Quasi-classic approximation in Markov's parabolic equation for spaced position and frequency coherency in the inhomogeneous background medium

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Abstract

The technique of quasi-classic approximation in Markov's parabolic equation for spaced position and frequency coherency has been recently developed in (Bitjukov, A.A., V.E.Gherm, and N.N.Zernov, On the solution of Markov's parabolic equation for the second order spaced frequency and position coherence function, *Radio Science, 37*(4), 10.1029/2001RS002491, 2002; and Bitjukov, A.A., V.E.Gherm, and N.N.Zernov, Quasi-classic approximation in Markov's parabolic equation for spaced position and frequency coherency, *Radio Science, 38*(2), 10.1029/2002RS002714, 2003). This technique employs the formalism of the quasi-classic complex paths in the extended complex-valued co-ordinate space and allows constructing of the second order coherencies for arbitrary realistic models of the structure function of the fluctuations of the refractive index of the medium of propagation. The cases of incident plane and spherical waves were considered and it was shown that the known rigorous solutions were automatically produced as particular cases.

In the present paper the technique of quasi-classic approximation in Markov's parabolic equation for spaced position and frequency coherency is generalized to the case of the non-homogeneous medium of propagation, where the ray paths are curve shaped, and more than one ray could arrive at the given point of observation. The technique takes account of the possible frequency dependency of the dielectric permittivity of the medium (e.g., ionosphere), when the ray trajectories depend on the frequency.

The parabolic equation for the coherence function is formulated in the curvilinear local ray-centered co-ordinate system along the reference ray in the smoothly inhomogeneous background medium. This reference ray is just the geometrical-optics ray corresponding to the central frequency and arriving at the observation point. The Lamé coefficients specifying the local co-ordinate system and curvature radii of the reference ray are defined at the ray points. Accordingly to the quasi-classic method, the coherence function is represented as the asymptotic series by the inverse powers of the large parameter kl_ε, where k is the vacuum wave number and l_ε is the characteristic scale of the fluctuations. It is also assumed that the condition $R_c >> l_\varepsilon$ takes place, where R_c is the curvature radius of the ray. Standard asymptotic procedure results in the eikonal equation for the phase function and the series of the transport equations for amplitudes of the different orders. Eikonal equation is then solved by the method of characteristics, and the appropriate Hamilton equations define the complex-valued trajectories which can be parameterized by a real parameter. The trajectories start at the complex-valued initial point and arrive at the real-valued point of observation. In the case of the frequency dependent permittivity the trajectories corresponding to the different frequencies are different even if they arrive at the same point. Once the trajectories have been found, the phase function and the main transport equation are determined along the trajectories giving the asymptotic solution to the space-frequency coherence function at the given point.

The generalized technique is capable of determining the time-space-frequency statistical moments (coherence functions) of the field propagating in the realistic conditions – inhomogeneous background medium with fluctuations having a realistic structure function, including the case of strong fluctuations of the field.

Suggested Sessions: B02, or B05 (I,C,P)