DIFFRACTION BY A SYSTEM CONSISTING OF A GRATING AND A LAYERED CHIRAL MEDIUM

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ABSTRACT

The analytical regularization procedure derived from the Riemann-Hilbert problem method was used to solve the vector problem of diffraction of an obliquely incident elliptically polarized wave by a periodic grating attached to a layered medium when one of the layers is chiral. The numerical investigation shows a number of the new diffraction features caused by the chiral medium presence. The processes of energy redistribution between the propagating harmonics, whose amount much varies from layer to layer, as well as the resonant phenomena occurring in the structure layers affect the diffraction character.

INTRODUCTION

A chiral inclusion can do more than vary one or another characteristic of the system which takes it in. In cases, it imparts novel properties even to well-known structures, for example a cross-polarized component in the reflected field of a linearly polarized wave incident normally on an ordinary strip grating, attached to the isotropic chiral half space [1]. In view of the circular polarization of the chiral medium eigenwaves and due to the boundary conditions, the chiral medium binds both linear polarizations. On the one hand, this gives rise to the new interesting effects, but on the other hand it complicates the problem, which becomes then a vector one.

PROBLEM FORMULATION

A periodic grating of infinitely thin and perfectly conducting strips parallel to the OX axis lies in the plane $z = h_1$ (Fig.1). The grating period is l, the slot width is d. The layered medium consists of the four layers $h_1 < z$, $0 < z < h_1$, $-h_2 < z < 0$, and $z < -h_2$ with the layer permittivity ε_j and permeability μ_j, where $j = 1,2,3,4$. The $j = 3$ layer is chiral one with the chirality parameter γ.

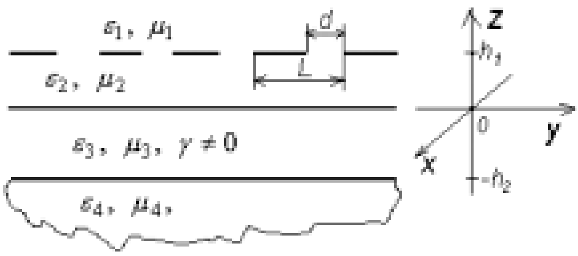

Fig.1. The structure profile

The wave $\mathbf{E}^i = \mathbf{E}_0 exp(i\mathbf{k}^i\mathbf{r})$, $\mathbf{H}^i = \mathbf{H}_0 exp(i\mathbf{k}^i\mathbf{r})$ with $\mathbf{E}_0 = (\tilde{e},0,0)$ and $\mathbf{H}_0 = (\tilde{h},0,0)$ ($\tilde{e}, \tilde{h}$ are complex) is obliquely incident on the grating so that $\mathbf{k}^i = -\omega\sqrt{\varepsilon_0\mu_0\varepsilon_1\mu_1}(0, sin\alpha, cos\alpha)$, where α is the angle between the incident wave vector $\mathbf{k}^i$ and the OZ axis. Time factor $exp(-i\omega t)$ is omitted. We seek to find the diffracted field.

In so far as the incident field is $x-$independent and the grating extends infinitely in the x direction, the problem can be solved in two-dimensional terms ($\partial/\partial x \equiv 0$). For existence and uniqueness of the solution to problem [2], the conditions to satisfy are: Maxwell's equations referred to the j th layer

$$rot\mathbf{E}=\begin{cases} i\omega\mu_0\mu_j\mathbf{H},\ j=1,2,4 \\ i\omega(\mu_0\mu_3\mathbf{H}-i\gamma\sqrt{\varepsilon_0\mu_0}\mathbf{E}),\ j=3 \end{cases} \qquad rot\mathbf{H}=\begin{cases} -i\omega\varepsilon_0\varepsilon_j\mathbf{E},\ j=1,2,4 \\ -i\omega(\varepsilon_0\varepsilon_3\mathbf{E}+i\gamma\sqrt{\varepsilon_0\mu_0}\mathbf{H}),\ j=3; \end{cases} \tag{1}$$

radiation condition; boundary conditions; quasiperiodicity condition, and condition of the field energy finiteness within any confined volume of space.

