THE ACTION OF NON-LINEAR EFFECTS IN A RESONATOR

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ABSTRACT

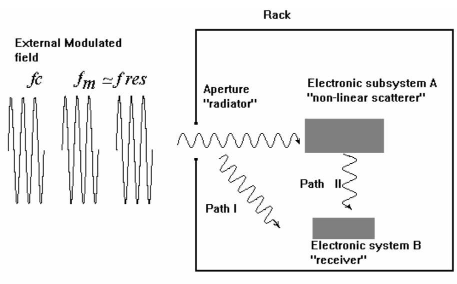
Experimental results for the excitation of a high-quality resonator system (mode stirred chamber) by repetitive sinusoidal pulses are presented. An empty as well as a cavity loaded by a non-linear scatterer are examined. In both cases, a strong low-frequency cavity excitation (correspondingly 30 and 85 dB) was observed when an integer multiple of the repetition frequency equals the lowest cavity resonance frequency. This additional strong effect for the non-linear loaded chamber was explained by the demodulation of the scattering current (which also exciting the resonator) in the non-linear scatterer.

INTRODUCTION

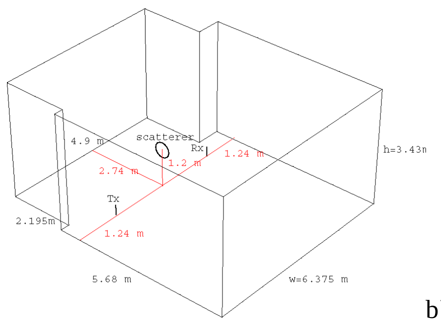
It is well known that the induced currents and voltages on semiconductor components of an electronic system may cause their upset or damage. This also holds if the interference signal contains lower frequencies [1], [2]. Usually, an electronic system is placed into a well shielded cavity which preferably can be excited by high frequency signals penetrating through slots and apertures [3]. Therefore, pulse modulated high frequency signals or repetitive unipolar pulses are used to achieve maximum coupling with an electronic system [2]. In particular, the non-linearly loaded circuits, excited by modulated pulse trains, exhibit a so-called high-to-low frequency conversion. Due to these peculiarities of non-linear systems an electromagnetic environment as well as coupling processes will be remarkably modified (see Fig. 1a). It is our main objective to describe this phenomenon in our paper.
To model this effect we experimentally examine and theoretically analyze the excitation of a rectangular cavity by a pulse modulated high frequency cw signal. An empty as well as a non-linearly loaded cavity was considered. In both cases, a strong low-frequency cavity excitation was observed when an integer multiple of the repetition frequency equals the lowest cavity resonance frequency. The effect is about 30 dB for the empty cavity. Another, additional 55 dB amplification of the resonating chamber was reached, when a non-linearly loaded loop was put into the chamber. We have explained the observed effect by the demodulation of the scattering current current in the non-linear scatterer.

EXPERIMENT

All presented measurements have been performed in the Magdeburg mode-stirred chamber with the mode stirred removed. The room dimension are $7.875\,\mathrm{m} \times 6.375\,\mathrm{m} \times 3.430\ \mathrm{m}$ ($l \times w \times h$, see Fig. 1 b). The base is not rectangular but is narrowed at one side. The lowest resonant frequency of the chamber is $f_1 = 30.78$ MHz. The corresponding quality factor is $Q(f_1) = f_1/\Delta f_{3dB} = 1424{,}8$.



a)



b)

Fig. 1: a). Electronic system in a cavity. b). Sketch of the experimental setup. Tx: radiating antenna (33 cm). Rx: receiving rod antenna (97 cm) The diameter of the non-linearly loaded loop (scatterer) is 40.7 cm.

The room is exited by a small rod antenna (Tx, $L = 0.33$m, $r_0 = 1$ mm). The signals are generated by a signal generator with pulse modulation from an external source. The carrier frequency was adjusted to $f_c = 900$ MHz for all shown results. The modulation frequency $f_m = 1/T_m$ (f_m: pulse repetition frequency, PRF; T_m: pulse repetition period) was changed over a wide range up to 20 MHz. Duty cycle was 50% always. Upper PRF limit and duty cycle are due to instrumentation limits, both. The chamber E-field was measured by means of a second rod antenna (Rx). The length of this antenna was increased to $l = 0.97$ m in order to achieve higher sensitivity. The antenna was connected to EMI receiver via a well shielded coaxial cable.

