MIMO SYSTEM ESTIMATION BASED ON SECOND-ORDER SPECTRA CORRELATIONS APPLIED TO ESTIMATION OF SPARSE CHANNELS

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ABSTRACT

We consider blind estimation of a MIMO system with a sparse impulse response, excited with cyclostationary inputs with known statistics. Such systems appear in cellular communications, HDTV, and underwater acoustic communications. We employ the blind system estimation method of [1], which yields each impulse response sample as the minimum of a function constructed based on frequency domain correlations of the system output. We propose a modification that exploits the sparseness of the channel.

1. INTRODUCTION

The blind identification of a $P \times M$ Multiple-input multiple-output (MIMO) system is of great importance in many applications, such as speech restoration in the presence of competing speakers, bioengineering, and multi-user multi-access communications. However, most of them become very inefficient and/or inaccurate in the case of sparse channels. Sparse channels are very often encountered in cellular communications, HDTV, and underwater acoustic communications, and characterized with large delay spreads but only a few non-zero taps. One reason for such channels being a challenge is the increased complexity introduced due to the increased channel length. Another reason is the large number of zeros in the convolution matrix, which is basis of time-domain methods. These zeros render time domain methods very sensitive. As a matter of fact, dealing with sparse channels is very difficult even in the case of single-input single-output (SISO) systems. Some results on estimation and/or equalization of sparse channels can be found in [3], [5], [4] and [2].

In this paper we consider the problem of blind MIMO identification of sparse channels for the case of cyclostationary inputs with known input statistics. For example, antenna-array CDMA system with spatial and/or temporal diversity can be formulated as such MIMO system. Some existing second-order statistics based algorithms for this problem can be found in [9], [10], [7], [1]. We here employ the method of [1], which yields each impulse response sample as the minimum of a function constructed based on frequency domain correlations of the system output. We propose a closed form solution to the problem based on frequency domain correlations, which by itself is not very robust, however, it can be used as initialization in the adaptive scheme of [1]. We also propose simple modifications of the algorithm in order to better exploit the sparseness of the channels.

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2. PROBLEM FORMULATION

Let us consider a $P-$input $M-$output linear time invariant FIR MIMO system with a $(M \times P)$ impulse response matrix $\mathbf{h}(l) = \{h_{ij}(l)\}$, where $h_{ij}(l)$ denotes impulse response between the $i-$th output and the $j-$th input. Let $e_j(k)$ and $x_i(k)$ be the $j-$th input and $i-$th output, respectively, with k denoting discrete time, and define $\mathbf{e}(k) \triangleq [e_1(k) \cdots e_P(k)]^T$ and $\mathbf{x}(k) \triangleq [x_1(k) \cdots x_M(k)]^T$. The system output vector equals:

$$\mathbf{x}(k) = \sum_{l=0}^{L-1} \mathbf{h}(l)\mathbf{e}(k-l) \quad (1)$$

where L denotes the length of the longest element of $\mathbf{h}(l)$ and the inputs are given as:

$$e_i(k) = \sum_{l=0}^{L_c-1} c_i(l)s_i(k-l), \; i = 1, ..., P \quad (2)$$

Here $s_i(k)$ is a cyclostationary process, white within a cyclic period, and $c_i(k), = 0, .., L_c - 1$ is the corresponding input color.

In the frequency domain it holds:

$$\mathbf{X}(\omega) = \mathbf{H}(\omega)\mathbf{E}(\omega) \quad (3)$$

where ω denotes continuous frequency in $[0, 2\pi]$, $\mathbf{X}(\omega)$ and $\mathbf{E}(\omega)$ are the Discrete-Time Fourier Transforms (DTFT) of respectively $x(n)$ and $\mathbf{e}(n)$, and $\mathbf{H}(\omega)$ is a $M \times P$ matrix whose ij-th element is the Fourier Transform of the unknown filter $h_{ij}(l)$, $l = 0, ..., L-1$, i.e.,

$$\mathbf{H}(\omega) = \sum_{n=0}^{L-1} \mathbf{h}(n)e^{-j\omega n} \quad (4)$$

We make the following assumptions:

(A1) The inputs $\{s_j(k)\}$ are unknown, cyclostationary, temporally white, and pairwise uncorrelated. For simplicity we will also assume that they have unit variances.

