi(\underline{r})\underline{E}_{inc}(\underline{r}) + k_b^2\chi(\underline{r})\int_V \mathbf{G}_i(\underline{r},\underline{r}')\underline{J}(\underline{r}')dr' = \chi(\underline{r})\underline{E}_{inc}(\underline{r}) + \chi\mathbf{A_I}[\underline{J}] \qquad \underline{r},\underline{r}'\in V \tag{1.a}$$

$$\underline{E}_s(\underline{r}_m) = k_b^2\int_V \mathbf{G}_e(\underline{r}_m,\underline{r}')\underline{J}(\underline{r}')d\underline{r}' = \mathbf{A_e}[\underline{J}] \qquad \underline{r}_m\in\Gamma,\underline{r}'\in V \tag{1.b}$$

wherein k_b is the background (complex) wave-number, $\mathbf{G}_i$ and $\mathbf{G}_e$ denote the dyadic Green's functions which give the field scattered by an electrical dipole in V and on the measurement surface Γ, respectively and $\mathbf{A}_\mathbf{I}$: $L^2(V)$à $L^2(V)$ and $\mathbf{A}_\mathbf{e}$: $L^2(V)$à $L^2(\Gamma)$ are the corresponding dyadic integral radiation operators.

By following similar reasoning as in [1] for the 2D scalar case, we rewrite Eq.(1.a) as

$$\underline{J}(\underline{r}) = \chi(\underline{r})\underline{E}_{inc}(\underline{r}) + \chi(\underline{r})k_b^2 \int_V \boldsymbol{G}_i(\underline{r},\underline{r}')d\underline{r}'\,\underline{J}(\underline{r}) + \chi(\underline{r})k_b^2 \int_V \boldsymbol{G}_i(\underline{r},\underline{r}')[\underline{J}(\underline{r}') - \underline{J}(\underline{r})]d\underline{r}' \,. \tag{2}$$

When losses are present in the background medium, due to the singularity of the Green's function in the origin and its exponential decay when moving apart from the origin, the first integral term at the right-hand side of (2) becomes the dominant one, as the second term is more and more negligible for increasing losses.

Now, let us introduce the dyadic function

$$\boldsymbol{F}_V(\underline{r}) = k_b^2 \int_V \boldsymbol{G}_i(\underline{r},\underline{r}')d\underline{r}' \,, \tag{3}$$

whose dominant contribution for each observation point $\underline{r}$ and for increasing losses is the one lying on the diagonal, owing to the Green's function behaviour discussed above. Taking this latter circumstance into account, we can further separate the dyad $\boldsymbol{F}_V(\underline{r})$ into two contribution as:

$$\boldsymbol{F}_V(\underline{r}) = F_{dm}(\underline{r})\boldsymbol{I} + \boldsymbol{\Delta F}(\underline{r}) \tag{4}$$

wherein the first term (F_{dm}) is a scalar quantity proportional to the average value of the diagonal terms of $\boldsymbol{F}_V$ (which is expected to be the dominant one), while $\boldsymbol{\Delta F}$ is given by the difference between $\boldsymbol{F}_V$ and $F_{dm}\,\boldsymbol{I}$.

By defining the auxiliary function

$$p(\underline{r}) = \chi(\underline{r})[1 - \chi(\underline{r})F_{dm}(\underline{r})]^{-1} \tag{5}$$

we can then rewrite Eq.(1.a) as:

$$\underline{J}(\underline{r}) = p\underline{E}_{inc}(\underline{r}) + p\mathbf{A}_{\mathbf{iMOD}}[\underline{J}], \qquad \mathbf{A}_{\mathbf{iMOD}}(\underline{J}) = k_b^2 \int_V \boldsymbol{G}_i(\underline{r},\underline{r}')\underline{J}(\underline{r}')d\underline{r}' - F_{dm}(\underline{r})\underline{J}(\underline{r}) \tag{6}$$

Eq.(6) descends, *without any approximation*, from the Electric Field Integral Equation (1.a), hence, together with Eq.(1.b), it can be used to define a new model for 3D scattering problems. As the new equation exploits the CS-EB concepts [1] and takes advantage of the auxiliary function p, defined by Eq.(5), which embeds the traditional contrast function and preserves its scalar nature, we have named the new scattering model Contrast Source Extended Born scalar (CS-EB-s) model.

