GENERALIZED MODEL OF THE SATCOMM CHANNEL FOR APPLICATIONS TO FADE COUNTERMEASURES

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ABSTRACT

The model is a generalisation of the Maseng–Bakken model targeting dual–site dual–frequency rain attenuated satellite links. The outcome is a consistent and comprehensive theoretical description of rain fade slope, expected long–term power spectral density of rain attenuation, frequency scaling factor, site diversity and a novel way to compute fade duration statistics using Markov Chains.

GENERALIZED MODEL OF RAIN ATTENUATION

This work is a generalisation of the stochastic dynamic model first proposed by Maseng and Bakken (MB) in [1] and is also inspired by [2] and [3]. The MB model is extended to two arbitrarily correlated satellite links at two different carrier frequencies. This allows to provide theoretical descriptions of, (i) of rain attenuation's power spectral density (PSD), (ii) dual-location site diversity systems, (iii) rain attenuation frequency scaling factor and (iv) rain fade slope statistics, [4]. Finally, (v) a new computational method based on Markov chains ([7]) is also applied for evaluating rain fade durations statistics.

There is agreement that point rainfall rate can be modelled as a lognormal variable, [5]. An extension is to consider that the joint distribution of rainrate at two points on the horizontal (x,y) plane is lognormal ([3]) with pdf:

$$f_{R_1,R_2}(R_1,R_2,r) \equiv \Lambda_{R_1R_2}(m_1,\sigma_1,r,m_2,\sigma_2) \tag{1}$$

(m_1,σ_1) and (m_2,σ_2) characterise the marginal lognormal statistics of point rainfalls, R_1 [mm/h] and R_2 [mm/h] at two locations, 1 and 2, of interests. Parameter r denotes the correlation factor between R_1 and R_2. The correlation factor, r, for dual-point rainfall rate is:

$$r = \frac{e^{r'\sigma_1\sigma_2} - 1}{\sqrt{e^{\sigma_1^2} - 1}\sqrt{e^{\sigma_2^2} - 1}} \equiv f(r')\ ,\ r' \in [0,1] \tag{2}$$

The quantity $r' = f(x,y,t)$ denotes the cross-correlation factor for the reduced variables, $u_1 \equiv (\ln R_1 - m_1)/\sigma_1$ and $u_2 \equiv (\ln R_2 - m_2)/\sigma_2$. It can be shown that the joint pdf of u_1 and u_2 is jointly Gaussian with zero mean and unit variance. $r' = f(x,y,t)$, depends on the space–time characteristics of rainfields Assuming isotropy and introducing the separation distance $\rho \equiv \sqrt{(|x-x_0|)^2 + (|y-y_0|)^2}$, the correlation may be chosen to be:

$$r' = \exp\left(-\frac{\beta_1}{\sqrt{L_{xy}}}\rho - \beta_2|t|\right) = \exp\left(-\frac{\beta_1\overline{V}}{\sqrt{L_{xy}}}|t| - \beta_2|t|\right) \tag{3}$$

where the second term uses Taylor's hypothesis assuming that the raincells are moving uniformly with an average velocity $\overline{V}$. The last term of Eq. (3) is $\propto\ |t|$ so we may compactly write:

$$r' = \exp(-\beta|t|) \approx \exp\left(-\beta\frac{\overline{\rho}}{\overline{v}}\right) \tag{4}$$

where β is a *compound constant* catering for birth/decay of raincells as well as their average speed at a particular location. We will see later that Eqs (2) to (4) are consistent with the MB model and that they imply point rainfall *and* indeed rain attenuation have a *first order PSD*, [8]. As an estimate of β, consider a mean storm velocity of 10 m/s and a mean raincell diameter of 5 km giving $\beta \approx 10/5000 = 2.0\cdot 10^{-3}$ [s^{-1}]. This compares well to $\beta = 1.68\cdot 10^{-3}$ s^{-1} and $1.85\cdot 10^{-3}$ s^{-1} in [1] and [9]. Note also that (1) and (4) show that the joint pdf can be expressed as $f_{R_{12}}(R_1,R_2,r(\rho))$ or $f_{R_{12}}(R_1,R_2,r(t))$ depending on whether our interest is placed on spatial or temporal variations of the rainfall field. We will use this *duality* to calculate below different first and second order characteristics of rain attenuation.