(A2) The input colors are known and pairwise non-identical.

(A3) $\mathbf{H}(\omega)$ is full column rank for all ω's in $[0, 2\pi]$.

For the covariance matrix of $\mathbf{X}(\omega)$ it holds:

$$\begin{aligned} \mathbf{R}_x(\omega_1, \omega_2) &= E\{\mathbf{X}(\omega_1)\mathbf{X}(\omega_2)^H\} \\ &= \mathbf{H}(\omega_1)\mathbf{R}_e(\omega_1, \omega_2)\mathbf{H}(\omega_2)^H \end{aligned} \quad (5)$$

where the superscript ^{H} denotes Hermitian transpose, and $\mathbf{R}_e(\omega_1,\omega_2)$ is a diagonal matrix denoting the covariance of $\mathbf{E}(\omega)$.

Evaluating (5) for $\omega_1=\omega_2=\omega$ and also for $\omega_1=\omega,\ \omega_2=\omega+\alpha$, we obtain:

$$\mathbf{R}_x(\omega,\omega)=\mathbf{H}(\omega)\mathbf{R}_e(\omega,\omega)\mathbf{H}(\omega)^H \tag{6}$$

$$\mathbf{R}_x(\omega,\omega+\alpha)=\mathbf{H}(\omega)\mathbf{R}_e(\omega,\omega+\alpha)\mathbf{H}(\omega+\alpha)^H \tag{7}$$

For cyclostationary non-white inputs with non-identical colors and known covariance, it was shown in [1] that equations (6) and (7) suffice for the recovery of $\mathbf{H}(\omega)$ within a complex diagonal matrix. The solution was obtained by matching the second-order spectra correlations and minimizing the following error with respect to the impulse response matrix $\mathbf{h}(n)$, for all n:

$$\Gamma_f \triangleq \sum_{k=0}^{N-1} \parallel \mathbf{D}_1(k) \parallel_F^2 + \sum_{k=0}^{N-1} \parallel \mathbf{D}_2(k;f) \parallel_F^2 \tag{8}$$

where $\mathbf{D}_1(k)$, $\mathbf{D}_2(k;f)$ are samples of $\mathbf{D}_1(\omega)$, $\mathbf{D}_2(\omega)$ obtained at $\omega=\frac{2\pi}{N}k$, $k\in[0,N-1]$, with

$$\begin{aligned}\mathbf{D}_1(k) &\triangleq \hat{\mathbf{R}}_x(k,k)-\mathbf{H}(k)\mathbf{R}_e(k,k)\mathbf{H}(k)^H\\ &= \hat{\mathbf{R}}_x(k,k)-\sum_{m=0}^{L-1}\sum_{n=0}^{L-1}\mathbf{h}(m)\mathbf{R}_e(k,k)\mathbf{h}(n)^H e^{[-j\frac{2\pi}{N}(m-n)k]}\end{aligned} \tag{9}$$

$$\begin{aligned}\mathbf{D}_2(k;f) &\triangleq \hat{\mathbf{R}}_x(k,k+f)-\mathbf{H}(k)\mathbf{R}_e(k,k+f)\mathbf{H}(k+f)^H\\ &= \hat{\mathbf{R}}_x(k,k+f)\\ &-\sum_{m=0}^{L-1}\sum_{n=0}^{L-1}\mathbf{h}(m)\mathbf{R}_e(k,k+f)\mathbf{h}(n)^H e^{[-j\frac{2\pi}{N}(m-n)k]+jnf}\end{aligned} \tag{10}$$

where $||.||_F$ denotes Frobenius norm, and f is an integer in $[0,\cdots,N-1]$ defined as $\alpha=\frac{2\pi}{N}f$.