EFFECTIVE SOLUTION OF FORWARD PROBLEMS VIA CS-EB-s

Solution of the forward scattering problem can be cast as:

$$\underline{J}(\underline{r}) = (\mathbf{I} - p\mathbf{A}_{\mathbf{IMOD}})^{-1}\, p\underline{E}_{inc}(\underline{r}) \tag{7}$$

which shows that the inverse of the dyadic operator ($\mathbf{I}$-p $\mathbf{A}_{\mathbf{IMOD}}$) has to be evaluated. On the other side, in the case wherein $\|p\mathbf{A}_{\mathbf{IMOD}}\|<1$, the inverse operator in (7) can be expand into a Neumann series and the solution of the forward problem can be computed in a very easy and efficient way by iterations of the kind:

$$\underline{J}_n = p\underline{E}_{inc} + p\mathbf{A}_{\mathbf{IMOD}}[\underline{J}_{n-1}], \tag{8}$$

where $\underline{J}_n$ is the partial sum (up to the n-th term) of the series expansion, which defines the CS-EB-s series.

To evaluate effectiveness of the model is now necessary to investigate the behaviour of the norm $\|p\mathbf{A}_{\mathbf{IMOD}}\|$ which plays a key role in the actual possibility of using (8). As p embeds the information related to the scatterers, one should in principle evaluate $\|p\mathbf{A}_{\mathbf{IMOD}}\|$ from case to case. However, since $\|p\mathbf{A}_{\mathbf{IMOD}}\|\leq\|p\|\,\|\mathbf{A}_{\mathbf{IMOD}}\|$, it is possible to achieve a sufficient criterium to establish applicability of the series expansion by separately studying $\|p\|$ and $\|\mathbf{A}_{\mathbf{IMOD}}\|$. By so doing, one can also split the effect of the objects from that of the scenario, since $\|p\|$, whose determination is anyway straightforward, depends on the scatterers, while $\|\mathbf{A}_{\mathbf{IMOD}}\|$ only depends on the electromagnetic background characteristics and the shape of the investigated volume. In particular, it is possible to build some "universal" plots by expressing $\|\mathbf{A}_{\mathbf{IMOD}}\|$ as function of the electrical dimension d of the region under test and the background's tangent loss. This "universal" plot is shown in Fig.2. To build this plot we have iteratively computed $\|\mathbf{A}_{\mathbf{IMOD}}\|$ by means of the *power method* [4], which allows exploiting the convolutional nature of dyadic operator $\mathbf{A}_{\mathbf{IMOD}}$ and to overcome the problem of storing a very large matrix, which would result in an unfeasible task as discretization increases

The achieved criterium, together with the universal plot of Fig.2 provide the tools to a priori determine the applicability of the CS-EB-s series in the solution of forward scattering problem.

It is worth to note that the CS-EB-s series favourably compares with the traditional Born series i.e., the series expansion arising from Eq.(1.a). In particular, not only the CS-EB-s converges faster but it also has a wider applicability range.

In addition, it is worth to remark that, as long as conditions for the applicability of the series are fulfilled, this expansion is a more effective strategy than usual Conjugate Gradient schemes to solve forward scattering problems, due to the extreme simplicity of its single iteration both in terms of computational times and memory requirements.

Accordingly, it makes sense to take advantage of suitable strategies to further improve convergence rate as well as enlarge convergence range of series expansions. The over-relaxation technique [5] consists in defining a new iterative process given by:

$$\underline{J}_n = \alpha p \underline{E}_{inc} + \left[\mathbf{I} - \alpha\left(\mathbf{I} - p\mathbf{A}_{\mathbf{iMOD}}\right)\right]\underline{J}_{n-1} = \alpha p \underline{E}_{inc} + \left[\mathbf{I} - \alpha \mathbf{L}\right]\underline{J}_{n-1} \qquad \underline{J}_0 \quad arbitrary\,, \tag{9}$$

