NEW, GENERAL THEORY OF MONOCHROMATIC AND SHORT IMPULSE PROPAGATION IN INHOMOGENEOUS MEDIA AND IN WAVE-GUIDE STRUCTURES

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ABSTRACT

In this paper a new solving method is presented, using the Method of Inhomogeneous Basic Modes (MIBM), that avoids all the former monochromatic ways of thinking, in order to obtain the complete solution of Maxwell's equations for real impulses. The paper presents new and general solutions for free-space propagation, in the case of ducted electromagnetic waves in wave-guides filled by vacuum or anisotropic plasma, and on inhomogeneous transmission lines. An application of the guided model is the impulse propagation in Earth's surface-ionosphere wave-guide (spheric). Further, the solutions for inhomogeneous transmission lines are important in fast-circuit development.

INTRODUCTION

"Any serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical concept with which the theory operates."[1] We must take into account this distinction in the case of the electromagnetic (or other) wave propagation in inhomogeneous media even if we use the phenomenological description of the problem. The "objective reality" in the case of wave propagation in (linear) inhomogeneous media is the fact that the original (e.g. "forward" propagating or "source") signal will attenuate during the propagation and simultaneously generates another, reflected (or scattered) signal propagating in other direction in every spatial point, in which the inhomogeneity exists. The "physical concept" is the description of this phenomenon by solving Maxwell's equations. The whole process of derivation of a real (full wave) solution of Maxwell's equations is a self-consistent field (SCF) process (see Fig. 1.in [2]). During this process, we must make two crucial decisions. The first is the supposition of the starting form of the solution, e.g. time harmonic [3-7] or an arbitrary shaped signal [8]. The second is the description of the medium – signal interaction in the investigated case of inhomogeneity and for the supposed form of the solution [5, 8-11].

The traditional theoretical descriptions of a monochromatic electromagnetic signal propagating in an inhomogeneous medium – e.g. eikonal-equation, W.K.B. method, generalised propagation-vector, etc. – have a common fundamental inaccurate assumption relating the physical concept of the structure of the signal [5]. The model of the solution in these approaches is an additional sum of the different signal-parts, propagating forward or backward (scattered), which parts are supposed to be solutions of Maxwell's equations independently of each other in linear models. But this way of description does not make possible to recognise the influence of the real structure of the propagating signals, in which only the total energy - the forward propagating and the reflected (scattered) signal parts together - is the solution of Maxwell's equations (or telegraph equations) in linear cases too. As for the real, full-wave solution of Maxwell's equations always has to contain all the existing modes simultaneously. It is a well-known fact that the additional sum of solutions has to fulfil the original equations, if a linear differential equation-system has some solutions. However, it does not mean that the parts of an existing solution fulfilling the equations – separated from each other by application of different theoretical points of views – would automatically be solutions of the full equation-system. The resultant and existing signal is a solution of Maxwell's equations, but its parts (the backward and forward propagating signal-parts) are not. The main question is how to handle the continuous generation of the reflected (scattered) signal and the energetic coupling between the existing signal-parts during the propagation.

The Method of Inhomogeneous Basic Modes (MIBM) is well applicable for such problems. The philosophy of this method [7] considers the form of the final solution to be determined as a sum of the so-called basic modes, which are not the full solutions of Maxwell's equations independently.

$$\overline{G}(\overline{r},t) = \sum_i G_i(\overline{r},t), \tag{1}$$

where $\overline{G} = \overline{E}, \overline{D}, \overline{H}, \overline{B}$ and i is the number of the existing modes. Substituting these basic modes into Maxwell's equations, those will disintegrate into two groups. One of them is valid even in a simple homogeneous medium; the other (the group of so-called coupling equations) characterizes the influence of the

inhomogeneous medium. Boundary conditions remain as unknown variables in the full form of Maxwell's equations. One can determine the final form of the solution by solving the coupling equations and describing these initial values.

A further theoretical problem, unanswered up to now, is originated from the fact, that a real physical signal excited by an impulse – or switching on-off transient – is always non-monochromatic and its description is not enough accurate by superposition of monochromatic signals, which was the common practice up to now [3, 5, 9, 11]. With other words, the problem of arbitrarily shaped signals does not make it possible to assume any $\exp(\mathrm{j}\omega t)$ -type starting form of the solution at the beginning of the derivation of Maxwell's equations [8].

One of the most commonly known theoretical approximations of wave-propagation in inhomogeneous media is the solution of the Stokes-equation by Airy integral functions [5]. It is useful to investigate the differences between the new method (based on MIBM) and the Airy-solution [12]. As it can be found e.g. in [5, Chapter 9 and 15], the Airy integral (Airy integral functions) is the mathematically correct solution of a type of differential equations (like Stokes-equation is):

$$\frac{d^2 E_y}{dz^2} + k_0^2 q^2 E_y = 0 \tag{2}$$

where

$$q^2 = n^2 \text{ (longitudinal propagation)}$$

n is the refraction index, as usual.

