ELLIPTICAL TO LINEAR POLARIZATION TRANSFORMATION BY A GRATING WITH A CHIRAL COMPOSITE

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ABSTRACT

The phenomenon of wave polarization conversion is found, when an incident electromagnetic wave of elliptical polarization is quasi totally transformed into a linearly polarized wave specularly reflected from the structure as a succession of layers in the order: strip grating, dielectric layer, chiral layer, and screen. Due to a chiral medium presence, the corresponding diffraction problem essentially is a vector boundary value problem. It has been solved using an analytical regularization procedure based on the Riemann-Hilbert problem method. The frequency range of polarization conversion is rather broadband, and the phenomenon occurrence depends weakly upon the elliptical polarization parameters of the incident wave and upon the incident wave angle.

INTRODUCTION

Chiral composites possessing spatial dispersion have unusual electromagnetic characteristics, including optical activity and circular dichroism [1]. In view of the circular polarization of the chiral medium eigenwaves, the chiral medium couples both linear polarizations and this gives rise to new and interesting polarization effects. However the presence of the chiral medium mathematically complicates the problem, making it a fully vectorial problem.

Even in well-known structures a chiral inclusion may impart novel properties. For example, in the case of a linearly polarized wave, normally incident on a strip grating attached to an isotropic chiral half-space, it was shown [2] that the reflected field contains cross-polarized harmonics, that the diffraction character depends on the direction of elliptic polarization of the incident wave, and that chiral medium losses are responsible for dichroism-induced peculiarities.

The structures considered in this paper have potential application in various microwave devices. The full transformation of an obliquely incident linearly polarized wave to a specularly reflected cross-polarized wave may be effected with this type of structure, and regimes of essentially autocollimated reflection and quasi-total nonspecular reflection with a high telescopicity factor have been found [3].

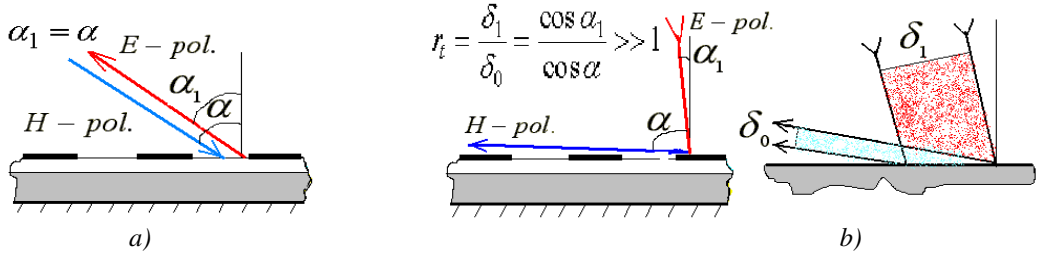

Fig.1. Nonspecular reflection regimes accompanied by polarization conversion

In the autocollimation regime (see Fig.1 (a)), the minus first harmonic of cross polarization propagates in the opposite direction to the primary incident wave of linear polarization. Thus in this regime the structure acts as a frequency-selecting mirror with polarization conversion. In the nonspecular reflection regime with high telescopicity factor (see Fig.1 (b)) there is almost complete conversion of the nearly normally incident wave into the first cross-polarized harmonic of the spatial spectrum, which skims above the grating surface. Both concentration of the wave energy and polarization conversion are realized in this regime. Thus, such a structure can be employed as an antenna component.

The determination of the necessary structure parameters, which lead to a particular desired regime of reflection and polarization conversion, faces serious calculational difficulties. Whilst structure optimization may be carried out by varying the controlling parameters over a wide band, the phenomena occur within narrow limits and often have a resonance nature. Therefore an effective and reliable numerical procedure is required.

PROBLEM FORMULATION AND FIELD REPRESENTATION

A periodic grating of infinitely thin and perfectly conducting strips lies parallel to the OX axis in the plane $z = h_1$, with grating period l and slot width d (Fig.2 (a)). The layers indexed by $j = 1$ ($h_1 < z$) and $j = 2$ ($0 < z < h_1$) are magnetodielectric with permittivity ε_j and permeability μ_j; the chiral layer indexed by $j = 3$ ($-h_2 < z < 0$) has chirality parameter γ, permittivity ε_3 and permeability μ_3. A perfectly conducting screen lies in the plane $z = -h_2$.