whose convergence properties are governed by the parameter α. A sub-optimal but effective [5] choice for this parameter is obtained by minimizing the residual error after the first iteration, by assuming $\underline{J_0}=p\underline{E}_{inc}$, i.e.:

$$\alpha = \langle p\mathbf{A}_{\mathbf{IMOD}}(p\underline{E}_{inc}), \mathbf{L}[p\,\mathbf{A}_{\mathbf{IMOD}}(p\underline{E}_{inc})]\rangle / \| \mathbf{L}[p\,\mathbf{A}_{\mathbf{IMOD}}(p\underline{E}_{inc})]\|^2 . \tag{10}$$

By exploiting (9) and (10) remarkable improvements in the solution of forward problems via series expansions are possible. Moreover, it is worth to note that since the overall computational complexity is unaltered, also this kind of iterative approach favourably compares with Conjugate Gradient schemes. Numerical evidence of the effectiveness of the iterative process described above will be given at the Conference.

APPLICATION OF THE CS-EB-s MODEL TO THE INVERSE PROBLEM

The inverse problem consists in determining the unknown contrast from the measurements of the scattered fields generated by a known collection of incident fields. Within the framework of the CS-EB-s model, the proposed inversion method is divided into two parts. The first one belongs to the class of modified gradient approaches and consists in finding the global minimum of the functional:

$$\sum_{v=1}^{V} \frac{\left\| \underline{J}^{v} - p\underline{E}^{v}_{inc} - p\mathbf{A}_{\mathbf{IMOD}}\left[\underline{J}^{v}\right]\right\|^2}{\left\|\underline{E}^{v}_{inc}\right\|^2} + \frac{\left\|\underline{E}_s - \mathbf{A}_{\mathbf{e}}\left[\underline{J}^{v}\right]\right\|^2}{\left\|\underline{E}_s\right\|^2} \tag{11}$$

with respect to p and the additional unknowns given by the contrast sources $\underline{J}^1,\ldots,\underline{J}^V$ (V being the number of illuminations). Exploitation of these latter allows to defeat the non-linearity of the problem but of course increases the overall computational burden. However, by making use of FFT routines in building the dyadic operators and exploiting a Conjugate Gradient minimization procedure, this task can be pursued quiet efficiently.

Ill-posedness of the problem is tackled by combining two different regularization strategies: the unknown functions are expanded into finite dimensional representations (in particular for the sake of comparison with the most widely adopted inversion schemes, a pixel based representation has been adopted), moreover, as the number of unknowns still exceeds the degrees of freedom of data, a further 'starting point induced' regularization has been used exploiting the back-propagation solution. The second step of the procedure consists in inverting point-wise Eq.(5) to determine the contrast function from the estimated value of p. As an interesting comparison with the widely adopted Contrast Source inversion method [3], let us consider the case of an inhomogeneous cubic scatterer. Such an object is composed by an inner cube of dimension $\lambda/2$x$\lambda/2$x$\lambda/2$, with contrast χ=0.5-j0.3, surrounded by an outer one of dimensions λxλxλ, with contrast χ=1.0-j0.3. The investigated domain is a volume with dimension 2λx2λx2λ (disctetized into 32x32x32 cells). The homogeneous embedding is chosen to be the free space. Some cross sections of the real and imaginary part of the reference contrast profile are shown in Fig.3. The investigated volume is uniformly illuminated by 12 plane waves whose wave-vectors lie in the z=0 plane and the scattered field is measured by 36 receivers located on 3 circumferences having radius $r_m=2\lambda$ and posed, as in Fig.1 at $z_1=-\lambda/2$, $z_2=0$, $z_3=\lambda/2$. The simulated data have been corrupted with a 5% additive Gaussian noise. Some cross sections of the reconstructed contrast profile through CS-EB-s method adopting a pixel based representation for p are depicted in Fig.4, while the result obtained by using the CS inversion method [3] is shown in Fig.5. By looking at these latter figures it appears that, due to the presence of losses in the target and according to the above theory, a more accurate reconstruction can be achieved with the CS-EB-s inversion method.

