CLOCK RATE ESTIMATION FOR A BETTER SYNTHESIZED ATOMIC TIMESCALE

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1. INTRODUCTION

The current definition of the second is based on the transition frequency of Cs atom. In a real Cs atomic clock, an output frequency includes a characteristic fluctuation. Therefore the outputs of many Cs clocks are averaged to decrease the total fluctuation. Such timescale is called average atomic timescale, and widely used as a high-stable standard time. UTC(NICT), standard time in Japan, is one of such average atomic timescales. It has been made and kept by National Institute of Information and Communications Technology (NICT) for more than 30 years.

UTC(NICT) is constructed from several Cs atomic clocks (HP5071A). During a long operation, these clocks are sometimes withdrawn from the ensemble since they need some maintenances. In such occasions, the rate of UTC(NICT) sometimes largely changed. We have found that the reason was an improper method of clock-rates estimation [1]. The amount of the change in the rate of the timescale however has not been estimated. It was supposed that the weight and rate of the withdrawn clock should be concerned, but a practical equation to express their relations was not acquired.

In this paper, we investigate the rate change of an average atomic timescale at a withdrawal of a clock. We describe some basic equations in section 2, and lead an equation to estimate the amount of rate change in an average atomic timescale at a clock's withdrawal in section 3.

2. BASIC THEORY AND EQUATIONS [2][3]

We obtain an average atomic timescale by the following equation (see Fig.1(a)).

$$TA_o(t) \equiv \sum_{i=1}^{N} w_i(t)h_i(t), \quad \sum_{i=1}^{N} w_i(t) = 1, \tag{1}$$

where $TA_o(t)$ is an average atomic time, i shows each clock in the ensemble, $w_i(t)$ is the weight of each clock, and $h_i(t)$ is the time difference between a clock and the ideal time. Usually the weight $w_i(t)$ is given as a function of a stability of a clock with respect to the timescale. Ideal time is a conceptual time which is made by an accumulation of the defined second.

Equation (1) is very simple but has a drawback. When one clock $i = 1$ is withdrawn from the ensemble at t_0, a discontinuity occurs in $TA_o(t)$. The larger $h_1(t_0)$ is, the larger this discontinuity becomes. Such discontinuity degrades the frequency stability of a timescale.

To avoid this problem, equation (1) should be modified. If a linear phase drift of each clock is removed previously, the discontinuity at the withdrawal of the clock becomes small and the timescale becomes stable. Therefore we introduce a new definition of an average atomic timescale as follows:

$$TA(t) \equiv \sum_{i=1}^{N} w_i(t)\{h_i(t) - \hat{x}_i(t)\}, \quad \sum_{i=1}^{N} w_i(t) = 1. \tag{2}$$

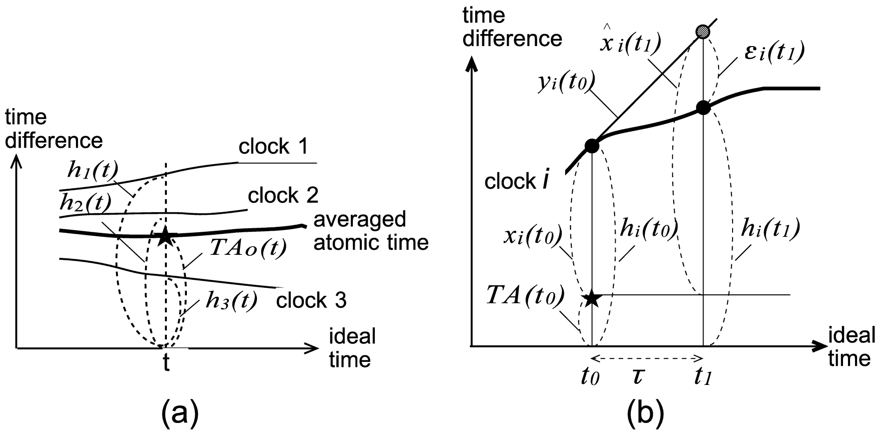

Figure 1: Parameter relations in average atomic timescales. (a) Simple timescale TA_o. (b) Modified timescale TA.

