A DISCRETE-TOPOLOGY METHOD FOR BOUNDED AND FRACTAL DOMAINS' ELECTROMAGNETIC PROBLEMS

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ABSTRACT

A computational method called Topological Calculus, based on discrete mathematical tools known as Algebraic Topology, is briefly introduced and its ability to represent a discrete counterpart to classical and quantum field equations is shown.

First of all the Topological Calculus setting is that of a certain discrete class of n-dimensional lattices, called simplicial complexes, whose constituting simplices are the n-dimensional analogue of triangles (0-simplices are points, 1-simplices segments, 2-simplices triangles, 3-simplices tetrahedra, etc.). Topological Calculus provides two nontrivial algebraic operators (cup '$\cup$' and cap '$\cap$' products) and two linear differential operators (boundary '∂' and coboundary 'δ'), from the combinations of whom almost any algebraic/differential equation can be imposed on the simplicial complex. They are, in fact, the discrete counterpart of usual scalar '$\cdot$', cross '$\times$' and exterior '$\wedge$' products, as well as divergence, gradient and curl I-order differential operators.

Once a suitable and accurate enough triangulation Σ of the continuum domain is performed, scalar, vector and tensor fields (or differential p-forms, according to the exterior formulation of Field Theories) are represented as p-chains: algebraic objects assuming different (e.g. time-dependent) values on each of Σ's p-simplices.

The diagonalization of Σ's incidence and adjacency matrices allows to both compute the number of 'holes' inside the domain. As the simplicial Laplace (and Helmholtz)'s operators are correlated to the adjacency matrices, this spectral decomposition also counts the number of linearly independent "harmonic functions". It is well known that the *qualitative* behaviour of electromagnetic fields on bounded domains strictly depends on the *topology* of the region (i.e., roughly, on the number of 'holes' and connected components), for example the number of a porous resonator's static modes, or a multiply-connected cross-sectioned waveguide's TEM modes. Such information can be easily extracted with this method, and is refinement-independent from the domain's triangulation: it can be computed even from a "coarse" simplicial complex, as long as it is topologically equivalent to the continuum domain.

Some examples are made with the spectral analysis of IFS prefractals of the Šerpinskij gasket and carpet. In the case of self-similar simplicial complexes, adjaceny and laplacian matrices are recursively built using a quasi-diagonal-block paradigm, physically interpreted as a renormalization of the multiscale electrodynamics within. Self-similar eigenmodes and eigenvalue distribution is then observed, and compared with the well-known multi-band properties of fractal antennas and resonators.