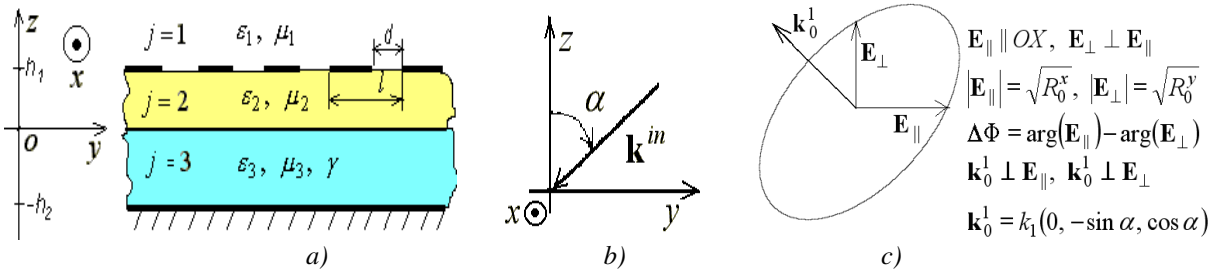

Fig.2. The structure profile and field configuration

The elliptically polarized wave $\mathbf{E}^{in} = \mathbf{E}_0 \exp(i(\mathbf{k}^{in}\mathbf{r} - \omega t))$, $\mathbf{H}^{in} = \mathbf{H}_0 \exp(i(\mathbf{k}^{in}\mathbf{r} - \omega t))$ is obliquely incident on the grating:

$$\mathbf{E}_0 = (\tilde{e},\ \rho_1 \tilde{h} \cos\alpha,\ -\rho_1 \tilde{h} \sin\alpha), \qquad \mathbf{H}_0 = (\tilde{h},\ -\tilde{e} \cos\alpha/\rho_1,\ \tilde{e} \sin\alpha/\rho_1), \tag{1}$$

where $\tilde{e}, \tilde{h}$ are complex amplitudes; $\mathbf{k}^{in} = -\omega\sqrt{\varepsilon_0\mu_0\varepsilon_1\mu_1}(0, \sin\alpha, \cos\alpha)$, α is the angle between the incident wave vector $\mathbf{k}^{in}$ and the OZ axis (Fig.2,b), $\rho_j = \sqrt{\mu_0\mu_j/\varepsilon_0\varepsilon_j}$. The diffracted field is to be found.

The sought solution has to meet Maxwell's equations, the radiation condition, boundary conditions, a quasi-periodicity condition, and a finiteness condition on field energy within any finite spatial volume.

An isotropic homogeneous chiral medium has the following constitutive relations [1]:

$$\mathbf{D} = \varepsilon_0\varepsilon_3\mathbf{E} + i\gamma\sqrt{\varepsilon_0\mu_0}\mathbf{H}, \qquad \mathbf{B} = \mu_0\mu_3\mathbf{H} - i\gamma\sqrt{\varepsilon_o\mu_o}\mathbf{E}. \tag{2}$$

The incident field and the grating are uniform in the x-direction so the problem can be solved in two-dimensional terms ($\partial/\partial x \equiv 0$). In the2D formulation, Maxwell's equations (2) yield the following field relationships in the chiral medium:

$$\mathbf{E} = \mathbf{E}^+ + \mathbf{E}^-,\ \ \mathbf{H} = \mathbf{H}^+ + \mathbf{H}^- = -i(\mathbf{E}^+ - \mathbf{E}^-)/\rho_3, \tag{3}$$

$$\Delta_{yz}u^\pm + k^{\pm 2}u^\pm = 0, \quad E_x^\pm = u^\pm(y,z), \quad E_y^\pm = \mp\frac{1}{k^\pm}\frac{\partial u^\pm}{\partial z}, \quad E_z^\pm = \pm\frac{1}{k^\pm}\frac{\partial u^\pm}{\partial y},$$

where $k^\pm = -k_3(1 \pm \eta)$, $k_j = \omega\sqrt{\varepsilon_0\varepsilon_j\mu_0\mu_j}$, and $\eta = \gamma/\sqrt{\varepsilon_3\mu_3}$. As seen, the eigenwaves of a homogeneous chiral medium are circularly polarized plane waves with propagation constants $k^\pm$. The discussed problem requires a vector approach because circularly polarized waves retain all of the components in (3) fully coupled.