The derivatives $\frac{\partial\Gamma_f}{\partial\mathbf{h}(i)}$, $i=0,\cdots,L-1$ can be derived analytically and are given as [1]:

$$\begin{aligned}\frac{\partial\Gamma_f}{\partial\mathbf{h}(i)^{(R)}} &= -2\sum_{k=0}^{N-1} Re\{\mathbf{D}_1(k)\mathbf{H}(k)\mathbf{R}_e^*(k,k)e^{j\frac{2\pi}{N}ik}\\ &+\mathbf{D}_1^T(k)\mathbf{H}^*(k)\mathbf{R}_e^*(k,k)e^{-j\frac{2\pi}{N}ik}\\ &+\mathbf{D}_2(k;f)\mathbf{H}(k)\mathbf{R}_e^*(k,k+f)e^{j\frac{2\pi}{N}i(k+f)}\\ &+\mathbf{D}_2^T(k;f)\mathbf{H}^*(k)\mathbf{R}_e^*(k,k+f)e^{-j\frac{2\pi}{N}i(k+f)}\}\end{aligned} \tag{11}$$

$$\begin{aligned}\frac{\partial\Gamma_f}{\partial\mathbf{h}(i)^{(I)}} &= -2\sum_{k=0}^{N-1} Im\{\mathbf{D}_1(k)\mathbf{H}(k)\mathbf{R}_e^*(k,k)e^{j\frac{2\pi}{N}ik}\\ &-\mathbf{D}_1^T(k)\mathbf{H}^*(k)\mathbf{R}_e^*(k,k)e^{-j\frac{2\pi}{N}ik}\\ &+\mathbf{D}_2(k;f)\mathbf{H}(k)\mathbf{R}_e^*(k,k+f)e^{j\frac{2\pi}{N}i(k+f)}\\ &-\mathbf{D}_2^T(k;f)\mathbf{H}^*(k)\mathbf{R}_e^*(k,k+f)e^{-j\frac{2\pi}{N}i(k+f)}\}\end{aligned} \tag{12}$$

where $\mathbf{h}(i)^{(R)}$ and $\mathbf{h}(i)^{(I)}$ represent real and imaginary parts of $\mathbf{h}(i)$, and superscript * denotes the complex conjugate.

As in all nonlinear optimization problems, the convergence of the proposed approach depends on initialization. In order to improve the convergence, it is possible to develop a close form solution based on the second-order spectra correlations of the output signals and use it for the initialization. Let us evaluate 7 at two different α's, i.e., α_1 and α_2. Then, following the idea of [8], proposed for the SISO case, we can solve the resulting equations with respect to $\mathbf{H}(\omega)$ and equate the right-hand sides, i.e.,

$$\begin{aligned}&\mathbf{R}_e(\omega,\omega+\alpha_1)\mathbf{H}(\omega+\alpha_1)^H\mathbf{R}_x(\omega,\omega+\alpha_1)^{-1}=\\ &=\mathbf{R}_e(\omega,\omega+\alpha_2)\mathbf{H}(\omega+\alpha_2)^H\mathbf{R}_x(\omega,\omega+\alpha_2)^{-1}\end{aligned} \tag{13}$$

Let $\mathbf{H}_i(\omega)$ denote the $i-$th column of matrix $\mathbf{H}(\omega)$, and $\mathbf{h}_i(n)$ be the corresponding inverse Fourier transform. Let $d_k(\omega;\alpha_i)$ denote the (k,k)-th element of the diagonal matrix $\mathbf{R}_e(\omega,\omega+\alpha_i)$. Based on (13), the following equation can be easily derived:

$$\begin{aligned}&\sum_{n=0}^{L-1}\mathbf{h}_i^H(n)e^{j\omega n}[e^{j\alpha_1 n}d_i(\omega;\alpha_1)\mathbf{R}_x(\omega,\omega+\alpha_1)^{-1}-\\ &-e^{j\alpha_2 n}d_i(\omega;\alpha_2)\mathbf{R}_x(\omega,\omega+\alpha_2)^{-1}]=0,\quad i=1,...,n\end{aligned} \tag{14}$$

Evaluating (14) for different $\omega's$, we can set up a system of equations with unknown the elements of $\mathbf{h}_i(n)$, $n=0,...,L-1$, i.e.,

$$\mathbf{F}_i\mathbf{h}_i=\mathbf{0} \tag{15}$$

where the matrix $\mathbf{F}_i$ consist of block of rows, with the k-th block given as: $e^{j\omega_k n}[e^{j\alpha_1 n}d_i(\omega_k;\alpha_1)\mathbf{R}_x(\omega_k,\omega+\alpha_1)^{-1}-e^{j\alpha_2 n}d_i(\omega_k;\alpha_2)\mathbf{R}_x(\omega_k,\omega_k+\alpha_2)^{-1}]$.