The cornerstone of the new method (based on MIBM) is the realization of the physical fact, that only and exclusively the resultant sum of the forward propagating and reflected (scattered) signals can be an existing, real solution of Maxwell's equations.

As it is well seen e.g. in Budden's argumentation, the supposed starting form of the solution to be determined contains the resultant sum of forward and backward propagating signal-parts:

$$E_y = A \cdot \mathrm{e}^{-\mathrm{j}k_z z} + B \cdot \mathrm{e}^{\mathrm{j}k_z z} \tag{3}$$

where

$$k_z = k_0 \cdot n = \frac{\omega}{c} \cdot n$$

Further, the detailed investigation of the derivation enlightens some important problems. Budden applies Maxwell's equations, and deduces the Stokes-equation from them. As he states, this should refer to the signal-form (3), and a result of this deduction is the known Stokes-equation. The Airy integral functions are valid for this differential equation type. But it is important to recognize, that Budden supposes by the introduction of the signal form of (3), that the substitution of the forward and backward propagating signal parts separately into Maxwell'equations results formally identical equations, and he deduces the Stokes-equation for only the forward or backward propagating part, independently. This assumption is not a special case of the new theoretical model presented in this paper, but means a fundamental contradiction between the former approximation and the new model.

In order to compare this solving method to the new one, let us control Budden's derivation. If the whole solution shown in (3) is written back into the Stokes-equation and one derives the equations on a correct way, it will be obvious, that the result presented by e.g. Budden cannot be yielded. Assuming (in concordance with Budden's work), that A and B are constant (it must be emphasized, that this precondition is a hard restriction of the validity limits in the case of spatial inhomogeneities) and substituting (3) into the Stokes-equation, the following will be obtained:

$$A = -B \cdot \mathrm{e}^{\mathrm{j}2k_z z} \tag{4}$$

This is in obvious and fundamental contradiction with the starting precondition (A and B have to be constant). It well can be seen, that Budden's formulas are valid only for the forward and the backward propagating signal-parts separately, so this way of thinking implicitly considers this signal-parts as independent solutions of Maxwell'equations. (eq.4 cannot be considered as some „reflection coefficient", because of the starting mathematical suppositions.)

If A and B are not constant, the result does also not lead back to the known formulas, from which the Airy functions are deducible, but gives a more complicated relation between A and B:

$$\left[-2\mathrm{j}\frac{\mathrm{d}A}{\mathrm{d}z}(\mp k_0 q) - \mathrm{j}A\left(\mp k_0 \frac{\mathrm{d}n}{\mathrm{d}z}\right) + \frac{\mathrm{d}^2 A}{\mathrm{d}z^2}\right] \cdot \mathrm{e}^{-\mathrm{j}k_z z} + \left[2\mathrm{j}\frac{\mathrm{d}B}{\mathrm{d}z}(\mp k_0 q) + \mathrm{j}B\left(\mp k_0 \frac{\mathrm{d}n}{\mathrm{d}z}\right) + \frac{\mathrm{d}^2 B}{\mathrm{d}z^2}\right] \cdot \mathrm{e}^{+\mathrm{j}k_z z} = 0 \tag{5}$$

This relation, on the one hand, does not coincide with results published by Budden (and others), and, on the other hand, it results an underdetermined description of the problem (one equation containing two unknown variables), which is unsolvable.

Because of these fundamental theoretical differences, the new method never results the Stokes-equation, but uses a new and theoretically different way in order to solve the problem of spatially inhomogeneous media. This new method delivers mathematically correct answers for the problems presented above. Furthermore, this fact is independent from the nature of the signal (monochromatic or impulse); this argumentation is equally valid for both cases. The direction of propagation (referring to the gradient of inhomogeneity; longitudinal, transversal or oblique) has also no influence on this theoretical difference. Summarizing, the new method never leads back to Stokes-equation during the solving process, because of this it becomes possible to avoid the theoretical contradictions presented above and to obtain an exact solution.

SHORT OVERVIEW OF THE RESULTS

Finally, some results of the new solving method will be presented, without the detailed derivations (that would extend far beyond the frame of this presentation). The reflected and the forward propagating signal forms for inhomogeneous medium and monochromatic excitation are (E_{10} is the source amplitude, i.e. constant) [2]:

$$E_2 = \frac{E_{10}}{2}\sqrt{Z_0(x)}\int_x^{x_M}\frac{\mathrm{d}(\ln Z_0)}{\mathrm{d}u}\mathrm{e}^{-\mathrm{j}2\int_0^u k(v)\mathrm{d}v}\mathrm{d}u\cdot\mathrm{e}^{\mathrm{j}\left(\omega t+\int_0^x k(v)\mathrm{d}v\right)} \tag{6}$$

$$E_1 = E_{10}\sqrt{Z_0(x)}\left\{1-\frac{1}{4}\int_0^x\frac{\mathrm{d}(\ln Z_0)}{\mathrm{d}u}\mathrm{e}^{\mathrm{j}2\int_0^u k\mathrm{d}v}\cdot\left[\int_u^{x_M}\frac{\mathrm{d}(\ln Z_0)}{\mathrm{d}w}\mathrm{e}^{-\mathrm{j}2\int_0^u k\mathrm{d}v}\mathrm{d}w\right]\mathrm{d}u\right\}\cdot\mathrm{e}^{\mathrm{j}\left(\omega t-\int_0^x k(v)\mathrm{d}v\right)} \tag{7}$$