Defining by D_i [km] $i = 1,2$, the horizontal projections of the slant paths with respective elevation angles θ_i, H_i as local rain heights [km] and h_i as the altitudes of the stations, we can calculate the projection of the paths on the xy plane with $D_i = (H_i - h_i)/\tan\theta_i$ The total attenuation is $A_i = \Gamma_i \times D_i$: where the specific attenuations are $\Gamma_1 = aR_1^b$, $\Gamma_2 = cR_2^d$. The use of $\{a,b\}$ for Γ_1 and $\{c,d\}$ implies that we generally consider two different carrier frequencies and/or drop size distributions. It can be shown that the distribution of rain attenuation is joint lognormal with pdf:

$$f_{A_1,A_2}(A_1,A_2,r_A) = \Lambda_{A_1A_2}(M_1,\Sigma_1,r_A,M_2,\Sigma_2)$$

$$\begin{cases} M_1 \equiv bm_1 + ln(a) + \ln(D_1),\ \Sigma_1 = b\sigma_1 \\ M_2 \equiv dm_2 + \ln(c) + \ln(D_2),\ \Sigma_2 = d\sigma_2 \end{cases}, \quad r_A = \frac{e^{r'\Sigma_1\Sigma_2} - 1}{\sqrt{e^{\Sigma_1^2} - 1}\sqrt{e^{\Sigma_2^2} - 1}} \tag{5}$$

ASYMPTOTIC PSD OF RAIN ATTENUATION AND IDENTIFICATION OF β

The MB model is extended here by providing *asymptotict solution* for the PSD, $P_{yy}(\omega)$, of rain attenuation:

$$P_{yy}^{\infty}(\omega) \approx \frac{2\beta\sigma^2 e^{2m} e^{2\sigma^2}}{\omega^2}, \quad \omega > \omega_c,\ P_{yy}^{0}(\omega) \approx 0.1745 \cdot \frac{e^{2m} e^{\sigma^2} e^{2.648\sigma}}{\beta}, \quad 0 < \omega \leq \omega_c$$
$$\omega_c \approx 3.3855 \cdot \frac{\beta\sigma e^{\sigma^2/2}}{e^{1.324\sigma}} \tag{6}$$

where $m = M_i$ and $\sigma = \Sigma_i$, $i = 1, 2$. This PSD has a first order characteristic ([8]). For fixed (m,σ), a large β will lower the plateau in the PSD while the cut-off frequency of the spectrum will move toward the higher frequencies. Thus second order properties such as fade slope and fade durations depend significantly on the actual value of β. Note that the cut-off ω_c [rad/s] is independent of the median value e^m. This gives an easy way to identify a value of β for particular events (or average PSD).This was applied to around 70 rain events of 1998 ITALSAT beacon data collected in Milan. This yielded the histogram shown in Fig. 1 showing that $3.16 \times 10^{-4} \leq \beta \leq 3.16 \times 10^{-3}$ at a particular location. This is also smaller than the values in [1] or [9] thus β probably depends on lthe ocal climate.

FREQUENCY SCALING FACTOR OF RAIN ATTENUATION

A new statistical model for the frequency scaling factor, $z \equiv A_1/A_2$, has been developed giving the conditional expected value of the frequency scaling factor given as a function of the base attenuation, i.e.:

$$E\{z|A_2\} = \int_0^{\infty} z \cdot \frac{f_{z,A_2}(z,A_2,r_A)}{\Lambda_{A_2}(M_2,\Sigma_2)} dA_2 \tag{7}$$

We assumed the frequency pair 32.12/20.1 GHz at Portsmouth (ITU-R model). r_A is kept as a free parameter to see its impact on $E\{z|A_2\}$. Fig. 2 shows that r_A plays an important role on the the scaling factor. Intesteringly, $E\{z|A_2\}$ can be assumed as constant for close–to–one correlation factors while it becomes strongly base-attenuation dependent for low values of r_A. As in most SatCom scenarios, the requirement is to frequency-scale attenuation on links which are geographically overlaping, the two attenuation time-series will be highly correlated. From Fig. 2, the scaling factor can therefore be considered as *constant*.