Let L_i be the greatest integer such that $\mathbf{h}_i(L_i)\neq\mathbf{0}$. Assuming that parameter L in (14) was selected so that $L-L_c<L_i\leq L$, the null space of $\mathbf{F}_i$ will have dimension 1, so we can obtain $\mathbf{h}_i$ within a scalar ambiguity. Finally, combining all columns, $\mathbf{h}_i$ can be obtained within a diagonal matrix with scalar elements.

3. BLIND ESTIMATION OF SPARSE CHANNELS

The frequency-domain blind system estimation method described above can easily be modified to exploit the sparseness of the channels. Let us call "non-zero taps" the samples of $\mathbf{h}(n)$ for which the matrix $\mathbf{h}(n)$ has at least one non-zero element.

Let us assume that non-zero taps are known in advance and that $n=0$ corresponds to the first non-zero tap. Let S be the set of indices that correspond to non-zero taps. Since we have an analytical expression for the gradient at any tap, we are able to focus the above described minimization on the non-zero taps only. In our experiments the steepest-descent method was used for the minimization of Γ_f, i.e.,

$$\tilde{\mathbf{h}}(i)^{k+1}=\tilde{\mathbf{h}}(i)^k-\mu_k\frac{\partial\Gamma_f}{\partial\mathbf{h}(i)}\qquad i\in S \tag{16}$$

where $\tilde{\mathbf{h}}(i)^k$ denotes the updated estimate of $\mathbf{h}(i)$ at k-th iteration and μ_k is the step size.

By concentrating on the non-zero taps only, we significantly reduce complexity. It is important to note that the time-domain blind MIMO algorithms cannot take advantage of the sparse nature of the channels. In addition, the fact that there are many zeros among the channel coefficients renders those algorithms very sensitive.

The problem is considerably more complicated in the case where the locations of non-zero taps are unknown. Usually it is assumed that an upper bound on the non-zero taps, B, is known. Then the above approach can be reformulated as follows.

Let us express the system frequency response as:

$$\mathbf{H}(\omega) = \sum_{m=k_1}^{k_B} \mathbf{h}(m) e^{-j\omega m} \tag{17}$$

where now the summation contains the non-zero taps of $\mathbf{h}(m)$.

The minimization of Γ_f can now be performed with respect to the non-zero taps and also the corresponding locations. This is highly nonlinear minimization problem. As it will be shown in the next section, our preliminary results indicate that the above described minimization can lead to the locations of non-zero taps and finding the corresponding channel coefficients.

The close form solution can also serve as indicator where the non-zero taps are located. For example, most of the non-zero tap locations can be determined comparing the norms of the estimated channel matrices for each tap. However, further work is needed in order to develop more accurate way of determining all non-zero tap locations.

4. SIMULATION RESULTS

In this section we demonstrate the performance of the proposed approach in the case of a 2×2 system with 4-level QAM inputs. The length of the input colors was assumed to be $L_c = 8$. Each cross-channel was generated as a 3-ray channel, and the locations of the non-zero taps were selected randomly (except for $n = 0$) from the set of integers $\{1, 2, \cdots, 63\}$ (thus, the maximal channel delay spread was assumed to be 8 symbol intervals). In this case, the locations of non-zero taps were assumed to be known. Figs. 1a and 1b show the real and imaginary parts of the reconstructed versus estimated channels, respectively, normalized with respect to the zero delay component. The solid line represents the estimated mean based on the 50 Monte Carlo simulations while the grey area represents the standard deviation around the estimated mean. Stars denote the true channel coefficients for the non-zero taps. $T = 256$ symbols were used at the receiving end to estimate the channels. The signal to noise ratio was assumed to be $SNR = 10dB$.

Based on the system estimate, an equalizer can be formed to recover the inputs. In our simulations a zero-forcing block linear equalizer was used [6], [1]. The received signal before equalization is shown in Figure 2. Typical signal at the output of the equalizer is shown in Figure 3. Both results are obtained for the case of 2×2 system described in the example above and $SNR = 15dB$. Channel estimates used for the equalizer are obtained based on $T = 256$ symbols. Finally, Fig. 4 represents the bit error rate (BER) of the recovered signals averaged over 50 Monte Carlo runs. The results were obtained based on two different data lengths used for system identification.