The reflected signal in an inhomogeneous medium for real impulse excitation is [12]:

$$E_{z2}(x,t) = -\frac{\mathrm{j}}{8\pi}\int_{-\infty}^{\infty}\left[\frac{C_0(\omega)}{\sqrt{k(x,\omega)}}\int_\xi^x\frac{1}{2k(u,\omega)}\frac{\partial k(u,\omega)}{\partial u}\mathrm{e}^{-2\mathrm{j}\int_0^u k(v,\omega)\mathrm{d}v}\mathrm{d}u\right]\mathrm{e}^{\mathrm{j}\left(\omega t+\int_0^x k(h,\omega)\mathrm{d}h\right)}\mathrm{d}\omega \tag{8}$$

where

$$C_0(\omega) = I_{x=0}(\omega)\frac{k_0(\omega)\sqrt{k(x=0,\omega)}}{k_0(\omega)+k(x=0,\omega)} \tag{9}$$

the transient excitation is

$$I_{x=0}(\omega) = \int_{-\infty}^{\infty}\left\{\int_{-x_0}^{0}J_0\left(l,t+\frac{l}{c}\right)\mathrm{d}l\right\}\mathrm{e}^{-\mathrm{j}\omega t}\mathrm{d}t \tag{10}$$

A measured spheric (propagating in the Earth's surface – ionosphere wave-guide) and a calculated result for homogeneous, vacuum-filled wave-guide and real impulse excitation is shown in Fig. 1.

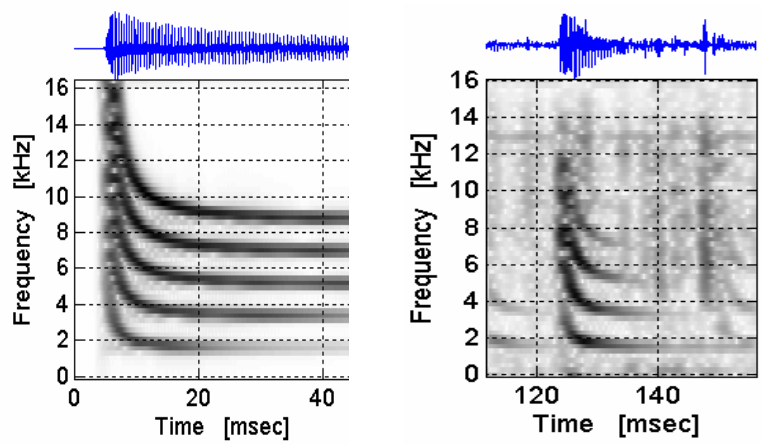

Figure 1. Spectrogram of a calculated and a measured spheric.

The exact solution for inhomogeneous transmission lines, for monochromatic excitation:

$$\gamma = \sqrt{Z(x)\cdot Y(x)},\ \ Z_0(x) \stackrel{\Delta}{=} \sqrt{\frac{Z(x)}{Y(x)}},\quad \text{and}\ \ C_0 = \text{constant}. \tag{11}$$

$$U_2(x,t) = -C_0\sqrt{Z_0(x)}\int_{x_M}^{x}\frac{1}{2}\frac{\mathrm{d}[\ln Z_0(u)]}{\mathrm{d}u}\cdot \mathrm{e}^{-2\int_0^u \gamma(v)\mathrm{d}v}\,\mathrm{d}u\cdot \mathrm{e}^{\mathrm{j}\left(\omega t+\int\gamma(u)\mathrm{d}u\right)} \tag{12}$$

and

$$U_1(x,t) = C_0\sqrt{Z_0}\left\{1+\frac{1}{4}\int_0^x \frac{\mathrm{d}(\ln Z_0)}{\mathrm{d}u}\mathrm{e}^{2\int_0^u \gamma\mathrm{d}v}\cdot\left[\int_{x_M}^{u}\frac{\mathrm{d}(\ln Z_0)}{\mathrm{d}v}\mathrm{e}^{-2\int_0^v \gamma\mathrm{d}w}\mathrm{d}v\right]\mathrm{d}u\right\}\cdot \mathrm{e}^{\mathrm{j}\left(\omega t-\int\gamma(u)\mathrm{d}u\right)}. \tag{13}$$

From the results it became obvious that the new theoretical model delivers exact solutions, importance of which is great, because they seriously differ from the former approaches and models. The application of these results for different wave-propagation problems seems to be useful, in order to obtain more exact description of the signals, in the case of fast circuits, wave-guides and free-space propagation.

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