FIELD REPRESENTATION

In terms of the two-dimensional formulation and harmonic time dependence, (1) for $j=3$ yield the field relationships [3] in the homogeneous chiral medium

$$\mathbf{E}=\mathbf{E}^+ +\mathbf{E}^-,\ \ \mathbf{H}=\mathbf{H}^+ +\mathbf{H}^- = -i\left(\mathbf{E}^+ -\mathbf{E}^-\right)/\rho_3,$$

$$\Delta_{yz}u^\pm + k^{\pm 2}u^\pm = 0,\ \ E_x^\pm = u^\pm(y,z),\ \ E_y^\pm = \mp\frac{1}{k^\pm}\frac{\partial u^\pm}{\partial z},\ \ E_z^\pm = \pm\frac{1}{k^\pm}\frac{\partial u^\pm}{\partial y},$$

where $k^\pm = -k_3(1\pm\eta), k_j = \omega\sqrt{\varepsilon_0\varepsilon_j\mu_0\mu_j}, \eta=\gamma/\sqrt{\varepsilon_3\mu_3}, \rho_j=\sqrt{\mu_0\mu_j/\varepsilon_0\varepsilon_j}$. Thus all the field components are expressed through the $E_x^\pm$. Eigenwaves are right and left circularly polarized waves with the propagation constants $k^\pm$. The discussed problem requires vector approach because the sought fields have all the components.

In view of that the medium interfaces coincide with the coordinate planes, our approach to this coordinate problem solution is by the method of separation of variables. Anticipating existence of the solution, the grating periodicity along the *OY* axis enables the problem solution to be expanded into Fourier series for each structural region. Substitution the series in Helmholtz equation derived from (1) ($\Delta_{yz}u + k_j^2 u = 0$ for the $j=1,2,4$ domains and $\Delta_{yz}u^\pm + k^{\pm 2}u^\pm = 0$ for $j=3$) gives the field representation, which coincides with the Rayleigh expansion of the diffracted field as an infinite series of partial waves of spatial spectrum, and satisfies the quasiperiodicity condition. The wave propagation character is clear from the considered field representation. The wave complex amplitudes are unknown Fourier coefficients. So the problem is to find the coefficients.

DETERMINATION OF THE FIELD COEFFICIENTS

Applying the boundary conditions to each surface one can relate the sought Fourier coefficients in the partial domains and obtain the two coupled systems of dual series equations involving trigonometric functions. The obtained systems are equivalent to an operator equation of the first kind in the Hilbert space given by the Meixner condition [2]. These systems are ill-conditioned, therefore the truncation technique is generally unappreciable. The analytical regularization can help us to get rid of this ill-conditioning and arrive at the form admitting effective numerical and analytical treatment It is proved that these systems can be reduced to

$$\begin{cases} \sum\limits_{\substack{n=-\infty \\ n\neq 0}}^{n=+\infty} X_n\, exp(in\varphi)+\theta\, X_0 = 0, \quad \delta<|\varphi|<\pi \\ \sum\limits_{\substack{n=-\infty \\ n\neq 0}}^{n=+\infty} \frac{|n|}{n} X_n\, exp(in\varphi)+\theta\, X_0 = F^1,\ |\varphi|<\delta \\ \sum\limits_{\substack{n=-\infty \\ n\neq 0}}^{n=+\infty} \frac{(-1)^n}{n+\theta} X_n + X_0 = 0, \qquad \varphi=\pi \end{cases} \qquad \begin{cases} \sum\limits_{\substack{n=-\infty \\ n\neq 0}}^{n=+\infty} Y_n\, exp(in\tilde{\varphi})+\theta\, Y_0 = 0, \quad \tilde{\delta}<|\varphi|<\pi \\ \sum\limits_{\substack{n=-\infty \\ n\neq 0}}^{n=+\infty} \frac{|n|}{n} Y_n\, exp(in\tilde{\varphi})+\theta\, Y_0 = F^2,\ |\varphi|<\tilde{\delta} \\ \sum\limits_{\substack{n=-\infty \\ n\neq 0}}^{n=+\infty} \frac{(-1)^n}{n+\theta} Y_n + Y_0 = 0. \qquad \varphi=\pi \end{cases} \tag{2}$$