Next we provide some preliminary results for the case where the case where the number of non-zero taps is known, but their locations are unknown. We consider the 2×2 system with 4-level QAM colored cyclostationary inputs, with the color lengths $L_c = 8$. The number of multipaths between any user and any receiver was 2 (zero-tap and tap selected from the set of integers $\{1, 2, \cdots, 23\}$. Let $\mathbf{s}_{ij}$ denotes the vector of locations of non-zero taps for the ij-th channel and let $\mathbf{r}_{ij}$ denotes the vector of the corresponding channel coefficients. The system analyzed then can be described as:

$$\begin{aligned}
\mathbf{s}_{11} &= [0, 9] \quad , \quad \mathbf{r}_{11} = [-0.786 + i0.679, -0.690 + i0.717] \\
\mathbf{s}_{12} &= [0, 7] \quad , \quad \mathbf{r}_{12} = [-0.583 + i0.610, -0.936 - i0.762] \\
\mathbf{s}_{21} &= [0, 6] \quad , \quad \mathbf{r}_{21} = [0.610 - i0.687, -0.551 + i0.695] \\
\mathbf{s}_{22} &= [0, 20] \quad , \quad \mathbf{r}_{22} = [-0.766 + i0.640, 0.897 + i0.745]
\end{aligned}$$

For illustration purposes, we assumed that the non-zero taps at locations $l = 6$ and $l = 7$ were known, while the taps $l = 9$ and $l = 20$ were unknown. For every possible value for the locations of two remaining non-zero taps the cost function was minimized with respect to the unknown taps. Again we considered $T = 256$ symbols and $SNR = 10dB$. The obtained normalized cost function (inverted value) is shown in Figure 5 for all possible combinations of l_1 and l_2. As it can be seen, the peak value ($l_1 = 20$, $l_2 = 9$) indicates the true location of the remaining two non-zero taps. In addition, the obtained values for channel coefficients for that combination were (normalized with respect to the zero delay component):

$$\begin{aligned}
\hat{\mathbf{r}}_{11} &= [-0.786 + i0.679, -0.628 + i0.661] \\
\hat{\mathbf{r}}_{12} &= [-0.583 + i0.610, -0.874 - i0.819] \\
\hat{\mathbf{r}}_{21} &= [0.610 - i0.687, -0.601 + i0.667] \\
\hat{\mathbf{r}}_{22} &= [-0.766 + i0.640, 0.824 + i0.718]
\end{aligned}$$

The above values indicate that the algorithm converges to the right values.

5. REFERENCES

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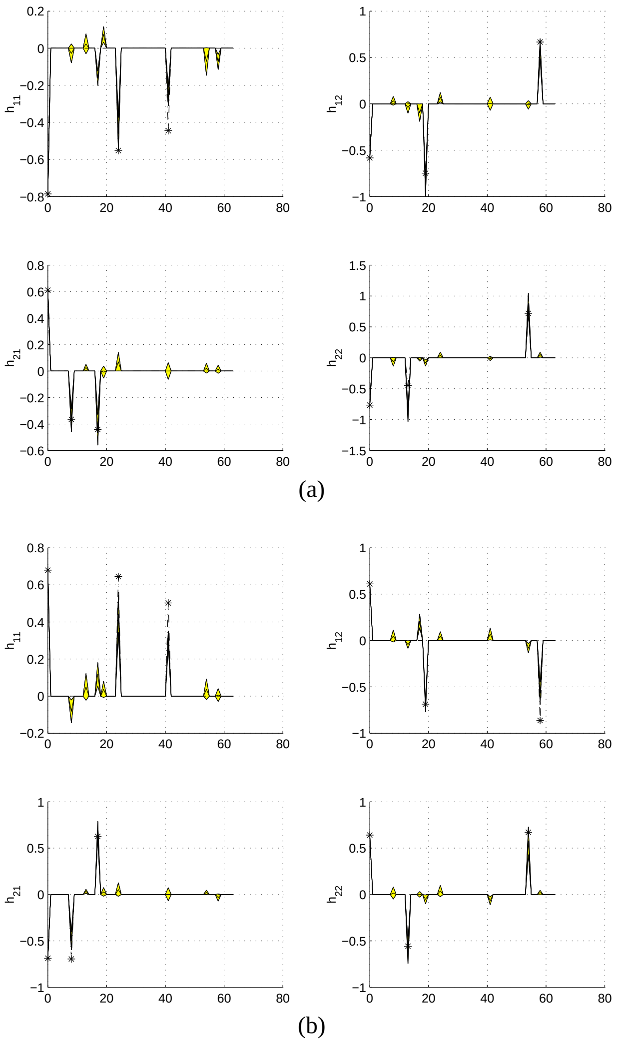

Figure 1: True versus estimated channels of a 2×2 system: (a) Real parts, (b) Imaginary parts.

