Diffraction Coefficients of Composite Wedge Constructed by Virtual Ray of Diffraction

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1. ITRODUCTION

In spite of significant advances in the application of numerical techniques in electromagnetics, there have not been comparable achievements in physical understanding of the diffraction by simple canonical scatterers. For example, no rigorous solution to the diffraction by penetrable wedge and cone is available. In this paper, we suggest a new method, the virtual ray of diffraction(VRD), to construct the diffraction coefficients of a composite wedge composed of perfect conductor and lossless dielectric[1]. The method consists of actual ray-tracing in the physical region and virtual ray-tracing in the complementary region. At first, the ordinary ray-tracing provides the geometrical optics(GO) field. Applying the physical optics(PO) approximation to the corresponding dual integral equations[2], one may obtain the complete form of the PO field consisting of the GO term and the edge-diffracted field[3]. The PO diffraction coefficients are expressed by finite series of cotangent functions, of which amplitudes and poles are equal to the amplitudes and propagation directions of the ordinary rays, respectively. It should be noted that there is one-to-one correspondence between the geometrical rays and the PO diffraction coefficients. However the PO diffraction coefficients cannot satisfy the boundary conditions at the wedge interfaces.

To correct the error posed in the PO diffraction coefficients, we need some additional cotangent functions. It is well known that the exact diffraction coefficients of the perfectly conducting wedge consists of four cotangent functions with angular period $2\pi\nu_\infty$, where ν_∞ is the minimum value satisfying the edge condition at the tip of the conducting wedge[4]. In contrast, the corresponding PO diffraction coefficients consist of two cotangent functions with angular period 2π. Then one may guess that the remaining two cotangent functions among the exact diffraction coefficients may be generated from two virtual rays, which can be obtained only by extending the ordinary ray-tracing technique in the complementary region. In this paper, both trajectories of actual rays in the physical region and virtual rays in the complementary region are illustrated in case that a plane wave is incident on a composite wedge. According to the one-to-one correspondence, one may routinely construct the diffraction coefficients consisting of the cotangent functions[5]. Then the angular period of the diffraction coefficients is changed from 2π to $2\pi\nu_\varepsilon$, where ν_ε satisfies the edge condition at the tip of the composite wedge. The validity of the above method is assured by showing that the VRD diffraction coefficients approach zero in the complementary region.

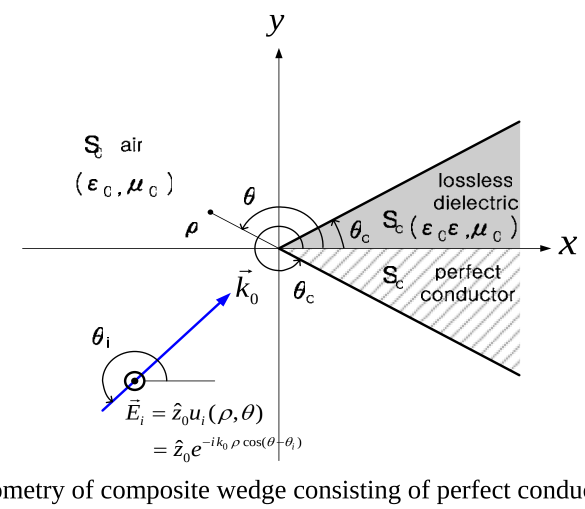
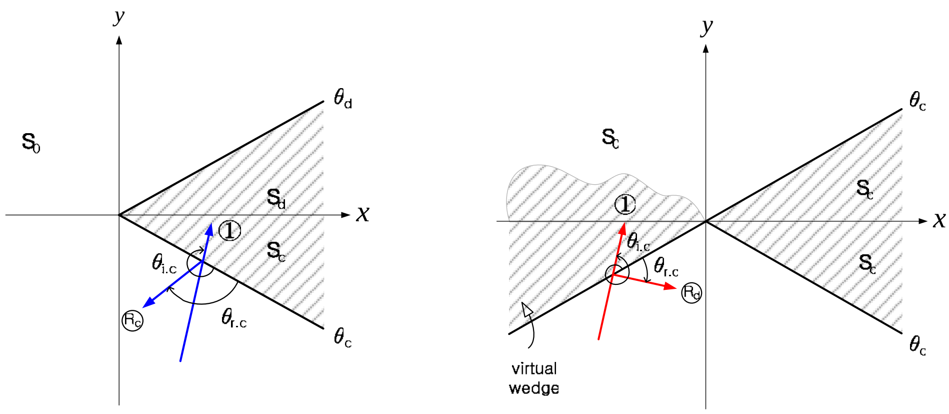


