INVESTIGATION OF HIGH-FREQUENCY COUPLING WITH UNIFORM AND NON-UNIFORM LINES: COMPARISON OF EXACT ANALYTICAL RESULTS WITH THOSE OF DIFFERENT APPROXIMATIONS

Sergey Tkachenko(1), Jürgen Nitsch(2)

Institute for Fundamental Electrical Engineering, Otto-von-Guericke University, Magdeburg
P.O.Box 4120, 39016 Magdeburg, Germany

1)Sergey.Tkachenko@ E-Technik.Uni-Magdeburg.DE, 2)Juergen.Nitsch@E-Technik.Uni-Magdeburg.DE

ABSTRACT

In this paper we consider the interaction of plane waves with infinite transmission lines and investigate the propagation of a TEM current wave along a semi-infinite line, both at very high frequencies where radiation effects become important. In both cases we obtain approximate analytical solutions, applying different methods: the radiation approach and a perturbation theory in terms of the parameter 1/2ln(2h/a)). The approximate results are compared with exact analytical solutions for the problems. The radiation approach gives satisfactory results for frequencies kh<=1.5 for both line configurations, whereas the open line is well described with perturbation theory only for frequencies kh<=10.

INTRODUCTION

The necessity to consider high-frequency coupling with transmission lines arises in many EMC problems which are connected with the interaction of an external electromagnetic field with electronic equipment. When the wavelength of the effecting electromagnetic field is comparable or to less than the cross section of the line, the classical transmission line theory (TL) is no longer applicable [1]. On the other hand, pure numerical methods, which are usually used to model transmission lines by antenna theory, do not give a clear physical description of the problems and require large computer resources. Thus, it is necessary to develop simple analytical methods based on the understanding of the physics of coupling processes.

An effective hybrid method to obtain a closed - form approximate analytical solution for the high – frequency field (plane wave) coupling to long terminated uniform lines was recently proposed in [2]. The method is based on a simple analytical representation of the solution for the Pocklington equation in the uniform part of the transmission line, far enough from the line terminals. The obtained solution contains reflection coefficients (TEM as well as non-TEM) for current waves. To obtain these reflection and transmission coefficients, in the general case some effective numerical procedure had to be developed. For simple cases, as for an open circuit line, an exact or approximate iteration (perturbation theory in terms of ‘a thin-wire parameter for a horizontal line $1/2\ln(2h/a)$’ , where h is the height and a is the radius of the line) solution can be obtained [3].

In papers [4,5] another approximate (numerical - analytical iteration) method was proposed ("radiation approximation"). This method uses a TL approximation and takes into account ohmic losses in the line. To account for nonuniform elements, as, e.g., non-parallel elements near the loads, line bends, etc., lumped capacitances and inductances were introduced. They were obtained from solutions of the electrostatic problem or the problem for constant current. After that, radiation losses of the system were calculated by numerical integration. These losses were added to the ohmic ones and thereby defined the losses of the line for the next iteration. The method [4, 5] requires extensive numerical calculations and contains the assumption, that the radiation losses are distributed along the whole line like the ohmic ones. Moreover, in our opinion, it is consistent to describe the radiation (caused by TEM and non - TEM current waves, scattered by line nonuniformities in long lines) by the introduction of corresponding lumped line impedances, which contain a reactive part as well as an active part connected with radiation.

In this paper we consider two problems, which are basic for the high frequency coupling with transmission lines: External plane wave coupling with an infinite transmission line and reflection of TEM current waves from the open end of a semi-infinite line. These problems have well known exact analytical solutions in the thin wire approximation. We obtain approximately analytical solutions by using the radiation approach and compare the results with exact ones. It has been shown that the radiation approach gives good results as long as $kh <\approx 1$.

INFINITE LINE EXCITED BY A PLANE WAVE

Consider a lossy infinite wire above perfect conducting ground which is excited by a plane wave (See Fig.1). For this problem one has a well known analytical solution [6]. From this solution one can derive the radiation losses along the line. On the other hand, there exist approaches to describe the radiation losses of such a line in an approximate way [4, 5]. Since the latter ones are easily calculated, it is of great interest to get to know the differences of the results of the different approaches and the exact one. In this section we therefore will compare and discuss radiation losses obtained by different methods.