Here $\varphi = 2\pi\, y/l$, $\delta=\pi d/l$, $\tilde{\delta}=\pi-\delta$ (then the inequality $|\varphi|<\delta$ indicates the grating slot and $\delta<|\varphi|<\pi$ refers to the strip); $-\theta = m_0 + l\, sin\alpha/\lambda_0$, $-1\le 2\,\theta<1$, m_0 is the nearest integer number to $-l\, sin\alpha/\lambda_0$, λ_0 is the wavelength in vacuum; $F^{1,2}=\sum\limits_{n=-\infty}^{n=+\infty} f_n^{1,2}(X_n,Y_n)exp(in\varphi)$; $X_n(x_n,y_n), Y_n(x_n,y_n)$ are the linear combinations of

the unknowns x_n, y_n which are the Fourier coefficients describing the amplitudes of the n partial harmonics of the spatial spectrum in the chiral medium. The all others Fourier coefficients can be obtained though x_n, y_n.
Using the asymptotic estimates, it is possible to show that

$$f_n^{1,2} \underset{|n|\to\infty}{=} \sigma_n^{1,2} n^{-2} + O(exp(-\sigma|n|)),$$

where $\sigma = 4\pi \frac{h_1}{l} \left| 1 - \frac{l\sqrt{\varepsilon_1\mu_1}\, sin\alpha}{\lambda_0 n} \right| \left(1 - \frac{1}{2} \left(\frac{l\sqrt{\varepsilon_2\mu_2}}{\lambda_0 n - l\sqrt{\varepsilon_1\mu_1}\, sin\alpha} \right)^2 \right) > 0$, and the values $\sigma_n^{1,2}$ satisfy the conditions $\sum_{n=-\infty}^{n=+\infty} \left|\sigma_n^{1,2}\right|^2 < \infty$. These $f_n^{1,2}$ representations allow one to use the regularization method.

Taking the right-hand sides of (2) as being known allows us to assume that the obtained functional systems are separated. So for the given problem, we not only have extracted the singularity in the left-hand sides of the equations but also managed to decouple the systems for the principal part of the operator.

The obtained systems of functional equations are equivalent to the Riemann-Hilbert problem [2] for the analytical function reconstruction by its limiting values on a unit circle arc. Applying the well-known method of this problem solution to each system individually yields the following infinite system of linear algebraic equations

$$\begin{cases} \theta X_0 = \sum_{p=-\infty}^{+\infty} V_{0p}\left\{\alpha_p^0 X_p + \beta_p Y_p\right\} + b_0, \quad X_n = \sum_{p=-\infty}^{+\infty} V_{np}\left\{\alpha_p^n X_p + \beta_p Y_p\right\} + b_n, \\ \theta Y_0 = \sum_{p=-\infty}^{+\infty} \tilde{V}_{0p}\left\{\tilde{\alpha}_p X_p + \tilde{\beta}_p^0 Y_p\right\} + \tilde{b}_0, \quad Y_n = \sum_{p=-\infty}^{+\infty} \tilde{V}_{np}\left\{\tilde{\alpha}_p X_p + \tilde{\beta}_p^n Y_p\right\} + \tilde{b}_n, \end{cases} \qquad (3)$$

where the values $V_{np}, \tilde{V}_{np}$; α_p^n, β_p and $\tilde{\alpha}_p, \tilde{\beta}_p^n$ are given in [4].

From the asymptotic estimates of the coefficients $\alpha_p^n, \tilde{\beta}_p^n = O(n^{-2})$ and $\tilde{\alpha}_p, \beta_p = O(exp(-\sigma|n|))$ and from the behaviour of $V_{np}, \tilde{V}_{np}$ as $|n|,|p| \to \infty$ it follows that (3) is a Fredholm system of the second kind. As well known such a system can be solved with any necessary accuracy by means of truncation procedure, as well as initial diffraction problem. Having chosen the smallness parameter, for example l/λ_0, an analytical solution may be found by the method of successive approximations.

NUMERICAL RESULTS

Consider the problem of the normally incident ($\alpha = 0$) wave diffraction by a grating placed at $z = h_1$ to be over the interface between the chiral ($\varepsilon_3, \mu_3,\ \gamma \neq 0$ for $z < 0$, i.e. $h_2 \to \infty$) and nonchiral ($\varepsilon_1 = \varepsilon_2, \mu_1 = \mu_2, \gamma = 0$ for $z > 0$) half spaces. It is assumed that $|\tilde{e}| = 1$ and $|\rho_1 \tilde{h}| = 1$.

Of interest there will be the reflection coefficients a_0^x and a_0^y to define the field average over the grating period. Since all the field components are present, the reflection coefficients will be considered referred to the x- and y- components of the electric field to give E- and H- polarizations, respectively (E-polarization when $\mathbf{E}||OX$ and H-polarization when $\mathbf{H}||OX$). Let us call major polarization that of incidence, then the cross one is that normal to it.