For the first part of experiments the scatterer was absent. The lower curves of Fig. 2 shows the results of our investigations of the coupling of the pulse modulated signal with the lowest empty - chamber cavity mode. For this experiments the PRF f_m was chosen such, that $n \cdot f_m$ (n is indicated in the figure; $2 \le n \le 10$) is in the vicinity of the lowest resonant frequency f_1. As expected, a peak in the measured spectrum occurs at $n \cdot f_m$ for each PRF f_m. The amplitudes of these peaks are plotted versus the frequency position. All these frequency - amplitude pair for a given n (n-th harmonic of PRF f_m) are combined to one curve. The measurements show, that the low frequency spectral component strongly increase, if n times the PRF f_m. The effect most pronounced for $n = 2$ with an accentuation of about 30 dB.

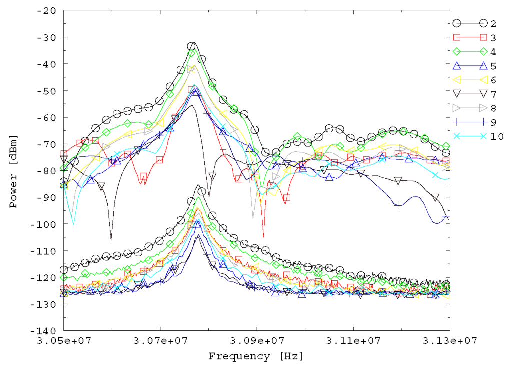


Fig. 2 Maxima of low frequency component for different PRF with (upper curves) and without (lower curves) non-linear scatterer. Symbols are plotted for every 10th data.

In the second series of the experiments we placed a non-linear scatterer in the chamber between radiating and receiving antenna (see Fig.1 b). (Obtained results are qualitatively independent of scatterer position). The scatterer is a loop made from 8 in series low capacitance Schottky barrier diodes separated 16 cm ($\approx l/2$) from each other. Thus, a loop diameter is about 40.7 cm. A sketch of the experimental setup is given in Fig.1. As before, the spectrum was measured for different PRF f_m. The results are plotted in the upper curves of Fig.2. The shape of the curves is more complicated, but the maxima occur if $n \cdot f_m = f_1$ as in the empty chamber curves shown in the same graph. Comparing to the empty chamber curves, a strong enhancement of low frequency spectral component was measured. The corresponding values for different are given Table I for $f = f_1$.

Table 1. Enhancement of the low frequency component due to presence of a non - linear scatterer (NLS).

N	2	3	4	5	6	7	8	9	10
with nls, dBm	-31.96	-48.08	-34.61	-50.20	-40.47	-55.50	-40.63	-49.23	-47.64
Without nls, dBm	-85.39	-94.23	-89.98	- 98.95	-93.66	-105.01	-95.95	- 104.105	-98.17
Δ dB	53.43	46.15	55.37	48.75	53.19	49.51	55.32	54.82	50.53

As shown, the amplitude of low frequency spectral component can be enlarged by about 30 dB by adjusting the PRF such, that $n \cdot f_m = f_1$. An additional increase of up to 55 dB was achieved by putting a simple non-linear scatterer into the chamber. Both effects together give an 85 dB gain for the low-frequency component. The relative amplitude compared with the carrier frequency is -18 dBc.

QUALITATIVE THEORY

Consider a resonator ($a \times b \times h$) exited by a small vertical monopole along z-axis located in point $\vec{r}_0$ (see Fig.2). Assume that the antenna is fed by a harmonic signal modulated with a rectangular pulse chain:

$$U(t) = U_0 F(t)\,, \qquad F(t) = \sum_{n=0}^{N} f(t - nT_m)\,, \qquad f(t) = \sin w_c t \cdot h(t) \cdot h(T_p - t) \qquad (1)$$

where $N >> 1$, $T_p \equiv N_p T_c$, $T_c \equiv 2p/w_c$, and $h(t)$ is the unit step function. The currier frequency w_c is assumed to be much larger than the lowest resonance frequency w_1. Then the response of the resonator can be represented can be a set of independent oscillators which are exited by external forces:

$$E_z(\vec{r}_1,t)=\sum_{\nu}E_{\nu,z}(\vec{r}_1,t)\,,\qquad \left(\partial^2/\partial t^2+2\gamma_\nu\,\partial/\partial t+\omega_n^2\right)E_{\nu,z}(t,\vec{r}_1)=E_{0,n}\omega_{\nu,r}^2\,F(t) \qquad (2)$$

