THE INPUT IMPEDANCE FUNCTION FOR GENERAL FRACTAL GRATINGS

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ABSTRACT

We use the impedance method to study the far field of a transmission grating characterized by a Cantor density function. This density is obtained from a multiplicative superposition of periodical functions. Intensity patterns for different configurations are shown when randomization and filtering of some components are introduced. These results relate the geometric-statistical properties with the scattered fields.

INTRODUCTION

The main challenge of rigorous solutions in electromagnetic scattering theory is the imposition of boundary conditions. This problem is exacerbated when dealing with a large volume of data typical of many numerical methods. This is one reason many approximation methods were developed. Hessel and Oliner [1] were one of first to introduce the impedance boundary condition to the problem of gratings. Afterwards, this concept was adapted for application to random rough surfaces [2, 3], planar structures [4] and also applied to non-speculars effect [5].

Here we are interested in characterizing multi-scale surfaces, such as those occurring with deterministic fractal objects [6, 7]. These are representative examples for the study of general properties for fractal electrodynamics. There are several works on this subject, which examine the scattering of electromagnetic waves by fractal surfaces from a scalar or vectorial point of view [8]. In this paper, we expand these ideas using the impedance approximation.

The results of the calculated scattering are shown using a density Cantor function [9-11] as a perturbation to constant impedance on the flat surface. This Cantor distribution is obtained as a multiplicative superposition of scaled periodical functions, similar to the way Walsh functions are used in optical processing [12]. These scaled periodic functions can be combined in different ways according with the dimension or lacunarity of the corresponding Cantor set.

MATHEMATICAL BASIS AND RESULTS OBTAINED

The impedance function Z is included into the relation between tangential components of electromagnetic fields ($\mathbf{E}_{//}$ and $\mathbf{H}_{//}$) at the interface between two media [13]:

$$\mathbf{E}_{//} = \mathrm{Z}\,(\hat{\mathbf{n}} \times \mathbf{H}_{//}) \tag{1}$$

If a decomposition of normal (according with geometrical optics) and diffuse transmitted fields are considered, when a plane wave is incident on the grating, we can deduce, using any perturbation function, the corresponding integral equations to calculate such diffuse fields:

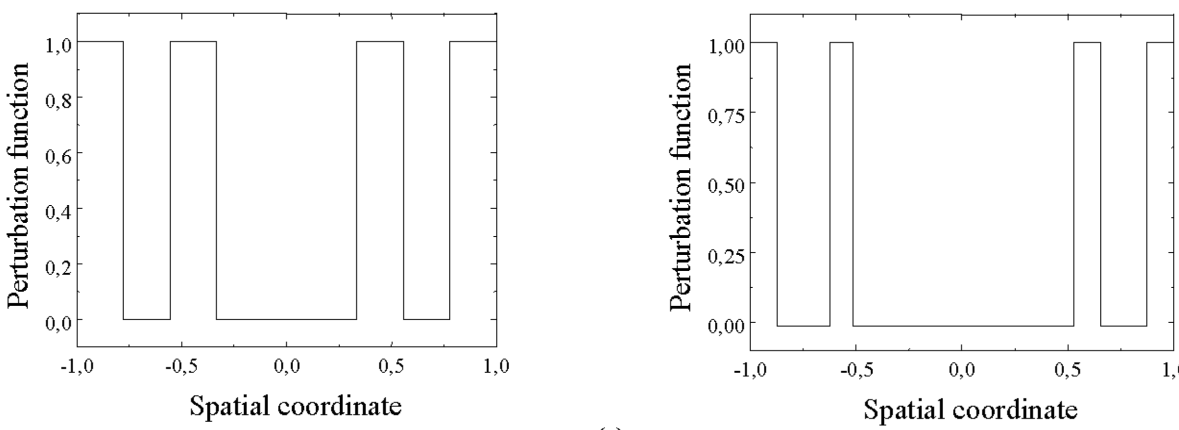
$$\begin{cases} E_t^D(\alpha) = \dfrac{1}{\beta(\alpha)\,Z_o + k}\,\dfrac{2k\,\beta_o\,Z_o}{\beta_o\,Z_o + k}\,Z[\alpha-\alpha_o] - \dfrac{Z_o}{\beta(\alpha)\,Z_o + k}\displaystyle\int_{-\infty}^{+\infty} \beta(\varsigma)\,\mathrm{Z}[\alpha-\varsigma]\,E_t^D(\varsigma)\,d\varsigma \\[2ex] H_t^D(\alpha) = \dfrac{1}{\beta(\alpha) + k\,Z_o}\,\dfrac{2\beta_o^2\,Z_o}{\beta_o + k\,Z_o}\,Z[\alpha-\alpha_o] - \dfrac{k\,Z_o}{\beta(\alpha) + k\,Z_o}\displaystyle\int_{-\infty}^{+\infty} \mathrm{Z}[\alpha,\varsigma]\,H_t^D(\varsigma)\,d\varsigma \end{cases} \tag{2}$$