Fig. 1 Geometry of composite wedge consisting of perfect conductor and lossless dielectric illuminated by an E-polarized plane wave

2. VIRTUAL RAY OF DIFFRACTION

Fig. 1 shows a composite wedge composed of perfect conductor in S_c and lossless dielectric with relative dielectric constant ε in S_d. Consider that the composite wedge is illuminated by an E-polarized plane wave with an arbitrary incident angle θ_i in S_0. As the permittivity of its dielectric part increases to infinite, the composite wedge may become a perfectly conducting wedge. In this limiting case, the exact diffraction coefficients are expressed by sum of four cotangent functions with the angular period $2\pi\nu_\infty$, where ν_∞ can be derived from the edge condition at the tip of the perfectly conducting wedge. In contrast, the PO diffraction coefficients consist of only two cotangent functions with the angular period 2π, which correspond to two actual rays one-to-one in Fig.2(a). Adjusting the angular period from 2π to $2\pi\nu_\infty$ and multiplying the factor $1/\nu_\infty$ to the cotangent functions, the PO diffraction coefficients become sum of two cotangent functions among the exact diffraction coefficients.

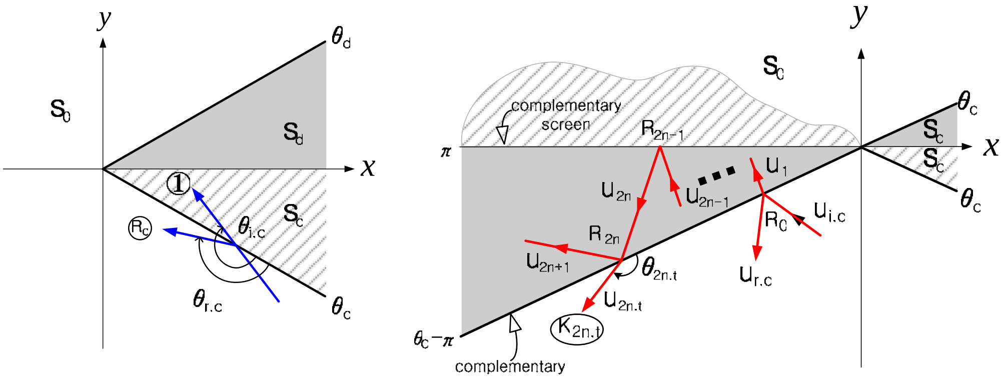
Then the arguing point is whether two additional rays exist in conjunction with two remaining cotangent functions of the exact diffraction coefficients or not. We assume that the incident field with the same amplitude but propagation angle of 2π angular shift generates the virtual reflection on the other boundary. Then one may easily trace two virtual rays in the complementary region, as shown in Fig. 2(b). Employing the one-to-one correspondence between geometrical rays and cotangent functions, two virtual rays in Fig. 2(b) provide the remaining two cotangent functions among the exact diffraction coefficients routinely.