FADE DURATION STATISTICS USING MARKOV CHAINS

Consider a discrete-time discrete amplitude Markov chain $X(n)$ representing the discretized version of rain attenuation. Assume a state space $\Omega = \{1, 2, ..., N\}$ dB. Then the evolution of $X(n)$ can be completely described by the transition matrix $\boldsymbol{P}$, whose elements are given by $P_{ij} = Prob\{X(n+1) = j \mid X(n) = i\}, \quad i,j \in \Omega$. $\boldsymbol{P}$ is such that for any of its row of (labelled by i) we have $\sum_j P_{ij} = 1$ where j is used to identify the columns of $\boldsymbol{P}$. The steady-state distribution linked to $\boldsymbol{P}$ will be denoted by the column vector $\boldsymbol{\pi}$ and satisfies, assuming an arbitrary initial state, $\boldsymbol{\pi}_0\boldsymbol{\pi} = \lim \boldsymbol{\pi}_0\boldsymbol{P}^n$ as $n \to \infty$. By setting $\Delta t = 1$ s, $m'_1 = m'_2 = m$, $\sigma'_1 = \sigma'_2 = \sigma$, and $A_1 = i$ dB and $A_2 = j$ dB, we rewrite the joint pdf (5) as:

$$f(i,j) = \frac{1}{2\pi\sigma^2 ij\sqrt{1 - r_A^2}} \exp\left[-\frac{1}{2(1 - r_A^2)}\left(\frac{(lni - m)^2}{\sigma^2} - \frac{2r_A(\ln i - m)(\ln j - m)}{\sigma^2} + \frac{(\ln j - m)^2}{\sigma^2}\right)\right]$$
$$r_A = e^{\sigma^2 \times e^{-\beta}} - 1/e^{\sigma^2} - 1 \tag{8}$$

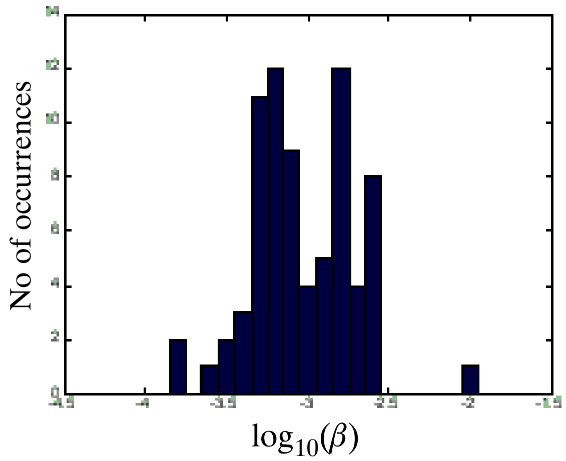

Fig. 1: Histogram of empirical values of beta (40 GHz)

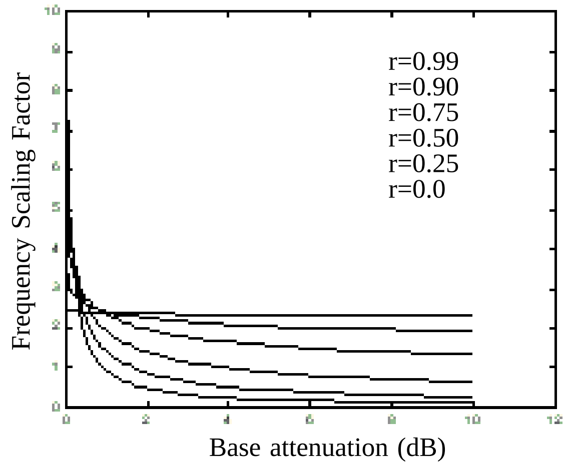

Fig. 2: Frequency scaling factor for 39.12 and 20.1 GHz.