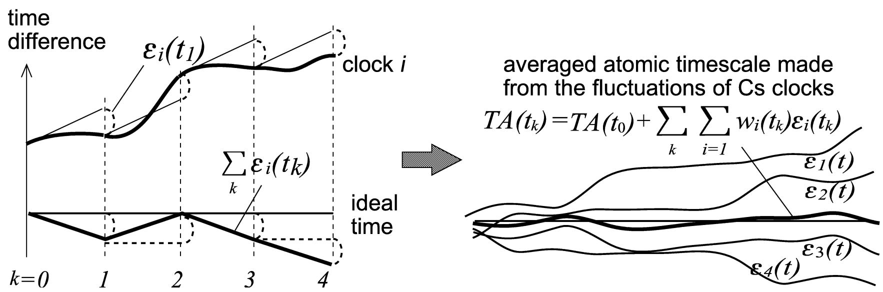

Figure 2: Construction process of a modified average atomic timescale TA.

Here $\hat{x}_i$ is the linear prediction of the time difference between TA and the clock$_{i=1}$, which is defined as follows:

$$\hat{x}_i(t_k) \equiv x_i(t_{k-1}) + y_i(t_{k-1}) \cdot \tau, \quad where \quad \tau = t_k - t_{k-1}. \tag{3}$$

The parameter x_i is defined as the time difference between TA and the clock$_{i=1}$:

$$x_i(t) \equiv h_i(t) - TA(t), \tag{4}$$

and the parameter y_i is defined as a phase drift rate of the clock$_{i=1}$ referred to TA:

$$y_i(t_k) \equiv \frac{x_i(t_k) - x_i(t_k - M\tau)}{M\tau}, \tag{5}$$

where τ is the interval for TA calculation, $M\tau$ is the span for the rate calculation, k shows the timing of TA calculation. We set $\tau = 1day$ and $M = 30$ in the algorithm of UTC(NICT) . Fig. 1(b) shows the relations between these parameters.

Using the above relations, we can express h_i as follows:

$$\begin{aligned} h_i(t_k) &= h_i(t_{k-1}) + y_i(t_{k-1}) \cdot \tau + \varepsilon_i(t_k) = TA(t_{k-1}) + x_i(t_{k-1}) + y_i(t_{k-1}) \cdot \tau + \varepsilon_i(t_k) \\ &= TA(t_{k-1}) + \hat{x}_i(t_k) + \varepsilon_i(t_k), \end{aligned} \tag{6}$$

where ε_i is a prediction error caused by a frequency fluctuation of each clock. We can obtain the following equation by substituting the equation (6) into the equation (2):

$$TA(t_k) = TA(t_{k-1}) + \sum_{i=1}^{N} w_i(t_k)\varepsilon_i(t_k) = TA(t_0) + \sum_{k=1} \sum_{i=1}^{N} w_i(t_k)\varepsilon_i(t_k). \tag{7}$$

From this equation, we can see that TA fluctuates by the total prediction errors of the clock-phase drifts (Fig. 2).

3. RATE CHANGES IN A TIMESCALE AT A WITHDRAWAL OF A CLOCK

Next we investigate the change of TA rate at the withdrawal of a clock by using the equation (7). At first we define the rate of a timescale TA as follows:

$$R(t_M) \equiv \frac{TA(t_M)-TA(t_0)}{M\tau}, \quad where \quad t_M=t_0+M\tau \quad and \quad M=30. \tag{8}$$

From the equation (7) and (8),

$$R(t_M) = \frac{1}{M\tau}\sum_{k=1}^{M} w_1(t_k)\varepsilon_1(t_k) + \frac{1}{M\tau}\sum_{k=1}^{M}\sum_{i=2}^{N} w_i(t_k)\varepsilon_i(t_k). \tag{9}$$

Here we consider a case that clock$_{i=1}$ is withdrawn from the ensemble just after the time t_0. In this case, the rate of the timescale is expressed by

$$R'(t_M) = \frac{TA'(t_M) - TA'(t_0)}{M\tau} = \frac{1}{M\tau}\sum_{k=1}^{M}\sum_{i=2}^{N} w'_i(t_k)\varepsilon_i(t_k), \tag{10}$$

where TA' shows the timescale without clock$_{i=1}$ and w'_i is the weights of the clocks in the ensemble without clock$_{i=1}$.