Here, $E_{\nu,z}(\vec{r}_1,t)$ are the electric field mode terms, $\gamma_\nu=\omega_\nu/(2Q_\nu)$ the modal damping factor, $\omega_\nu=c\sqrt{k_{\nu,x}^2+k_{\nu,y}^2+k_{\nu,z}^2}$, $k_{\nu,x}=\pi n_x/l$, $k_{\nu,y}=\pi n_y/w$, $k_{\nu,z}=\pi n_z/h$, $|\nu\rangle=|n_x n_y,n_z\rangle$, $E_{\nu,0}=-U_0\cdot C_A\cdot\Psi_\nu(\vec{r}_1)\Psi_\nu(\vec{r}_0)\cdot L/2\varepsilon_0\sim U_0/L\cdot L^3/V$, $\Psi_\nu(\vec{r})=\sqrt{2(1+\delta_{n_z,0})/V}\,\sin(k_{\nu,x}a)\sin(k_{\nu,y}b)\cos(k_{\nu,z}h)$, $C_A\approx 2\pi\varepsilon_0 L/(\ln(2L/r_0)-2)$ the antenna capacitance, r_0 is the antenna radius.

Now, we consider a periodic excitation function $F(t)$ which a repetition frequency ω_m.

When the lowest resonance frequency ω_1 of the cavity is an integer multiple of the repetition frequency $\omega_m\equiv 2\pi/T_m$, approximately, i.e. $\omega_1\approx n\omega_m$, then only the term with $|\nu\rangle=|1,1,0\rangle$ is important for the low-frequency part ($\omega<<\omega_c$) of the electric field. The solution for the for $E_{z,1}(t,\vec{r}_1)$ can then be written as convolution integral

$$E_{1,z}(t)=E_{1,0}\cdot\omega_1^2\cdot\int_0^t F(t')K_1(t-t')dt',\ \text{where}\ \ K_1(t)=\tilde{\omega}_1^{-1}\exp(-\gamma_1 t)\sin(\tilde{\omega}_1 t)\eta(t)\,,\ \ \tilde{\omega}_1=\sqrt{\omega_1^2-\gamma_1^2} \qquad (3)$$

First, we consider the excitation by a single pulse $U\cdot f(t)$, $0\le t\le T_m$. During the excitation, the solution is a sum of forced oscillations (ω_c) and eigen-oscillations ($\tilde{\omega}_1$). After T_p the system oscillates with its eigen frequencies. It is necessary to note that during the integration we have to integrate fast oscillating function $f(t)$ with slowly oscillating function $K_1(t)$ (see Fig. 3) that brings addition smallness for the eigen-oscillations as $\sim\omega_1/\omega_c$. After that it is easy to calculate an action of a chain of N rectangular wave packages (1), using an approach which was proposed recently [4] (by the summarizing corresponding geometrical series). Assuming that the time t is in an eigen-oscillations zone $T_m\cdot(N-1)+T_p<t<T_m\cdot N$ we get

$$E_{1,z}(t)=E_{\nu,0}\,\mathrm{Im}\left[\frac{\omega_1^2\cdot\omega_c/\tilde{\omega}_1}{\omega_0^2-2j\tilde{\omega}_1\gamma_1-\omega_1^2+2\gamma_1^2}\cdot\left(1-e^{-j\Omega_1 T_p}\right)e^{j\Omega_1 t}\frac{1-e^{-j\Omega_1 NT_m}}{1-e^{-j\Omega_1 T_m}}\right],\ \text{where}\ \ \Omega_1=\tilde{\omega}_1+j\gamma_1 \qquad (4)$$

For $\gamma_1 T_m>>1$ the system respond to the last pulse only. For $\gamma_1 T_m\le 1$ eigen-oscillations of several previous pulses add together. If the repetition frequency is tuned to the system's resonance frequency, i.e. $\tilde{\omega}_1=2\pi/T_m$, the eigen oscillations are in the same phase, and resonance excitation of the system is observed. The system response increases for $\gamma_1 T_m N\to 1$. For $\gamma_1 T_m N\approx 1$ a saturation of the system response occurs. From the (4) it is possible to see that in the near resonance frequency region $\omega_m=\omega_1/n+\Delta\omega$ the detuning frequency dependence for the amplitude of a low – frequency oscillations in a resonator has a Lorentz resonance form with maximum value $\left(E_{\nu,z}(t,\Delta\omega=0)\right)_{\max}\sim E_{1,0}\omega_1^2/2\pi\, n\omega_c\gamma_1=E_{1,0}\omega_1 Q_1/\pi\, n\omega_c$ and width of the resonance $\Delta_{3dB}((E_{1,z}(t))_{\max}(\Delta\omega)\approx\gamma_1/n$. For the high quality resonances (large Q) this is much larger than the response in the non-resonant case $\left(E_{\nu,z}(t)\right)_{\max}\sim E_{\nu,0}\omega_1/\omega_c$. These equations give a qualitative description of the experimental results: a strong resonance increase of the low-electric field in the chamber is observed when $f_1=n\,f_m$, these simple formulas approximately explain the differences between resonant and non –resonant values and relative position of the maximums in the Fig. 2. The more rigorous description of the linear resonator exiting by the repetitive illumination can be was done by the method of electrically small antenna [5].