In particular, the normalized reconstruction error is equal to 18% for the CS-EB-s inversion method and 68% for the CS one.

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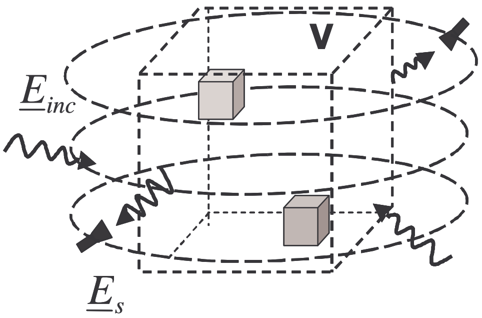


Fig.1: Reference Geometry

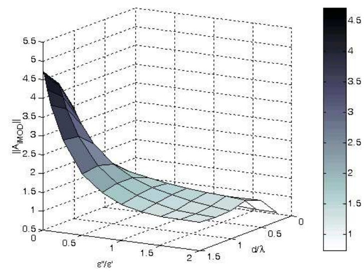
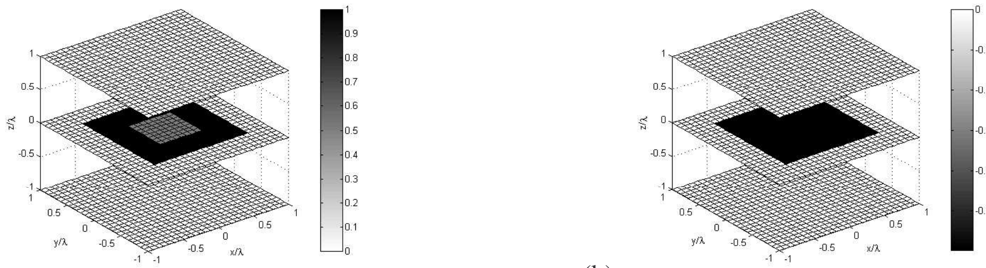
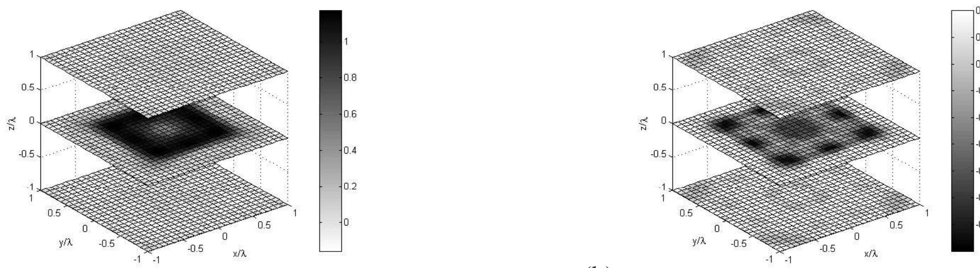


Fig.2: The "universal" plot of $\|\mathbf{A}_{iMOD}\|$



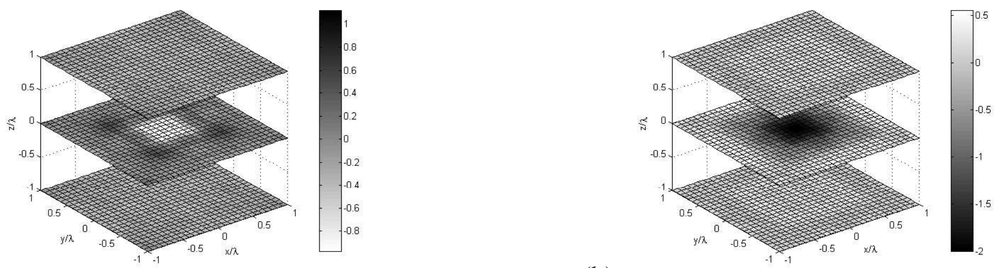
(a) (b)

Fig.3 : Reference contrast profile. (a) Real part; (b) Imaginary part.



(a) (b)

Fig.4: Estimated contrast profile with CS-EB. (a) Real part; (b) Imaginary part.



(a) (b)

Fig.5 : Estimated contrast profile with CS. (a) Real part; (b) Imaginary part.