(a) actual rays (b) virtual rays

Fig. 2 Extended ray-tracing for the incidence of E-polarized plane wave only on one boundary of a perfectly conducting wedge

The above extended ray-tracing procedure was applied to the diffraction by a composite wedge in Fig.1. As shown by blue lines in Fig. 3(a), the conventional ray-tracing provides the ordinary GO field as sum of actual rays in the physical region. No actual rays exist inside the real dielectric part. Following the same procedure in Fig. 2(b), virtually multiple reflections inside the imaginary dielectric part generate a number of virtual rays, as shown by red lines in Fig. 3(b). But it is not clear how to terminate the virtually internal reflections. We impose two additional conditions to virtual rays[4]. The first condition is so clear that all of the virtual rays should be located only in the complementary regions. The second condition is that the final reflection of virtual rays occurs on the conducting boundary. The last condition is required to satisfy the boundary condition at the conducting boundary. According to the one-to-one correspondence between rays and cotangent functions, the VRD diffraction coefficients are constructed directly by sum of cotangent functions, of which total number is equal to the total number of actual and virtual rays. The amplitude and pole of each cotangent function are taken by the amplitude and propagation angle of the corresponding ray, respectively. And the angular period of the cotangent functions is adjusted to $2\pi\nu_\varepsilon$, where ν_ε can be derived from the edge condition at the tip of the composite wedge with relative permittivity ε in its dielectric part.



(a) actual rays (b) virtual rays

Fig. 3 Extended ray-tracing for the incidence of E-polarized plane wave only on the conducting boundary of a composite wedge

3. DIFFRACTION COEFFICIENTS AND FIELD PATTERNS

To show the validity of the suggested method, the VRD diffraction coefficients are compared with the PO diffraction coefficients for $\theta_d = 45^\circ$, $\theta_c = 350^\circ$, and $\theta_i = 240^\circ$ as ε increases from 1.01 to 101. In Fig. 4, the diffraction coefficients for $\varepsilon = 1$ and infinite are exact in cases of the perfectly conducting wedges corresponding to $\theta_d = 0^\circ$ (dotted black line) and 45° (bold black line), respectively. The blue line in Fig. 4 illustrates the PO diffraction coefficients, which suffer from no change even if ε varies from 1.01 to 101. According to our formulation of dual integral equations[3], the exact diffraction coefficients have to become zero in the complementary regions. However, in Fig. 4, one may find that the PO diffraction coefficients suffer from large deviation from zero in the complementary regions of $0^\circ < w < 45^\circ$ and $350^\circ < w < 360^\circ$. In contrast, the VRD diffraction coefficients marked by red lines in Fig. 4 approach the corresponding exact diffraction coefficients monotonically as ε decreases to 1.01 or increase to 101. Compared with the PO diffraction coefficients, the VRD diffraction coefficients in Fig. 4 satisfy the null-field condition in the complementary regions quite well.

