PHASE RESONANCES IN DIFFRACTION GRATINGS

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ABSTRACT

We give numerical evidence of resonances that appear in infinite perfectly conducting gratings comprising a finite number of grooves in each period, when illuminated by a normally incident *p*-polarized plane wave. The resonances appear when a particular distribution of the phase of the electromagnetic field inside the cavities takes place, and are identified as sharp peaks in the specularly reflected efficiency. These phase resonances are accompanied by a significant intensification of the interior field. We explore and discuss the influence of the geometrical parameters of the structure in the generation of these resonances.

INTRODUCTION

The scattering from structures comprising a finite number of elements has been lately studied in connection to the superdirective property [1-6]. In these references the authors consider different structures formed by arrays of passive elements such as cylindrical dipoles [2], slotted cylinders [5] or rectangular cavities [6] to be used as superdirective antennas. The occurrence of superdirectivity can be attributed to the excitation of modes inducing a phase reversal in the adjacent scatterers. It has been found that when the structure is illuminated by *p*-polarized waves (magnetic field perpendicular to the plane of incidence) of certain resonant frequencies, there is a very high level of stored electromagnetic energy in the interior of the array elements [5,6]. This enhancement is associated with the excitation of high-Q resonances in the system [2] and is accompanied by a particular distribution of the phase of the field in the structure. In such a case, the phase difference between adjacent cavities is 0 or ð radians.

In a recent paper [7], we provided numerical evidence of a resonant effect that appears in infinite periodic gratings formed by a group of grooves in each period (compound gratings). When the structure is illuminated by a *p*-polarized plane wave of certain wavelengths, the phase of the electromagnetic field inside the corrugations is distributed in such a way that generates a significant enhancement of the interior field, and, at the same time, the specular efficiency is maximized. The resonances described in [7] (phase resonances) could be then added to the list of already known anomalies that occur in infinite gratings. These anomalies can be classified depending on their origin: i) the excitation of surface polaritons in metallic shallow gratings [8-10]; ii) Rayleigh anomalies, that occur when a new diffracted order appears [11]; iii) the surface shape resonances that depend strongly on the particular profile of the grooves, and influence the reflected pattern mostly when the cavities are deep (see [12]-[16] for lamellar profiles and [17,18] for multivalued profiles).

To study the phase resonances in compound gratings we use the modal approach. This simple and efficient method proves to be particularly suitable for the rectangular profile. It allows us to understand the resonant character of the grating in terms of the eigenmodes of the grooves, which are known to play an important role in the generation of the phase resonances. Besides, since this shape is easy to manufacture, we believe that experiments acknowledging these resonances could be performed.

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The phase resonances in infinite compound gratings could be used in practical applications which involve selective processes, such as polarizers and filters. Since we are interested in exploring this possibility, we investigate the behavior of the phase resonances when the different parameters of the system are varied.

THE COMPOUND GRATING

We consider a grating with rectangular grooves and analyze the evolution of the phase resonances when varying their depth (h) and width (a), the distance between grooves (b), and the period (d) of the grating. In particular, we have performed a detailed study for a grating with five grooves per period. The configuration of the diffraction problem is sketched in Fig. 1. A p-polarized plane wave impinges normally upon the compound grating, which consists of a finite number of grooves in each period. The formalism used to solve the diffraction problem from an infinite compound grating is an extension of the modal method presented in [12] for a perfectly conducting grating with rectangular grooves (see [7] for details on the method).

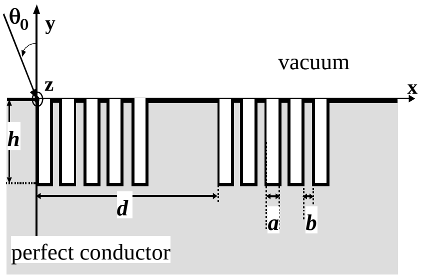


Fig. 1. Configuration of the problem

We study in detail the dependence of the phase resonances with the geometrical parameters of the grating. We found that only for p polarization there are certain resonant wavelengths associated with differences of π or 0 rad between the phases of the magnetic field inside the cavities (Fig. 2(b)). Such resonances appear as sharp peaks in the plots of the specular efficiency and the modal amplitude versus kh, where $k = 2\pi / \lambda$ (Figs. 2(a) and 2(c)). We call π resonance to the resonance in which the fields in adjacent grooves are in counterphase. In Fig. 2 it corresponds to $kh \approx 1.44$, where the most narrow peak is located.