The periodicity of the grating along the OY axis allows us to expand the solution in a Fourier series for each j-layer. Substitution in the Helmholtz equation gives a field representation which coincides with the Rayleigh expansion of the diffracted field as an infinite series of the partial harmonics of the spatial spectrum. Denote the propagation constant of the n-harmonic by ξ_n in the y-direction and by $\zeta_n^j\big|_{j\neq 3}$, $\zeta_n^\pm\big|_{j=3}$ in the z-direction:

$$\xi_n = \frac{2\pi}{l}n - k_1\sin\alpha = \frac{2\pi}{l}(n - \chi\sqrt{\varepsilon_1\mu_1}\sin\alpha), \quad \zeta_n^j\big|_{j\neq 3} = \sqrt{k_j^2 - (\xi_n)^2}, \quad \zeta_n^\pm\big|_{j=3} = \sqrt{(k^\pm)^2 - (\xi_n)^2}, \tag{4}$$

where $\mathrm{Im}(\zeta_n^{j,\pm}) \geq 0$, $\chi = l/\lambda_0$. The incident wave on the grating is changed by the structure into a superposition of spatial spectrum waves. The reflected field may be represented as a sum of a finite number of propagating harmonics (with $\zeta_n^j, \zeta_n^\pm \in R$) of linear E- polarization (when $\mathbf{E} \parallel OX$) and H- polarization ($\mathbf{H} \parallel OX$), and an infinite number of surface harmonics decaying in the OZ direction (with $\zeta_n^j, \zeta_n^\pm \in C$). The propagation direction and decay rate of these waves are given by (4), while the wave amplitudes and phases are determined by the unknown Fourier field coefficients.

DETERMINATION OF THE FIELD COEFFICIENTS

Enforcement of the boundary conditions on each surface relates the sought Fourier coefficients in the various domains and two coupled systems of dual series equations for the unknown Fourier coefficients (representing complex amplitudes of the spatial harmonics in the chiral medium) are obtained. These functional systems involving trigonometric functions are equivalent to an operator equation of the first kind in the Hilbert space given by the Meixner condition [2]. Such first kind equations are ill-posed and cannot be reliably and effectively solved. To overcome this difficulty, an original methodology has been developed [2] to extract the singular part of the operator corresponding to the coupled systems of dual series equations, and this makes it possible to use the Riemann-Hilbert problem method [4]. This analytical regularization approach reduces the operator equation of the first kind to the following infinite system of linear algebraic equations of the second kind

$$\{\mathbf{I}+\mathbf{H}\}x=b, \tag{5}$$

where $\mathbf{H}$ is a compact operator, $\mathbf{I}$ is the identity operator. This system has a reliable and effective numerical solution of any pre-assigned accuracy, even in the resonant domain, which abounds with energetically important effects and where, as a rule, direct solution techniques fail.

NUMERICAL RESULTS

Due to the presence of the grating, different harmonics from all layers can interact each other, and energy redistribution between harmonics takes place. With certain structure parameters significant energy redistribution may be achieved, in which the field of one polarization dominates over fields of other polarizations.

Denote the structure efficiency coefficients in the n-order of spectrum by R_n^x, R_n^y; these determine the fraction of energy scattered from the structure to the upper half-space by the propagating n-harmonic with the wave vector $\mathbf{k}_n^1=(0,\ \xi_n,\zeta_n^1)$. The upper indexes respectively relate to the field of E- and H-polarizations. The value $\Delta R_0=R_0^x-R_0^y$ describes the efficiency of elliptical to linear polarization transformation. If the energy density of the incident elliptically polarized wave $W_{in}^{el}=\tilde{e}^2+(\rho_1\tilde{h})^2$ is 100%, then when $\Delta R_0=100\,\%$, for example, total transformation of the incident elliptically polarized wave into the specularly reflected zero harmonic of linear E-polarization with energy density $W_0^E=100\,\%$ occurs.