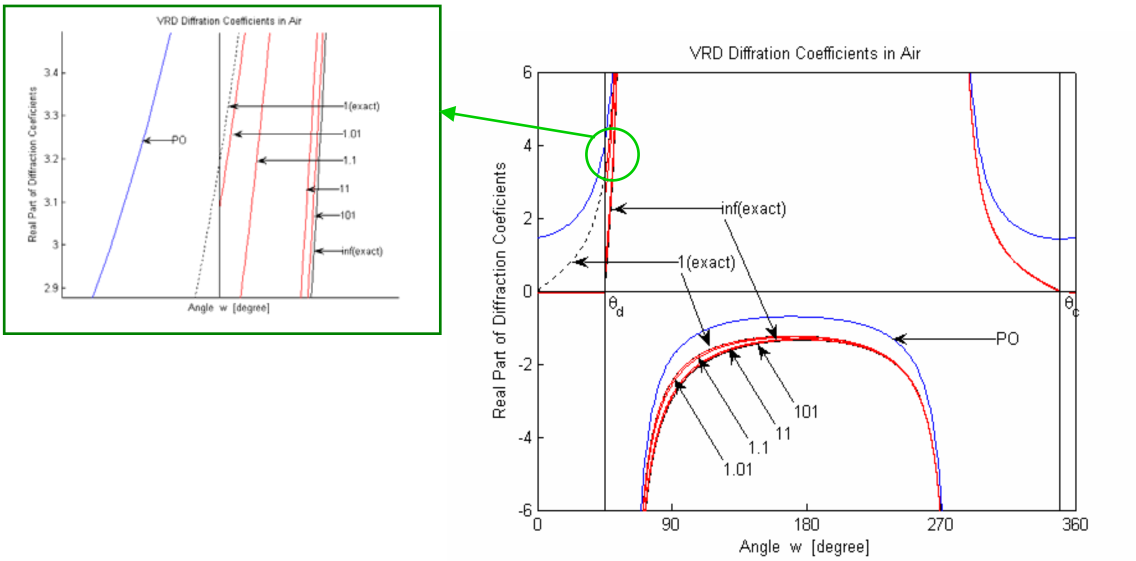
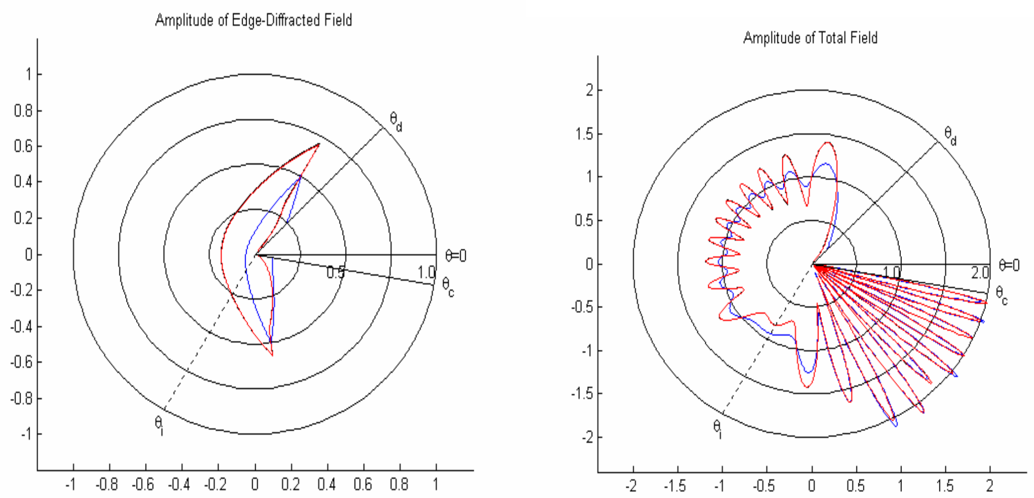


Fig. 4 Comparison between PO(blue line) and VRD(red lines) diffraction coefficients for $\theta_d = 45^\circ$, $\theta_c = 350^\circ$, $\theta_i = 240^\circ$, and $\varepsilon = 1.01$, 1.1, 11, and 101

Fig. 5(a) and (b) illustrate the amplitude patterns of edge-diffracted and total fields at 5 wavelength away from the wedge tip for $\varepsilon = 101$ in the same case of Fig. 4. Unlike the PO field pattern (blue line), the VRD field pattern (red line) satisfies the boundary condition, as shown in Fig. 5. Hence one may conclude that the VRD technique provides highly accurate diffraction coefficients of a composite wedge consisting of arbitrary dielectric and perfect conductor.



(a) Edge diffracted field (b) Total field

Fig. 5 Comparison between PO(blue line) and VRD(red line) field patterns calculated at $\rho = 5\lambda$ for $\varepsilon = 101$ in case of Fig. 4

4. CONCLUSION

A new method, the virtual ray of diffraction(VRD), is implemented by including not only the conventional tracing of actual rays in the physical region but also the additional tracing of virtual rays in the complementary region. The VRD method provides an analytical expression on the E-polarized diffraction coefficients of composite wedge composed of perfect conductor and lossless dielectric. In comparison with the conventional PO solution, the VRD diffraction coefficients approach to the exact solutions in two limiting cases more closely. The accuracy of the VRD method is assured by showing that in view of the formulation of dual integral equations[3], the VRD diffraction coefficients of a composite wedge become nearly zero in complementary regions.

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