The weight w'_i is related to w_i by the following consideration. A clock weight depends on its frequency stability referred to TA. Whether one clock is withdrawn from the TA ensemble, the relative weights of other clocks are not affected if TA is still stable after this event. TA can keep its frequency stability if the ensemble clocks are enough and their maximum weights are limited to protect a concentration. UTC(NICT) is such a time scale. In this case, relative weights of clocks basically don't change but the normalization constant will change when one clock is withdrawn. Thus w'_i is expressed as follows:

$$w'_i(t_k) \simeq A w_i(t_k), \ i \geq 2, \tag{11}$$

where A is a normalization constant. From the requirement such as $\sum_{i=1}^{N} w_i(t_k)=1$ and $\sum_{i=2}^{N} w'_i(t_k)=1$, the constant A becomes as follows:

$$A = \frac{1}{1-w_1(t_k)}. \tag{12}$$

In addition, we adopt another approximation here:

$$w_i(t_k) \simeq w_i(t_0). \tag{13}$$

In the case of UTC(NICT), this approximation is acceptable because the frequency stability of Cs atomic clock HP5071A does not change so much during 30 days. From the equation (11), (12) and (13), the weight w'_i is approximated as

$$w'_i(t_k) \simeq \frac{w_i(t_0)}{1 - w_1(t_0)}. \tag{14}$$

Using the equation (14), the equation (10) is expressed as follows:

$$R'(t_M) \simeq \frac{1}{M\tau}\frac{1}{1- w_1(t_0)}\sum_{k=1}^{M}\sum_{i=2}^{N} w_i(t_0)\varepsilon_i(t_k). \tag{15}$$

From the equations (9), (13) and (15), we can calculate the difference of the change of TA rate as follows:

$$\begin{aligned}
\{R(t_M) \quad &- \quad R'(t_M)\} \\
&\simeq \frac{1}{M\tau}\sum_{k=1}^{M} w_1(t_0)\varepsilon_1(t_k) + \frac{1}{M\tau}\{1-\frac{1}{1-w_1(t_0)}\}\sum_{k=1}^{M}\sum_{i=2}^{N} w_i(t_0)\varepsilon_i(t_k) \\
&= \frac{1}{M\tau}\sum_{k=1}^{M} w_1(t_0)\varepsilon_1(t_k) - \frac{w_1(t_0)}{1-w_1(t_0)}\{R(t_M) - \frac{1}{M\tau}\sum_{k=1}^{M} w_1(t_0)\varepsilon_1(t_k)\} \\
&= \frac{w_1(t_0)}{1-w_1(t_0)}\{\sum_{k=1}^{M}\frac{\varepsilon_1(t_k)}{M\tau} - R(t_M)\}.
\end{aligned} \tag{16}$$

Now we look back to the previous relations to express the right side of the equation (16) by using the computable parameters instead of the incomputable ε_1 and R . Using the equation (4) and (6),

$$x_1(t_k) - x_1(t_{k-1}) = y_1(t_{k-1}) \cdot \tau + \varepsilon_1(t_k) - \{TA(t_k) - TA(t_{k-1})\}. \tag{17}$$

Accumulating the equation (17) from t_1 to t_M, the following equation is acquired.

$$x_1(t_M) - x_1(t_0) = \sum_{k=1}^{M}\{y_1(t_{k-1}) \cdot \tau + \varepsilon_1(t_k)\} - \{TA(t_M) - TA(t_0)\}. \tag{18}$$

By substituting the equation (8) and (18) into the equation (16), we obtain the final expression:

$$R(t_M) - R'(t_M) = \frac{w_1(t_0)}{1 - w_1(t_0)}\{\frac{x_1(t_M) - x_1(t_0)}{M\tau} - \frac{1}{M}\sum_{k=1}^{M} y_1(t_{k-1})\}. \tag{19}$$

This is the equation that estimates a rate-change of TA caused by a withdrawal of clock$_{i=1}$. As $w_1(t), x_1(t)$ and $y_1(t)$ are obtained in the TA calculation, the right side of the equation (19) is computable.

We compared the result of the equation (19) and the actual rate change in a simulated UTC(NICT). We made an average atomic timescale by using eight HP5071A clocks at NICT, then removed one clock from this timescale. We compared the average rate change in 30 days and the result of equation (19), and confirmed the difference was a few parts in 10^{-15}. This value is comparable to the stability of the average timescale itself, which shows the availability of equation (19).

4. SUMMARY AND CONSIDERATION

We investigated an effect of a clock's withdrawal from an average atomic timescale. We started from a basic equations of TA construction and lead an equation to estimate a change in TA rate at a clock is withdrawn from the ensemble. We adopted two approximations about the clock weights in this process and obtained a simple expression. These approximations are reasonable in such conditions as the ensemble clocks are enough and stable, and do not have too large weights. The calculated value by using the final equation (19) showed a good agreement with the actual rate change in the simulated UTC(NICT). This means that the approximations are adequate and the equation (19) is available to estimate the rate change in UTC(NICT). More consideration is progressing to generalize this result.

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