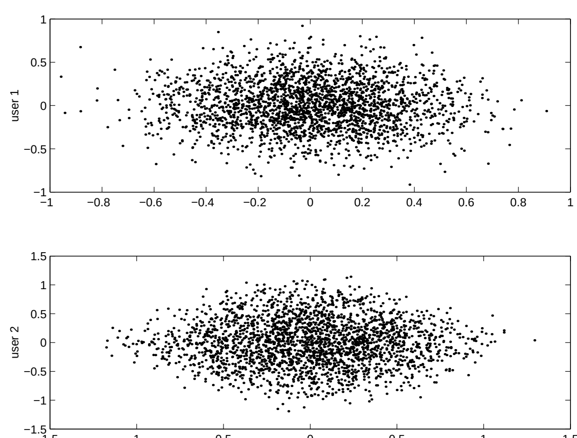

Figure 2: Received signal without equalization for 4-QAM input signals and $SNR = 15dB$.

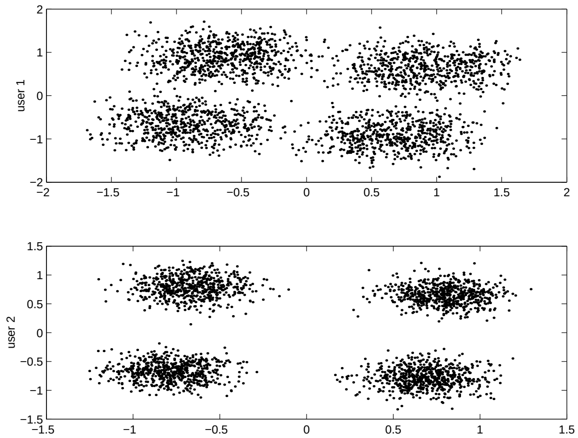

Figure 3: The output of the equalizer for 4-QAM input signals, $SNR = 15dB$ and data length $T = 256$ symbols.

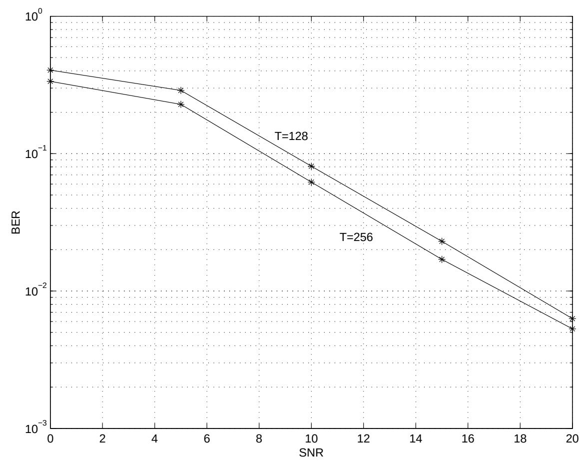

Figure 4: BER of the recovered signal.

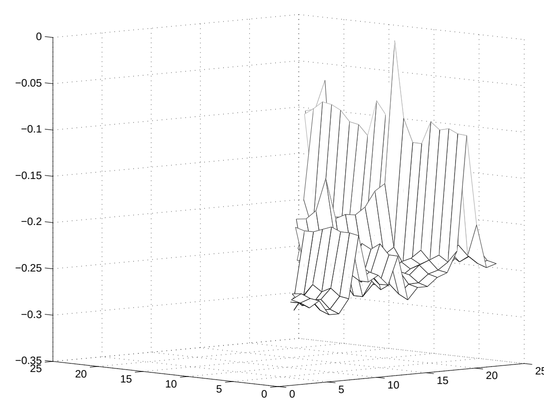

Figure 5: Estimated normalized cost function