The conditional pdf $f(j|i)$ can be found from Bayes' theorem yielding:

$$P_{ij} = \int_{i-0.5}^{i+0.5} \int_{j-0.5}^{j+0.5} f(i,j)didj \Bigg/ \int_{i-0.5}^{i+0.5} \Lambda_i(M_i,\Sigma_i)di \tag{9}$$

Output from (9) is shown in Fig. 3. As most of the elements of $\boldsymbol{P}$ are equal to zero except around the diagonal $i=j$, *the* that transitions from i in dB to j dB can only be small over one second. Now let ϵ [dB] be an arbitrary attenuation threshold relevant to the FMT scenario being considered.We call $\mathfrak{B}$-states the bad states belonging to the attenuation subset $\mathfrak{B} = \{\epsilon + 1, ..., N\}$ and $\mathfrak{G}$-states the desired states belonging to $\mathfrak{G} = \{1, 2, ..., \epsilon\}$, [12]. The conditional exceedance probability of rain fade durations is the most commonly estimated quantity in propagation experiments ([10]) and can be computed using the machinery developed in [7].For this let us define a bad period as having a duration $T_{\mathfrak{B}} \equiv d$ [s] if a transition from a good state ($\mathfrak{G}$-state) to a bad state ,$\mathfrak{B}$-state, is followed by t - 1 consecutive $\mathfrak{B}$-states, in turn followed by one $\mathfrak{G}$-state. Zorzi showed that the duration statistics is:

$$\Pr\{T_{\mathfrak{B}} \geq d | A_i \geq \epsilon\} = \frac{\boldsymbol{\pi}_{\mathfrak{G}}^T \boldsymbol{P}_{\mathfrak{B}}^{d-1} \boldsymbol{P} \boldsymbol{e}_{\mathfrak{B}}}{\boldsymbol{\pi}_{\mathfrak{G}}^T \boldsymbol{P} \boldsymbol{e}_{\mathfrak{B}}} \tag{10}$$

where T denotes the transpose operator, $\boldsymbol{\pi}_{\mathfrak{G}}$ is obtained from the steady-state distribution by setting to zero all entries corresponding to states which are *not* in $\mathfrak{G}$, $\boldsymbol{P}_{\mathfrak{B}}$ is obtained from $\boldsymbol{P}$ by setting to zero all P_{ij} with $j \in \mathfrak{G}$, and $e_{\mathfrak{B}}$ is a column vector whose i^{th} entry is one if $i \in \mathfrak{B}$. The results in Fig. 4 on a lognormal plot for $M_i = -1.40$, $\Sigma_i = 1.498$ and $\beta = 1.65 \cdot 10^{-4}$. The graph shows that the exceedance probability of rain attenuation is lognormal in the range of durations between 10 and 1000 [s]. The predicted shape and dependency on attenuation thresholds seems to fit quite well with ACTS results given in [10] but more detailed comparison to experimental results is needed.

DUAL-LOCATION SITE DIVERSITY MODEL

Site Diversity (SDV) consists in the selection of the least attenuated link between two Earth stations pointing toward a same satellite at any particular point in time. SDV results in selecting the best signal according to $A_d(t) = \min(A_1(t), A_2(t))$. From this it is clear that the CCDF of the (balanced) site diversity system is given by:

$$Prob\{A_d \geq w\} = \int_w^{\infty} \int_w^{\infty} f_{A_1,A_2}(A_1, A_2, r_A) dA_1 dA_2 \tag{11}$$