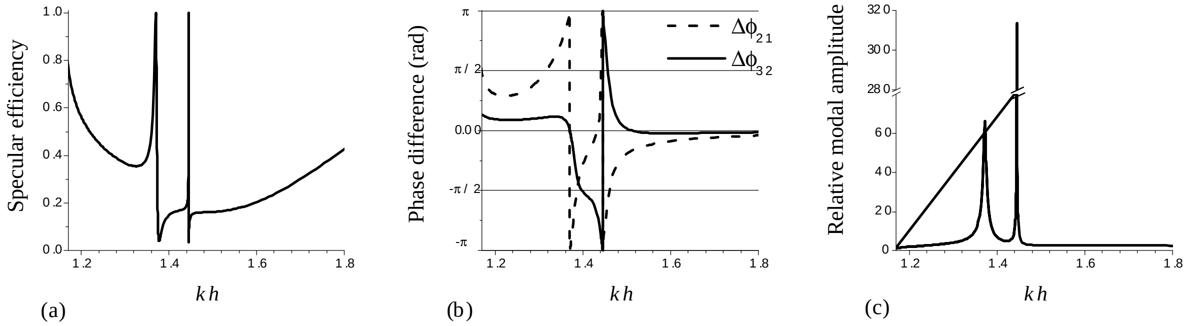


Fig. 2. Grating with N = 5, a/h = 0.2, b/h = 0.1 and d/h = 5.4. (a) Specular efficiency versus kh; (b) Phase difference versus kh; (c) Relative modal amplitude versus kh.

We studied the dependence of the resonant peak on the depth of the grooves, by keeping the other parameters fixed. We observed that as the ratio h/a increases, the resonant value of kh for both resonances become closer to each other and verges on $\pi / 2$ (not shown). As the depth is increased the electromagnetic field tends to be confined and enhanced inside the grooves, whereas outside them the fields keep roughly the same values. In the limit case of $h/a \to \infty$ the value of the resonant wavelength is $4h$. The quality of the resonance can be evaluated not only by its maximum modal amplitude but also by the width of the peak. Better quality resonances are observed when the two phase resonances existing for N = 5 become closer to each other, which is the case for large values of h/a. Therefore, we conclude that the quality of the resonance is strongly dependent on the h/a ratio.

On the other hand, we have also observed that for *h/a* < 3 the quality of the resonance decreases and the peak of specular efficiency becomes wider and lower than unity.

In Fig. 3 we show the contour plots of the magnetic field in the vicinity of the structure corresponding to four situations in which *b/a* = 0.5, *d/a* = 27, λ/*a* = 17.75, and *h/a* = 3, 4, 5, 8. Taking into account these parameters, we have three propagating orders in all the figures. However, the wavelength is resonant only in Fig. 3(b). In this case the efficiency of the specular order is ≈1, and in the far field we see the interference pattern between the incident and the reflected waves. A similar effect can be observed in Fig. 3(d), where the depth of the grooves is approximately half of a wavelength, which is close to that of the next π resonance ($kh \approx 3\pi/2$). On the other hand, a different behaviour can be observed in Figs. 3(a) and 3(c), where the specular efficiency is less than 1, and consequently the power diffracted in the directions of the +1 and –1 orders is not negligible. Therefore, the interference pattern of four waves is obtained. It must be noticed that the internal field in the resonant case (Fig. 3(b)) is strongly intensified and the amplitudes of the evanescent waves are sufficiently large to modify the interference pattern close to the structure.

