H-POLARIZED DIFFRACTION FROM 2-D ARBITRARY STRUCTURES WITH CAVITIES OR EDGES: A RIGOROUS APPROACH

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ABSTRACT

A mathematically rigorous method for numerical simulations of two-dimensional boundary value problems (BVP) of H-polarized electromagnetic wave diffraction by closed or open perfectly conductive cylindrical screens, of arbitrary shape, has been suggested in a previous paper [1], in which a regularisation procedure reduces the initial BVP to an infinite algebraic system of the second kind. Such equations can be solved numerically to any prescribed accuracy by numerically stable truncation methods. However their efficient numerical implementation requires the resolution of a number of non-trivial numerical problems which are solved in this paper, thus creating an effective numerical tool for the above-mentioned diffraction problem.

INTRODUCTION

Accurate mathematical simulation of wave-scattering problems for scatterers with cavities and edges is of great importance in many practical applications, where the reflectivity of objects with complex scattering mechanisms must be accurately evaluated. It is desirable to develop methods that are uniformly valid in a wide frequency range for scatterers of various geometrical shapes and also are of guaranteed accuracy, especially for highly resonant structures (such as cavities with high Q-factor resonances). One such approach is the so-called "Method of Regularisation" (MoR) [2]. The key idea behind this analytical-numerical technique is the analytical transformation of an ill-conditioned first kind Fredholm integral equation, determining the unknown surface current density, into a well-conditioned second kind Fredholm equation for the same unknown expanded in a Fourier series. The final well-conditioned matrix equation is effectively solved numerically by a truncation method; this results in a reliable, stable, numerically accurate and efficient algorithm. It is particularly advantageous for electrically large structures with edges or cavities for which general purpose methods such as the method of moments or geometrical theory of diffraction may be difficult to apply. Until recently the MoR was restricted to diffraction problems from canonically shaped screens [2], such as spherical, spheroidal, or circular cylindrical shells with apertures. A major generalisation [3, 4] that permitted rigorous analysis of scattering from 2-D open screens of an arbitrary profile was successfully used by the authors to analyse E-polarised diffraction from two important classes of scatterers: a cavity having the typical form of an aircraft engine duct, and a 2-D flanged parabolic reflector [5].

In this paper we extend our analysis to the H-polarised case of diffraction from these classes of scatterers. In this case the mathematical development of the algorithm is more complicated then the E-polarisation case because the integro-differential equation, which is equivalent in an exact mathematical sense to the initial boundary-value problem, has a strongly singular kernel. A mathematically rigorous approach that lead to numerically efficient methods for two-dimensional boundary value problems (BVP) of H-polarized electromagnetic wave diffraction by open or closed perfectly conducting cylindrical screens of arbitrary shape, with a Neumann boundary condition, have been suggested in the paper [1]. In [3, 4] it is shown how to transform the given BVP to an equivalent infinite algebraic system of the second kind, that is, of the form $(I+H)x=b$, where I and H denote the identity and some compact operator, respectively, in the Hilbert space l^2 of square summable sequences. Such an equation can be solved numerically with any prescribed accuracy by means of a simple truncation method; this is known to be numerically stable for equations of the second kind. However, for a long time this method has been awaiting an efficient numerical implementation, precisely because such an implementation is a non-trivial numerical problem in itself. This paper is devoted to filling that gap, namely, the creation of an efficient tool for the numerical simulation of the above-mentioned diffraction problem.

Amongst the problems that arise on the way are the efficient calculation of the kernel of the corresponding integro-differential equation and particularly its Fourier coefficients, the efficient summation of the Fourier series for the current density on the screen, and so on. The essential part of the efficient numerical implementation suggested herein is the systematic usage of the Fast Fourier Transform, and the acceleration of certain slowly convergent series (see [2,3]) by means of an analytical transform of the most singular part of the corresponding function.