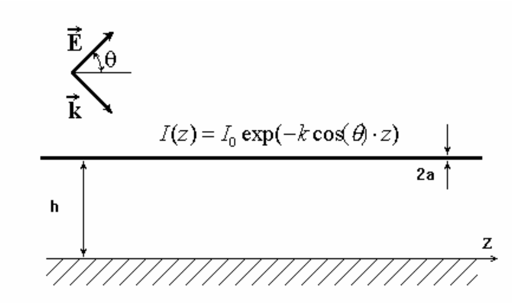


Fig.1 Sketch of the geometry for the infinite line.

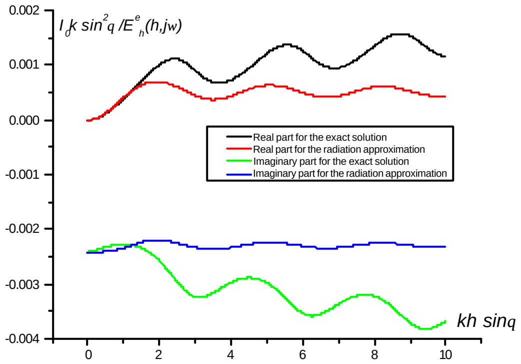


Fig 2. Frequency dependence of the normalized induced current amplitude for an infinite line. Exact an approximate solution ($a/2h = 0.001$, internal wire impedance $Z_w(jw) \to 0$).

We start with the algorithm of [4]. In a first step a zero-iteration current $I^{(0)}(z) = I_0^{(0)} \cdot \exp(-k_1 z)$ is defined by the usual telegrapher equations for the line with finite conductivity and non-zero per-unit length wire impedance $Z'_w(jw)$ [1].(Here the wave number $k_1 := k\cos(q)$ is connected with the elevation angle of the incident field, see Fig. 1. The azimuth angle j is chosen to zero). In a second step, the radiation power P'_S is calculated in the usual way using the zero - order solution $I^{(0)}(z)$. With the aim of this radiation power one can define first order radiation losses, $R'^{(1)}{}_{rad}(z)$, which are assumed to be distributed homogeneously along the line. After that these radiation losses are considered as additional ohmic losses and are added to wire resistance. Thus we get a new wire impedance, say $Z'^{(2)}_w(z)$, which can be inserted into the telegrapher equations, and we repeat the above procedure. It is interesting to note, that for an infinite line $R'^{(n)}_{rad}$ does not depend on the order of the iteration. (This is because the current amplitude is divided out in the corresponding expression for $R'^{(n)}_{rad}$). The corresponding value for the current then is given by

$$I^{(rad)}(z) = I_0^{(rad)} e^{-jk_1 z} \quad \text{with amplitude} \quad I_0^{(rad)} = E_h^e(h, jw)\cdot\left\{ jk\sin^2(q)\left[Z_C - j\frac{h_0}{4}\left(1 - J_0(2kh\sin q)\right)\right] + Z_w(jw)\right\}^{-1} \qquad (1)$$

Here $E_h^e(h, jw)$ is an amplitude of the exciting tangential electric field at the height h, $E_h^e(h, jw) = E^{inc} 2j\sin(jkh\sin q)$.

We obtained the result (1) using the procedure applied in [4]. It has to be compared with the well-known exact solution for an infinite wire above a perfect conducting plane (see [6]). For a thin wire ($ka << 1$) this solution is

$$I_0^{(exact)} \approx E_h^e(h, jw)\cdot\left\{ jk\sin^2(q)\cdot\left\{ Z_C + \frac{h_0}{4}\left[Y_0(2kh\sin q) - \frac{2}{p}\ln(g\, kh\sin(q))\right] - j\frac{h_0}{4}\left[1 - J_0(2kh\sin q)\right]\right\} + Z_w(jw)\right\}^{-1} \qquad (2)$$