Figs. 2-4 report the absolute values $|a_0^x|, |a_0^y|$ of the reflection coefficients versus $\chi = l/\lambda_0$ defining the excitation frequency. The case of simultaneous incidence of E- and H- polarized waves is considered in Fig.4, where $\delta\phi = arg(\rho_1 \tilde{h}/\tilde{e})$ and if the incident waves are in phase $(\delta\phi = 0)$, they give the plane-polarized wave, if $\delta\phi = \pm\pi/2$, the resultant is right or left circularly polarized. Fig.5 presents the reflection coefficients versus the chirality parameter, where $\chi_1^- = 1/\sqrt{\varepsilon_3\mu_3}\,(1-\eta)$.

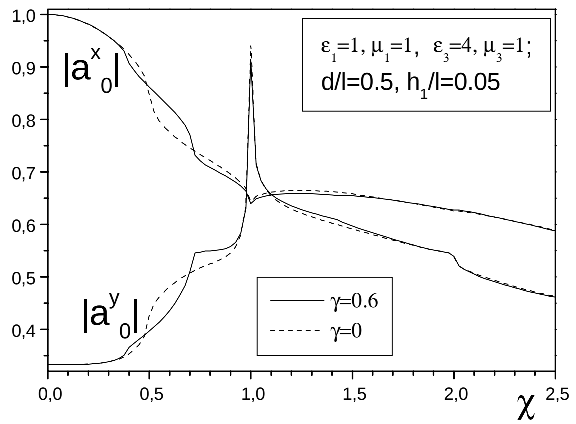

Fig.2.Major polarization

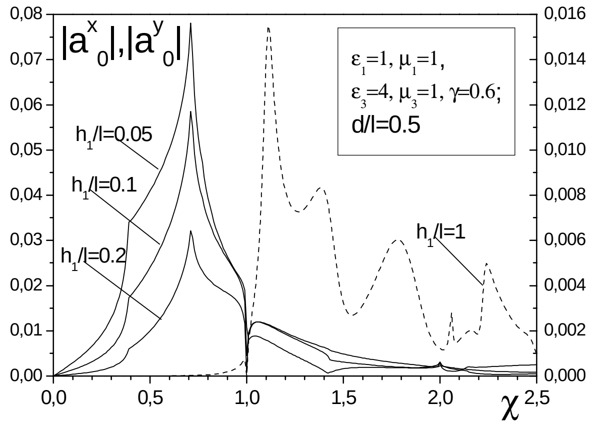

Fig.3. Cross polarization: the solid curves refer to the left axis, the dashed curve refers to the right axis

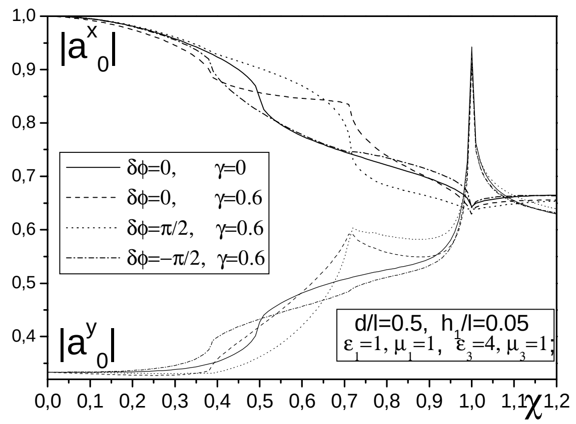

Fig.4. Simultaneous incidence of E- and H-waves

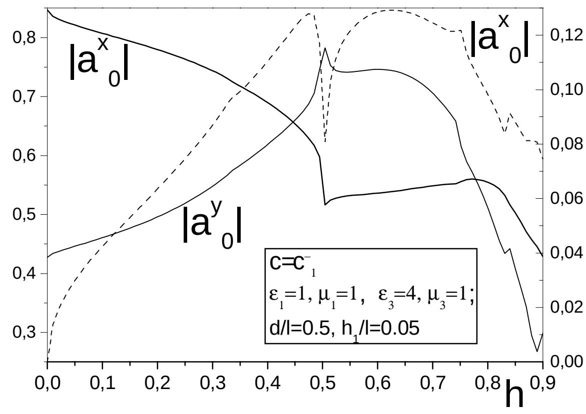

Fig.5. Major polarization (solid curves) refer to the left axis; cross polarization (dashed curve) to the right axis

CONCLUSION

The analytical regularization procedure for solving the vector problem considered has been constructed. Our approach based on the Riemann-Hilbert problem method lends as a reliable and effective tool. There has been revealed a series of the new diffraction features associated with the chiral medium presence.

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