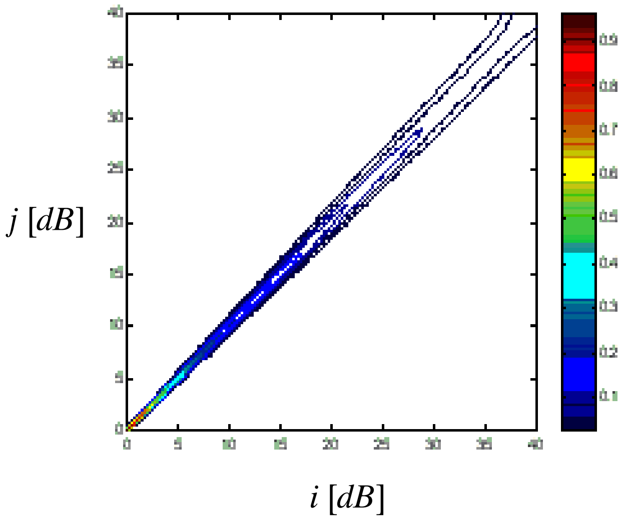

Fig.3: Contour plot of the transition matrix

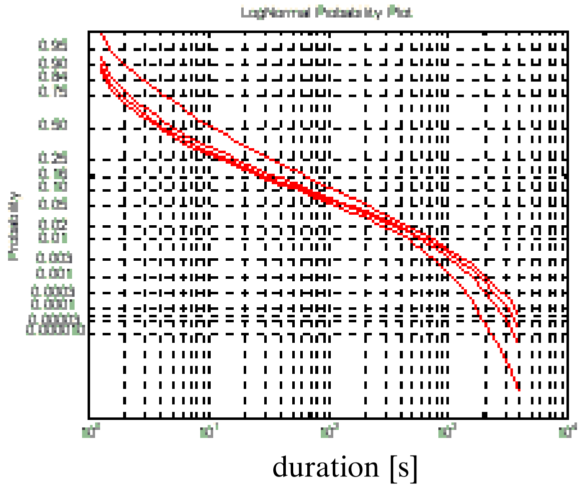

Figure 4: Fade duration statistics for attenuation thresholds of 1 (top), 3, 5, and 10 (bottom) dB

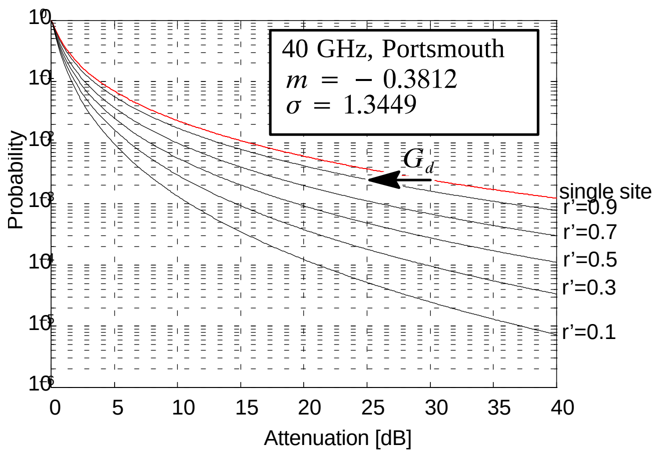

Fig. 5:Diversity gain as a function of reference attenuation (Portsmouth @40 GHz)

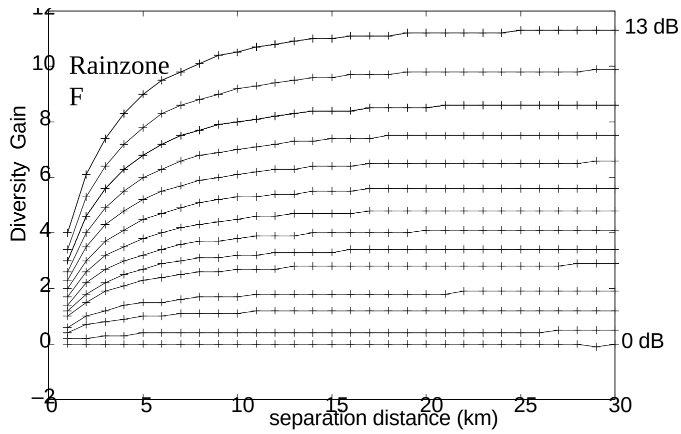

Fig. 6:Diversity gain as a function of separation distance for different values of reference attenuation (1 dB steps)