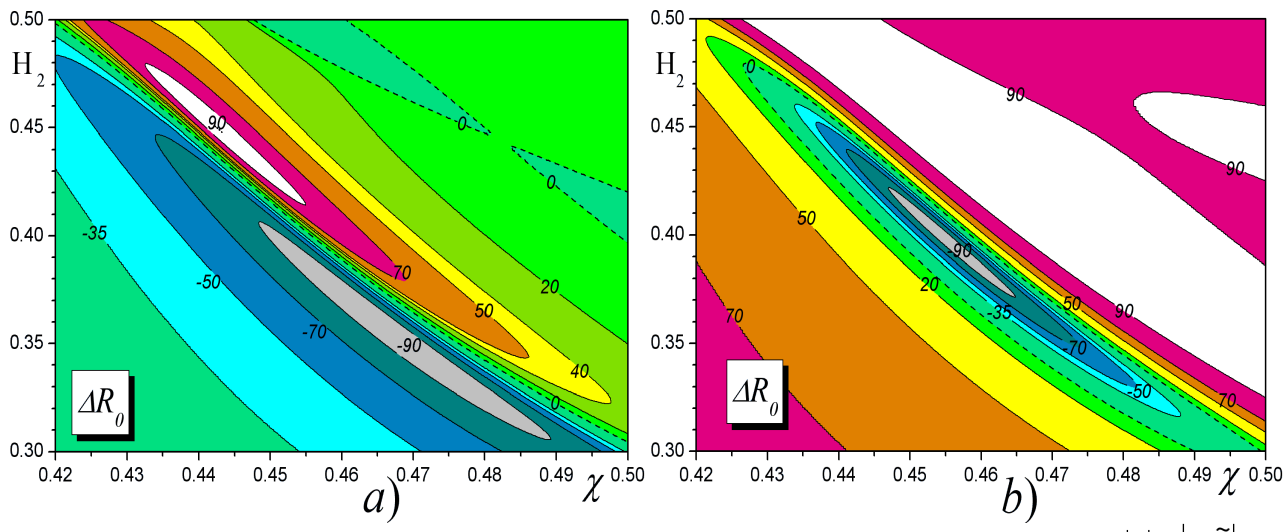

Fig.3. Efficiency ΔR_0 versus χ and H_2 for different polarization parameters of incident wave: *a)*-$|\tilde{e}|=|\rho_1\tilde{h}|$; *b)*-$|\tilde{e}|=4|\rho_1\tilde{h}|$ ($\varepsilon_1=1.0;\ \mu_j=1.0;\ \ \varepsilon_2=\varepsilon_3=4.0;\ \ \eta=0.3;\ \ d/l=0.5; h_1/l=0.015;\ \ \alpha=15^o; \delta\phi=-\pi/2$)

The value ΔR_0 as a function of the frequency parameter χ and the relative thickness of the chiral layer $H_2=h_2/l$ is represented in Fig. 3. The polarization transformation domains (with $|\Delta R_0|>90\%$) are exhibited for different relations of E- and H- field component in the incident wave. The domains of transformation into E- and H- polarized specularly reflected waves are closely situated to each other, and their bandwidth depends upon the content of E- and H-field com-

ponent in the incident wave. When the value $|\Delta R_0| \neq 100$, i.e., the scattered field has both E- and H- polarization, a wave with some elliptical polarization is reflected from the structure. Elliptical polarization is described by the Stokes' parameters, which may be expressed in terms of ΔR_0 and $\Delta\Phi_0$, where $\Delta\Phi_0$ is the phase difference between the zero harmonics of E- and H-polarizations, see Fig. 2(c). The elliptical to linear polarization transformation for different incident wave angle is illustrated in Fig. 4. The quasi-total $(|\Delta R_0| > 97\%)$ polarization transformation is seen to take place in a wide band $(\Delta\alpha \approx 20^o \div 25^o)$ of incident wave angle.