$\ln(g) = C = 0.577...$ -Euler's constant

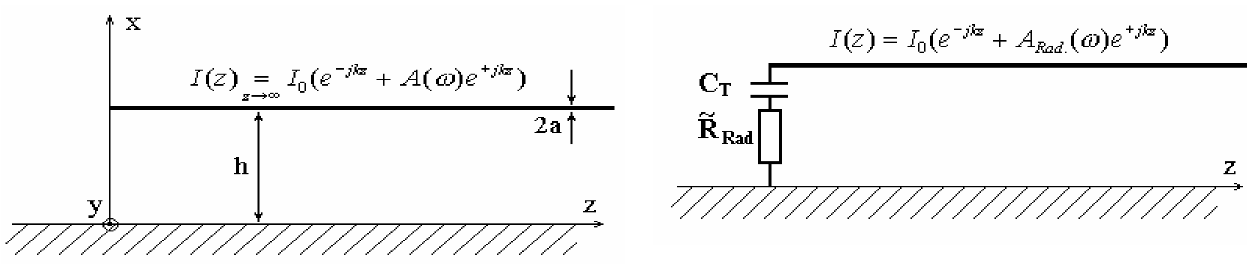
Looking at the results in (1) and (2) it becomes immediately obvious that their denominators are different. Whereas the real parts are the same, their imaginary parts are quite different and coincide only for smaller frequencies. This means that the reactive energy of the electric and magnetic fields which is stored in the close neighbourhood of the wire is different in both cases. In Fig. 2 we display these differences (we assume that $a/2h = 0.001$). From the graph we can conclude, that the results of the approximate solutions are quit satisfactory as long as $kh\sin q <\approx 1.5$ (or $h\sin q <\approx l/4$) is valid. For a height of $h = 0.5\,m$ and a radius of 1mm ($a = 1\,mm$) the agreement occurs for frequencies lower than 150 MHz ($f <\approx 150\,MHz$).

SCATTERING PROBLEM FOR CURRENT WAVES IN A SEMI-INFINITE LINE

In this Section we consider a well-conducting semi - infinite wire above the perfect conducting ground, without external exciting electromagnetic field (see Fig.3 a). There is, however, a current wave, $I_0 \exp(jkz)$, which came from $z = +\infty$ and returns after scattering at the open circuit left end of the line. Sufficiently far from the termination the current is defined by the solution of the uniform Pocklington equation for an infinite line [1]. It represents the TEM wave and does not radiate. At the far distance from the termination the current is given by

$$I(z) \underset{z\to\infty}{=} I_0(e^{jkz} + A(jw)e^{-jkz}) \tag{3}$$

where $A(jw)$ is a complex reflection coefficient. Near the end, the wave is not a TEM wave and can radiate.



a). Geometry of the physical problem b). Circuit model
Fig. 3 Sketch of the TEM current wave scattering problem for a semi-infinite line

The exact solution for the current reflection coefficient in a two – wire system was derived by Weinstein using the Wiener – Hopf technique [7]. This expression reduces in our case to the following form

$$A_{Exact}(jw) = -\exp\left\{\frac{4y}{p}\left[-\int_0^{2y}\frac{W(x)dx}{x\sqrt{4y^2-x^2}} - j\int_{2y}^{\infty}\frac{W(x)dx}{x\sqrt{x^2-4y^2}}\right]\right\}, \qquad \text{with } W(x) = \arctan\left[-\frac{J_0(ax/h)-J_0(x)}{Y_0(ax/h)-Y_0(x)}\right] \tag{4}$$

Here $J_0(x)$ and $Y_0(x)$ are zero-order Bessel functions of the first and second kind, respectively.

In [3] an approximate result for $A(jw)$ has been obtained by using a perturbation expansion in terms of the 'thin-wire parameter for a horizontal line' $1/2\ln(2h/a)$. The zero-order solution for the current and for the reflection coefficient is the usual transmission line solution. The first-order result for the reflection coefficient turns out to be

$$A^{(1)}_{Per.}(jw) = -1 + j(\ln(2h/a))^{-1}\cdot Si(2kh) + (\ln(2h/a))^{-1}\cdot(C + \ln(2kh) - Ci(2kh)) \tag{5}$$

where $Si(x)$ and $Ci(x)$ are sine and cosine integral functions, $C = 0.577...$ is Euler's constant.

For the thin wire, when $1/2\ln(2h/a) << 1$, the perturbation approach brings a good agreement with exact results for $A(jw)$ as long as the wavelength l is less or about several h, i.e., when diffraction effects are more important (see Fig.4).

Next we describe the results of application of the radiation approximation [5, 6] to the case of the TEM scattering problem for a semi-infinite line. In this case the radiation appears near the termination only. Therefore, it is necessary to introduce some effective lumped impedance $Z_{eff}(jw)$ on the end of the line (see Fig. 3b), where the radiation occurs. Then we can formally describe the scattering problem by a transmission line approximation using the standard TL expression for the reflection coefficient.