A typical output from (11) is shown in Fig. 5 (40 GHz located in Portsmouth assuming $D_i = 3.5\ [km]$). The SDV gain, G_D, is defined as the *equi-probability* difference between the path attenuation encountered on a reference link and that obtained on a diversity system ([13]). Quite expectedly, high correlation factors only provide a small diversity gain. Finally, as indicated in (4), the cross-correlation factor is simply an image of the impact of the separation between the two sites. Increasing separation distance will result in lower cross-correlation and therefore large diversity gains. This is best displayed by considering (11) as a function of r_A where w simply becomes a parameter. Inverting (4) allows us to express r_A (see (5)) as a function of $\bar{\rho}$. ($\bar{\rho} = \bar{v}/\beta \ln r' = -5000 \ln r'\ [\mathrm{m}] = 5 \ln r' [\mathrm{km}]$ for $\bar{v} = 10\ \mathrm{m/s}$ and $\beta = 2 \cdot 10^{-3}\ \mathrm{s}^{-1}$). This is shown in Fig. 6 for two links at 20 and 40 GHz in the Portsmouth area. The improvement in diversity gain tends to stabilize for site separation above 20-25 km, [13]. Clearly, this asymptotic behavior depends on the ratio $\bar{v}/\beta$. Thus we would need to to estimate the average raincell velocity ([14]) for good modelling.

CONCLUSIONS

A generalized model of rain attenuation has been introduced. It allows the theoretical calculation of first and second order charateristics of the rain fading process.and requiries five main input parameters. The first classical four characterise the first order lognormal statictics of rain attenuation They can easily be fitted to either experimental ccdfs or from global prediction nodel like the ITU–R's. The last parameter (β) is a space–time parameter used to characterise the properties of the rainfield. β was found to vary over one order of magnitude for a particular satellite link and it will vary with actual location and climate. Thus maps of β should ultimately be provided on top of the standard ITU–R distributions. To that effect, a method to estimate β from classical experimental time–series has been presented.

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REFERENCES

[1] T. Maseng, P.M. Bakken,"A Stochastic Model For Rain Attenuation", IEEE Trans, Commun., Vol. COM-29, No5, May 1981, pp 660-669.

[2] E, Matricciani, "Prediction of site diversity performance in satellite communications systems affeted by rain attenuation: Extension of the two-layer model", European Transactions on Telecomms and related technologies, Vol. 5 No 3, May-June 1994.

[3] R.K. Crane, " Electromagnetic Wave Propagation through Rain", John Wiley & Sons, 1996.

[4] B.C. Gremont, D.L. Ndzi, "First Order and Conditional Statistics of Rain Attenuation Fade Slope", IEE Electronics Letters, Vol. 36 No 20, 28/09/00, pp 1733–1734.

[5] S.H. Lin, "Statistical Behavior of rain attenuation", Bell Syst. Technical Journal, Vol. 52, No 4, pp 557-581, April 1973.

[6] R.M. Manning, "A Unified Statistical Rain-Attenuation Model for Communication Link Fade Predictions and Optimal Stochastic Fade Control Design using a Location Dependent Rain-Statistics Database", Int. Journal of Satellite Communications, Vol. 8, pp 11-30, 1990.

[7] Michele Zorzi, "Outage and error events in Bursty channels", IEEE Trans. on Communications, vol. 46 No 3, March 1998, pp 349-356.

[8] E. Matricciani, "Physical–mathematical model of the dynamics of rain attenuation with application to power spectrum", Electronics Letters, Vol. 30, 1994, pp 522–524.

[9] A. Burgueño, E. Vilar, M. Puigcerver, "Spectral Analysis of Rainfall Rate And Relation to Fade Dynamics ", IEEE Trans, Commun., Vol. COM-38, No 9, Sept. 1990, pp 1359-1366.

[10] H. Helmken, R.E. Henning, J. Feil, L.J. Ippolito, C.E. Mayer, "A three-site comparison of fade-duration measurements", Proc. IEEE, Vol. 85, No 6, June 1997, pp 917-924.

[11] E.Vilar, A Burgueño, "Analysis and modeling of time intervals between rain rate exceedances in the context of fade dynamics", IEEE Trans. Comms, Vol. 39, No 9, sept. 1999, pp 1306-1312.

[12] A. Paraboni, C. Riva, "A new method for the prediction of fade duration statistics in satellite links above 10 GHz", Int. J. Satellite Communications, Vol. 12, 1994, pp 387-394.

[13] D.B. Hodge, "An empirical relationship for path diversity gain", IEEE Trans. Ant. Prop., Vo; 24, No 3, March 1976, pp 250-251.

[14] Enjamio, E, Vilar and F P Fontan, "Micro Scale Rain Variations: Application to Correlation", UK National URSI Symposium, Malvern, 11–12 December 2001.