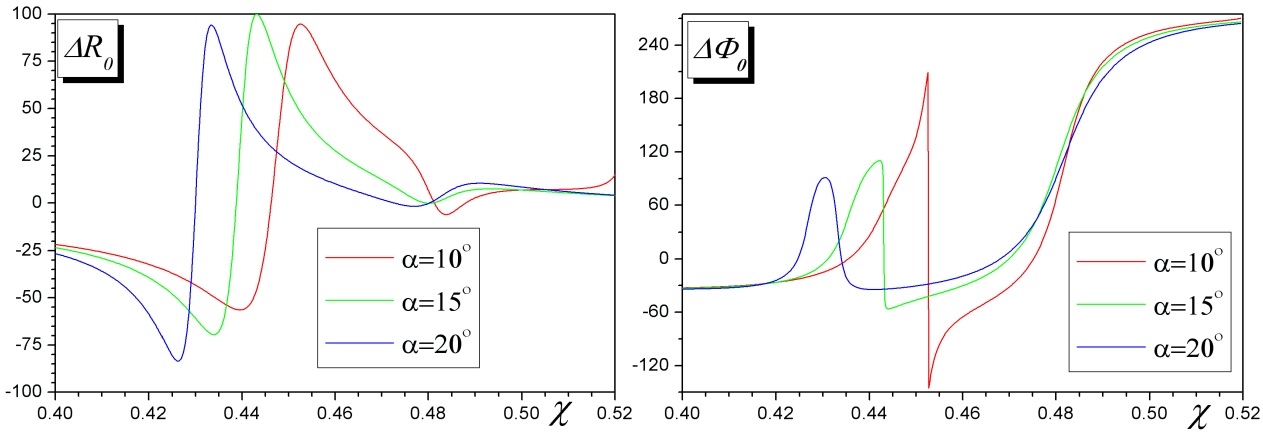

Fig.4. Efficiency ΔR_0 and phase difference $\Delta\Phi_0$ versus χ for different incident wave angle
$(\varepsilon_1 = 1.0;\ \mu_j = 1.0;\ \ \varepsilon_2 = \varepsilon_3 = 4.0;\ \ \ \eta = 0.3;\ h_2/l = 0.448;\ \ d/l = 0.5; h_1/l = 0.015;\ \ |\tilde{e}| = |\rho_1 \tilde{h}|; \delta\phi = -\pi/2)$

The phenomenon of polarization transformation is caused by the presence of the chiral medium. The discussed structure can be considered as an open resonator where the grating and the screen act as the mirrors and the chiral and dielectric layers are resonator filling. On the one hand, being a periodically perturbed inhomogeneity, the grating converts the incident wave into an infinite superposition of partial harmonics of the spatial spectrum and thus excites higher oscillations in the resonator layers. On the other hand, the grating allows different oscillations to interact with each other and so establishes an electrodynamical coupling between the different Floquet harmonics. Due to the circular polarization of the chiral medium eigenwaves, the E- and H-polarized waves are coupled in the chiral layer, and that makes possible the polarization transformation. At certain frequency and structure parameters, the wave interference causes such an energy redistribution between harmonics that results in the observed elliptical to linear polarization transformation. This effective polarization transformation occurs when the number of the harmonics propagating in the "resonator layers" is more than that of free space. This phenomenon has a resonance character, and it is a response to the excitation of oscillations which are close to the structure eigenmodes.

CONCLUSION

The phenomenon of quasi-total transformation of an elliptically polarized incident wave into a specularly reflected linear polarized wave was demonstrated. The scattering is treated in terms of the corresponding diffraction vector problem and takes advantage of an analytical regularization procedure that produces a solution which allows accurate and effective numerical treatment. The frequency range of polarization transformation is rather broadband, and the conditions of the phenomenon realization do not depend greatly either upon the elliptical polarisation parameters or upon the angle of the incident wave. The polarization conversion may be effectively controlled by a suitable choice of structure parameters.

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