Similar to the case of the chamber excitation by a (linear) radiating antenna, also the excitation of the chamber by a scattering current induced in a non-linear scatterer (non-linearly loaded loop) can be seen to be physically equivalent to an excitation of a set of oscillators, driven by external force, which is proportional to the time derivative of scattering current (the corresponding equation look like (2) with some changes). Then the solution results in a convolution integral of driving force, with kernel $K_1(t)$ (see (3)).

The essence of the used non-linear scatterer is the fact that the current propagates along the wire essentially different in different directions. For the simplest case of an electrically small loop ($c/f_c>>2R_0$, R_0 is a radius of the loop) loaded with an ideal diode, the current $J_s(t)$ induced by an incoming magnetic field $H^{inc}(t)$ (by numerical calculations it can be shown for the small loop, that reflected fields from the resonator walls can be omitted), turns out to be

$$J_s(t) = m_0 S/2L_a \left[\left|H^{inc}(t)\right| - H^{inc}(t)\right]\cos(a) \qquad (5)$$

Here S depends the loop area, L_a is the inductance of the loop, and a is an angle between the incoming magnetic field and the loop's normal vector.

The convolution integral (3) now contains a slowly changing function $K_1(t)$ and a quickly oscillating current function (5). But, unlike in the linear case, we have to take into account the negative part of the fast oscillating function. Therefore, the excitation of the chamber (resonator) by the non-linear loop is about the ratio (w_c/pw_1) larger than in the case of an excitation by the same linear scatterer (just the loop). In addition, a periodical repetition of the excitation pulse, which observes the condition $n \cdot f_m = f_1$, leads to the enhancement (like in the case of empty chamber) of the chamber response.

Using a simple coupling model of a radiating antenna with non-linear scatterer (we assume a direct coupling of the radiating antenna with small non-linear loop and don't take into account the self-action of the loop scattering current by signal reflection from the cavity walls) it can be shown, that the ratio of the amplitudes of electric field in the same point $(x_1, y_1, 0)$ with and without non-linear scatterer (which is positioned in the point (x_1, r_0, R_0)) is

$$\frac{\left(E_{1,z}(\vec{r}_1,t)\right)_{nlsc}}{\left(E_{1,z}(\vec{r}_1,t)\right)_{empty}} \approx \frac{f_{sc}(w_c, a, q=0)}{r_0}\cdot\frac{w_c}{w_1}\cdot\frac{2}{p}\cdot\frac{w_{1,y}}{w_1}\cdot\frac{\cos(k_{1y}y_1)}{\sin(k_{1y}y_1)} \sim \frac{f_{sc}(w_c, a, q=0)}{r_0}\cdot\frac{w_c}{w_1} \qquad (6)$$

where $f_{sc}(w, \vec{n}) = p\, k^2 R_0^3 \cdot \cos(a)\sin(q) / 4\left(\ln\left(2R_0/r_0\right) - 2\right) \sim k^2 R_0^3$ is a scattering amplitude of electromagnetic field by a non-loaded small loop in a free space (a is an angle between the normal of the loop and incident magnetic field, q is an angle of scattering).

From (6) it is possible to see that the amplitude of an electric field exited in a chamber containing an additional nonlinear scatterer can be much larger comparing with the case of empty chamber. This is observed in experiment. On another hand, the dependence of the electric field amplitude from frequency detuning Δw in this simple approximation has the same form as for the excitation of an empty chamber. This is also approximately satisfied in the experiment (see Fig.2). Of course, the result in (6) depends on the used simple model. For electrically small scatterers we expect smaller amplitudes than for largest ones. Since we used a larger scatterer in our experiment, the value, obtained in (6) is too small.

CONCLUSION

We have exited strong low frequency oscillations in a high quality resonator by a burst of pulses with a high carrier frequency and a pulse repetition rate which have chosen as an integer part of the lowest chamber resonance frequency. In this way we obtained an increase of the chamber excitation of about 30 dB. The exited low frequency oscillations amplify dramatically – about 55 dB, when a non-linearly loaded loop was put into the chamber. This happened due to the demodulation of the scattering current in the non-linear scatterer. Our result can be generalized to the behaviour of more complex non-linear electronic equipment, sitting in a well-conducting box. Moreover, this new interaction mechanism opens the way for more critical look at equipment excitations in resonators, including the interpretation of electromagnetic effects caused in mode-stirred chambers.

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