$$A(jw) = (Z_C - Z_{eff}(jw))\cdot(Z_C + Z_{eff}(jw))^{-1} \tag{6}$$

Omitting intermediate calculations, we present the result of the first iteration for $Z_{eff}(jw)$

$$Z_{eff.}(jw) \approx \frac{1}{jwC_T} + \tilde{R}_{Rad.}(w), \text{ where } C_T \approx \frac{2pe_0 h}{(\ln(2h/a))^2} \text{ and } \tilde{R}_{Rad}(w) \approx \left(\frac{\ln(2h/a)}{2kh}\right)^2 \cdot \frac{h}{p} \cdot \int_0^{2kh}\frac{(1-\cos x)}{x}dx \tag{7}$$

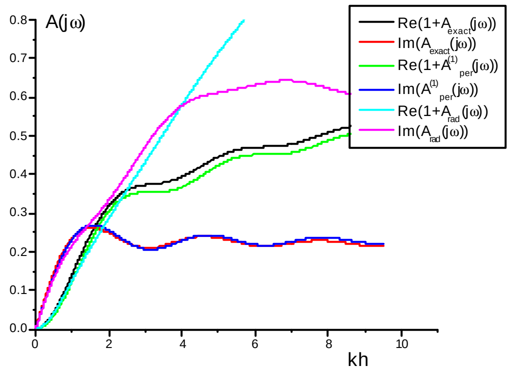


Fig. 4 Reflection coefficient of a semi-infinite wire above perfectly conducting ground

Thus the impedance $Z_{eff.}(w)$ can be represented as a sum of the capacitance and resistance (see Fig. 4 b). The first term in (7), $1/jwC_T$, corresponds to the zero order solution of the radiation approximation. This capacitance is physically caused by the differences in charge distribution in an infinite line and near the end of a semi-infinite line. This term was obtained by King [8], who considered a semi – infinite two line of wires for very small frequencies $kh << 1$. The second term in (7), $\tilde{R}_{rad}(w)$, corresponds to the first order solution of the radiation approximation. Physically it is caused by the radiation losses from the end of the line.
Numerical comparison of (6) with (7) and with the exact value (4) is shown in Fig. 4. From this figure we observe that, as in a previous case of an infinite line excited by a plane wave, the approximate theory shows a good agreement with exact results for low frequencies $kh <\approx 1.5$ (or $h <\approx l/4$).

CONCLUSION

In this paper, we obtain approximate analytical solutions by different methods for two line configurations which are basic for the high-frequency coupling with uniform and non-uniform transmission lines (plane wave coupling with an infinite line and TEM current wave reflection from the end of a semi-infinite line). We compare the results with known exact analytical solutions. For the second problem, a perturbation theory in terms of $1/2\ln(2h/a)$ leads to a good agreement with the exact solution for frequencies $kh <\approx 10$. For the both problems a radiation approach was used. It has been shown that this approach gives satisfactory results for intermediate frequencies ($kh <\approx 1.5$). Because these two problems contain main features of the coupling with uniform and non-uniform lines, we can assume, that the radiation approach can also describe coupling with long ($L >> h$) finite lines in this frequency region. Work in this direction is in progress.

We are grateful to Dr. C.E. Baum for helpful discussions.
The research is supported by the Deutsche Forshungsgemeinschaft DFG under contract number FOR 417.

REFERENCES

1. F.M. Teche, M.Ianoz, T.Karlsson, *EMC Analysis Methods and Computational Models*, Willey, 1996.
2. S. Tkachenko, F. Rachidi, M.Ianoz, "High-frequency electromagnetic field coupling to long terminated lines", IEEE Transaction on Electromagnetic Compatibility, vol. 43, No. 2, May 2001, p.117-129.
3. S.Tkatchenko, F.Rachidi, M.Ianoz, L.M. Martynov, "Exact Field - to Transmission Line Coupling Equation for Lines of Finite Length," **EMC'96 ROMA** September 17-20, 1996, Rome, Italy, pp.59-63.
4. D.O. Wendt, Ter-Hazeborg, "Radiation losses representation in the transmission line theory (TLT)", **EMC'94 ROMA**, Sept. 13-16,1994, Rome, Italy, pp.713-717.
5. D.O. Wendt, J.L. ter-Hazeborg, "Representation of the effects caused by inhomogeneities of transmission lines in the TLT", **EMC'96 ROMA**, Sept. 17-20, 1996, Rome, Italy, pp. 48-53.
6. E.F.Vance, "*Coupling to shielded cables,*" Willey , New York, 1978.
7. L.A.Weinstein,"*The Theory of Diffraction and the Factorization Method*," Chapter VI , Golem, 1969.
8. R.W.King, *Transmission Line Theory*, McGrawHill Book Comp., New York, Toronto, London, 1955.