NON-STATIONARY AND FREQUENCY-MODULATED SPACE-TIME FOCUSINGS IN IONOSPHERIC PLASMA

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ABSTRACT

The formation opportunity of three-dimensional space-time focusings of electromagnetic fields such as three-dimensional catastrophes in ionospheric plasma is studied. These focusings are caused by a non-stationarity of a distribution medium and a wavefront space curvature. The formation requirements of corank three wave catastrophes in non-stationary media as in a case originally stationary radiopulses and frequency-modulated signals are considered. By means of the catastrophe theory the classification of such space-time focusings and the subordination diagrams are fulfilled. In this paper the dependence of the focusing order on the type of a catastrophe is analyzed.

INTRODUCTION

In this paper three-dimensional space-time (ST) radiation focusings in non-stationary dispersive media such as ionospheric plasma are considered. In [1] it has been devoted to the examination of three-dimensional ST focusings, induced an initial wavefront flexure and a frequency modulation of a signal. In our case the time part of the focusing is caused by the non-stationarity of a dispersive medium, as which the cold homogeneous plasma with a variable electron concentration is taken.

INTEGRAL FIELD REPRESENTATION

To obtain the signal distribution in non-stationary media we take up Klein-Gorgon's equation:

$$\Delta U(\vec{r},t) - \frac{1}{c^2}\frac{\partial^2 U(\vec{r},t)}{\partial t^2} = \frac{\omega_{\mathrm{p}}^2(t)}{c^2} U(\vec{r},t), \tag{1}$$

with boundary conditions in the next form:

$$U(\vec{r},\eta)\Big|_{r_3 = f(r_1,r_2)} = A_0(r_1,r_2,\eta)\exp(i\omega_0\eta), \tag{2}$$

where $\vec{r}$ — radius-vector of an observation point, $A_0(r_1,r_2,\eta)$ — amplitude distribution on the initial phase front, given by the equation $r_3 = f(r_1,r_2)$, $\omega_{\mathrm{p}}(t)$ — time-dependent plasma frequency, c — velocity of light, ω_0 —

working signal frequency. The case of a linear time dependence of an electron concentration in plasma is considered. Then the plasma frequency is presented in the following form:

$$\omega_{\mathrm{p}}^2(t) = \omega_{\mathrm{n}}^2 (1 + \delta\, t), \tag{3}$$

where ω_{n}, δ — constant coefficients. In this case the solution of the equation (1) and initial conditions (2) given the uniform field distribution in any point on the right of initial wave front is defined by the multiple integral:

$$U(\vec{r},t)=\frac{\exp\left(-i\frac{\pi}{4}\right)}{(2\pi)^{3/2}}\sqrt{\left|\frac{\omega_{\mathrm{n}}^2\delta}{2\omega_0}\right|}\iiint k\, B(\xi_1,\xi_2,\eta)\exp(i\Phi(\vec{r},\, \xi_1,\, \xi_2,t,\, \eta))\, d\xi_1 d\xi_2 d\eta, \tag{4}$$

in the integral we used the next parameters:

$k = \left(\sqrt{\omega_0^2 - \omega_{\mathrm{p}}^2(\eta)}\right)\Big/ c$ — wave number,

$$\Phi(\vec{r},\, \xi_1,\, \xi_2,t,\, \eta)=\frac{2}{3\delta\alpha\,\omega_{\mathrm{n}}^2}\left[\left(\omega_0^2 + \delta\,\omega_{\mathrm{n}}^2(t-\eta)\right)^{\frac{3}{2}} - \omega_0^3\right]+\omega_0\eta - kQ, \tag{5}$$

— the phase function of the integrand, $Q(\vec{r},\, \xi_1,\, \xi_2\,)$ — the distance between a point on initial wave front and a point of observation,

$$B(\xi_1,\xi_2,\eta) = A_0\,(\xi_1,\xi_2,\eta)\sqrt{\left(f''_{\xi_1\xi_1}\, f''_{\xi_2\xi_2} - f''^2_{\xi_1\xi_2}\right)\Big/\left(1+ f'^2_{\xi_1} + f'^2_{\xi_2}\right)}.$$

CONDITIONS OF SPACE-TIME FOCUSINGS

Conditions of the radiopulse focusing on space coordinates were obtained in [1]. To obtain the time focusing conditions it is necessary to consider first and second derivatives of phase function (5) with respect to initial time η. By equating to zero first derivative of the phase (5) with respect to time, we obtain the equation of space-time rays:

$$Q = c(t-\eta)\sqrt{\varepsilon_0(\eta)}, \tag{6}$$

where $\varepsilon_0(\eta) = 1-\omega_{\mathrm{p}}^2(\eta)\big/\omega_0^2$.

Equating to zero the second derivative of eiconal we obtain the next condition of the impulse time focusing in non-stationary media:

$$2\, k^3 c^4 \omega_0 + \omega_{\mathrm{n}}^4\, Q\delta = 0. \tag{7}$$

Note that the formula (6), (7) are deduced on the assumption $\delta << 1$, that is the time dependence of an electron concentration in plasma was considered weak. In this paper the conditions under which there is no time focusing in case of frequency-modulated signal propagation in non-stationary media were deduced, that is requirement of compensating of signal squeezing by the medium non-stationarity. The conditions of the radiopulse focusing in case of a square-law time dependence of plasma electron concentration are obtained too.

CLASSIFICATION OF SPACE-TIME CATASTROPHES

To fulfill the classification of the stable ST focusings and the subordination diagrams the wave theory of singularities of differentiable mappings (the catastrophe theory) was applied. The three-dimensional singularities (catastrophes) with disintegrated space and time parts were considered. In this case the symbol of three-dimensional catastrophe consists of symbols of two-dimensional space and one-dimensional time singularities. The form of function $f(\xi_1, \xi_2)$ assigning the initial front defines the type of a space focusing. The type of a time focusing is determined by higher derivatives of phase function (5) with respect to time [1]. The total multiplicity of three-dimensional ST singularities is equal to product of the multiplicity of space and time focusings. The uniform asymptotic field distribution in such three-dimensional focal areas is described in terms of the special functions of wave catastrophes (SWC) and its derivatives. In this paper the dependence of the focusing order (the power of major parameter is explored to which the value of a field in a focal point of a system is proportional) on the catastrophe type is investigated [1,2].

CONCLUSION

In this paper we considered the propagation of originally stationary radiopulse and frequency-modulated signal in non-stationary media of ionospheric plasma type. It is obtained the uniform solution of this problem in terms of three-dimensional oscillatory integral. The curvature of initial wave front and non-stationary propagation medium result in the arising of simultaneous three-dimensional space and time focusings. Classification of such three-dimensional ST singularities with disintegrated space and time parts was fulfilled by means of the catastrophe theory. The conditions of these focal regions formation have a form of differential equation connecting the derivatives of the phase function of the integrand (5). The field amplitude in focal regions is proportional the frequency raised to a power equal the focusing order.

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