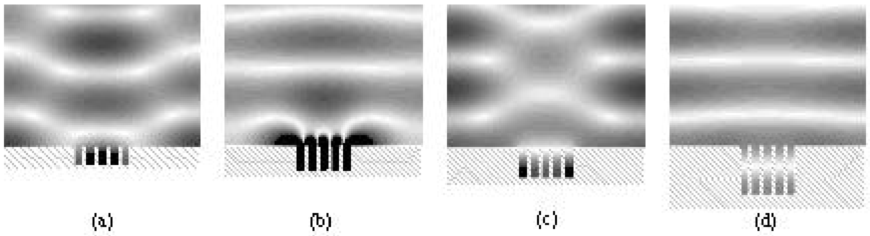
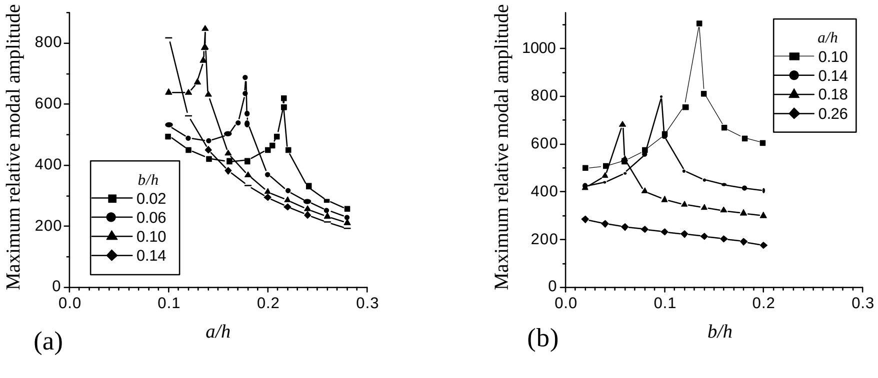

(a) (b) (c) (d)

Fig. 3. Contour plots of the relative magnetic field in the vicinity of the structure for N= 5, *b/a* = 0.5, *d/a* = 27 and λ/*a* = 17.75. (a) *h/a* = 3; (b) *h/a* = 4 (resonant situation); (c) *h/a* = 5; (d) *h/a* = 8.

We also consider variations of the width and the distance between grooves. To do this we keep the occupancy factor constant, which is the rate of the corrugated zone in relation to the flat one. We found that the position of the *kh* value for the π resonance remains almost constant when *d* is changed, but the maximum field intensity depends strongly on the occupancy factor [19]. Then, in the following analysis we keep the occupation factor fixed.

We analyze the influence of *a/h* and *b/h* on the maximum relative modal amplitude, what is shown in Figs. 4(a) and 4(b). We have mentioned above that as *h/a* increases, the quality of the resonance is improved, and this is so if the number of propagating orders remains constant. However, in Fig. 4(a) we observe that the curves show quite a different behaviour. Starting from *a/h* = 0.28 (where there are three propagating orders for all values of *b/h* considered) and going downwards, the modal amplitude grows up to a maximum, which is always found in the value of *a/h* where the diffracted orders +1 and -1 propagate at grazing angles. From that point onwards, there is only one propagating order, and the relative modal amplitude reaches a minimum, but as we expected, it increases when *a/h* falls to zero. Therefore, we conclude that if we keep the number of propagating orders fixed, we obtain the best quality of the resonance for the minimum value of *a/h*. The curves of maximum modal amplitude vs. *b/h* (Fig. 4(b)) show that if there are three propagating orders, a decrease in *b/h* improves the resonance. On the contrary, if there is only one propagating order, a decrease in *b/h* worsens the quality of the resonance.



(a) (b)

Fig. 4. Maximum relative amplitude of the fundamental mode at the π resonance for a grating with N = 5 and *d/h* = 5.4, (a) versus *a/h* for several values of *b/h*; (b) versus *b/h* for several values of *a/h*.

CONCLUSION

We have analyzed the phase resonances that take place when a *p*-polarized plane wave impinges on a diffraction grating comprising several grooves per period. The study was focused on finding the influence of the geometrical parameters of the grating, taken separately, on the characteristics of the resonances. We found that when a phase resonance occurs, both the reflected efficiency and the amplitude of the field inside the grooves are maximized, and the phase difference between the fields at adjacent grooves is 0 or π radians. If the number of propagating orders is fixed, an increase in the depth of the grooves produces a more significant enhancement of the internal field and a narrowing of the efficiency peak, which improves the quality of the resonance. We also found that in resonant conditions the near field is modified.

We studied the dependence of the modal amplitude at the π resonance on the width and on the distance between grooves for a constant occupancy factor, and found that for three propagating orders, the best resonance is obtained when the diffracted orders +1 and –1 propagate at grazing angles.

Since we are interested in the application of these resonances in selective devices, we are now modelling a more realistic metallic grating with finite conductivity. The results of this study will be the subject of a future work.

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