DESCRIPTION OF THE DIFFRACTION PROBLEM

Let us consider an open or closed contour L that is the cross section of an infinitely long perfectly conducting cylindrical screen; in this 2-D scattering problem, the cross-section is independent of axial position and is assumed to form part of a simple (non-self-crossing) closed contour S of finite length (including the possibility $L = S$). We suppose that the contour S is parametrised by a smooth vector-function $\eta(\theta) = \{x(\theta), y(\theta)\}, \theta \in [-\pi, \pi]$ in such way that after its 2π-periodic continuation, it is smooth on $(-\infty, \infty)$. We assume that : (i) the condition

$$l(\theta) \equiv \left\{[x'(\theta)]^2 + [y'(\theta)]^2\right\}^{1/2} > 0, \qquad (1)$$

which guarantees that $\eta = \eta(\theta)$ is one-to-one mapping, holds; (ii) the point $\eta(\theta)$ is moving in an anti-clockwise direction along the contour S, when θ is increasing; and (iii) the contour L is defined by the choice $\theta \in [-d, d]$ for some $d \le \pi$ (the case $d = \pi$ corresponds to $L = S$).

Let an incident harmonic field $u^0(q)$ be given. A time factor of $e^{-i\omega t}$ is assumed and subsequently suppressed. The scattered field $u^s(q)$ satisfies the homogeneous Helmholtz equation

$$\left(\Delta + k^2\right) u^s(q) = 0 \qquad (2)$$

with wave number k, and satisfies the Sommerfeld radiation condition, the edge condition of finite energy in the vicinity of sharp edges, and the Neumann boundary conditions,

$$\lim_{h \to 0+} \frac{\partial u^s\left(q \pm h n_q\right)}{\partial n_q} + \frac{\partial u^0(q)}{\partial n_q} = 0, \qquad (3)$$

where n_q is a unit vector which is the outward normal to the contour L at the point *q*.

ANALYTICAL REGULARIZATION

We use below the well-known Green function of free space R^2,

$$G(q,p) = -\frac{i}{4} H_0^{(1)}\left(k|q-p|\right), \quad q, p \in R^2 \qquad (4)$$

where $H_0^{(1)}$ is the Hankel function of first kind and order zero. Let us consider now the following functions

$$D_0(\theta,\tau) = \left[\frac{\partial^2 G(q,p)}{\partial n_q \partial n_p}\right]_{q=\eta(\theta), p=\eta(\tau)}, \qquad (5)$$

$$K(\theta,\tau) = 2\pi l(\theta) l(\tau) D_0(\theta,\tau) + \left[4\sin^2\frac{\theta-\tau}{2}\right]^{-1}, \qquad (6)$$

$$K_s(\theta,\tau) = K(\theta,\tau) - \frac{kl(\theta)kl(\tau)}{2} \ln\left|2\sin\frac{\theta-\tau}{2}\right|. \qquad (7)$$

It can be shown (see [1, 3, 4]) that the function $K(\theta,\tau)$ has only a logarithmic singularity (as written in formula (7)), that the function $K_s(\theta,\tau)$ is smooth with all its derivatives of the first order, that all its derivatives of the second order have only logarithmic singularities in the plane $(-\infty,\infty)\times(-\infty,\infty)$ after the 2π -periodic continuation of the function $K_s(\theta,\tau)$ with respect to both variables.

The diffraction problem formulated above may be equivalently reduced, as in [1,3], to the following integro-differential equation

$$\frac{d^2}{d\theta^2}\int_{-d}^{d} z(\tau)\ln\left|2\sin\frac{\theta-\tau}{2}\right|d\tau + \int_{-d}^{d} z(\tau)K(\theta,\tau)d\tau = F(\theta),\, \theta\in(-d,d) \quad (8)$$

for the unknown function $z(\tau)$, $\tau\in(-d,d)$. The second step in our procedure is the transformation of equation (8) into the following dual series equations:

$$\begin{aligned}&\sum_{n=-\infty}^{\infty}\left\{|n|\xi_n + 2\sum_{m=-\infty}^{\infty}k_{n,-m}\xi_m - f_n\right\}e^{in\theta} = 0,\, \theta\in(-d,d)\\&\sum_{n=-\infty}^{\infty}\xi_n e^{in\theta} = 0,\, \theta\in[-\pi,\pi]\setminus(-d,d)\end{aligned} \quad (9)$$

with column vector of unknowns $\xi=\{\xi_n\}_{n=-\infty}^{\infty}$; here $\{k_{n,s}\}_{n,s=-\infty}^{\infty}$ are the Fourier coefficients of the function $K(\theta,\tau)$. The third step in our procedure employs the Method of Analytical Regularization (see [2-6]) to transform such dual series equations [1, 3] to an infinite linear algebraic system of the second kind $(I+H)\hat{\xi}=\hat{f}$ with unknown column-vector $\hat{\xi}$ and known vector $\hat{f}\in l^2$. The coefficients of the matrix-operator $H=\{h_{s,n}\}_{s,n=-\infty}^{\infty}$ satisfy the inequality

$$\sum_{s=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}(1+|s|)(1+|n|)|h_{sn}|^2<\infty, \quad (10)$$

so that H is a compact operator in l^2.

As is well known, the solution of such a linear algebraic system of the second kind can be obtained with any predetermined accuracy by means of a truncation procedure. The solving of such a finite dimensional truncated algebraic system can be done by means of LU-decomposition of the system matrix. Thus the determination of the surface current density depends upon an effective calculation of the Fourier coefficients of the kernel, and efficient summation of the Fourier series representing the surface current density after it has been obtained by solution of the finite dimensional matrix system.

CALCULATION OF THE KERNEL FOURIER COEFFICIENTS

The first numerical problem is the calculation of the Fourier coefficients of the kernel $K(\theta,\tau)$. The singularities in $K(\theta,\tau)$ mentioned above present an obstacle to the fast and accurate calculation of its Fourier coefficients. However the function $K_s(\theta,\tau)$ is much smoother and its Fourier coefficients may be very effectively calculated by the two-dimensional fast Fourier transform (see [3]). The Fourier coefficients of the second term on the right hand side of formula (7) may be calculated analytically with the help of the convolution theorem, once the Fourier coefficients of the function $l(\theta)$ are known. The 2π -periodic continuation of the function $l(\theta)$ is smooth on $(-\infty,\infty)$; hence its Fourier coefficients $\{l_n\}_{n=-\infty}^{\infty}$ decay faster any negative power n^{-p} of its index, its coefficients $\{l_n\}_{n=-\infty}^{\infty}$ may be efficiently calculated by means of the fast Fourier transform.

This approach provides an efficient algorithm for the calculation of the coefficients $\{k_{n,s}\}_{n,s=-\infty}^{\infty}$. Further improvement of the algorithm efficiency by an analytical acceleration technique, in which the next (minor) singularities of the function $K_s(\theta,\tau)$ are extracted and their Fourier coefficients are analytically calculated – see further details in [3].

It should be noted that this calculation of the coefficients $\{k_{n,s}\}_{n,s=-\infty}^{\infty}$ necessarily involves calculation of the function value $K_s(\theta,\theta)$ at $\theta=\tau$. Such a calculation is impossible by directly using definitions (5-7), because the function $D_0(\theta,\tau)$ has strong algebraic and logarithmic singularities: the expression for $K_s(\theta,\theta)$ with arbitrary parameterization $\eta(\theta)$ may be derived analytically and is the one used in our calculations.

CURRENT DENSITY: FOURIER SERIES SUMMATION

Having obtained the Fourier series for the current density on the contour L, the next important problem is its efficient summation. This is necessary for calculation of various physical characteristics of the diffraction problem. Direct summation is very inefficient because of slow (and unstable) convergence of the series. We used asymptotic values of the Fourier coefficients constructed in [3] together with formulae for the analytical summation of the series involving these asymptotic values. The series formed from the difference between the Fourier coefficients and their asymptotic values is more rapidly convergent. As a result, a fast and reliable algorithm for the calculation of current density was constructed.

CONCLUSION

The Method of Regularisation transforms the H-polarised diffraction problem considered to an equivalent infinite linear algebraic system of the second kind: $(I+H)x=b$ with a compact operator H. An efficient and numerically stable numerical algorithm has been constructed. Numerical results thus obtained demonstrate the efficiency and reliability of the numerical method constructed herein. Radar cross-sections are computed for the cavities (of the form of an aircraft engine duct form) of up to 150 wavelengths for normal and oblique incident plane waves. Numerical results will be presented at the meeting.

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