with $\alpha = k \sin \theta$ and $\beta = k \cos \theta$ for the intensity distribution. There is a central component α_o for the incident radiation defined through the angle θ_o, as $\alpha_o = k \sin \theta_o$. Also, $\widetilde{Z}[\alpha - \varsigma]$ is the Fourier transform of the perturbation function and $\beta_o = k \cos \theta_o$. The Eqs. (2) are Freedholm integral equations of second kind.

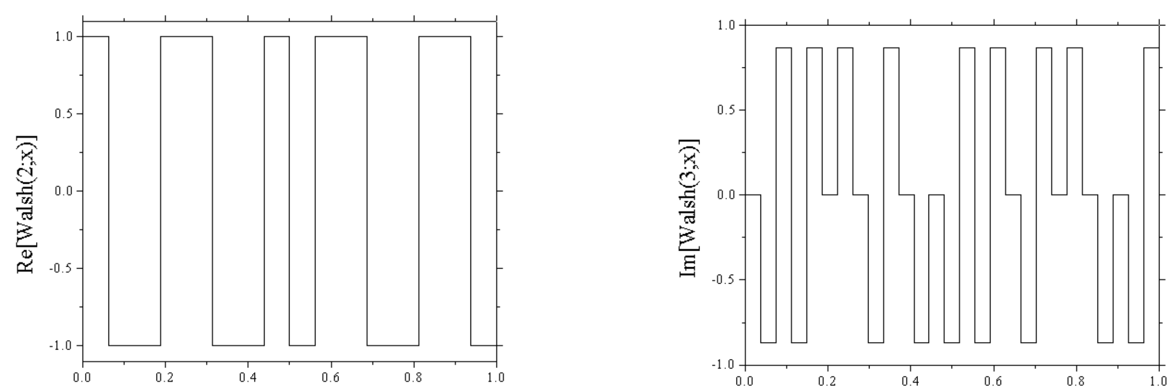
The density Cantor function *C(x)* is defined in a way that allows us to represent gratings with different dimension and lacunarity:

$$C(x) = \prod_{l=1}^{N} \left\{ \sum_{i=1}^{n_1(l)} R_i \, rect\left[\frac{x - x_i}{\Delta_1}\right] \right\}^{g_l} \left\{ \sum_{j=1}^{n_2(l)} S_j \, rect\left[\frac{x - x_j}{\Delta_2}\right] \right\}^{h_l} \qquad (3)$$

R_i and S_j being constant values into the interval defined for each rectangular function and centered at x_i and x_j respectively. Here, g_l and h_l can take the values within the set $\{0, 1\}$ and Δ_1, Δ_2 are the corresponding periods of the rectangular scaled functions. A grating with a Cantor distribution can be obtained if all exponents are equal to 1. In this way, different Cantor densities and generalized Walsh functions (see Fig. 1) can be obtained as a product of periodical functions, which represents a generalization of previous results [10,14].



(a)



(b)

Fig. 1. (a) Cantor density function for fractal dimension *D=ln2/ln3* and *D=0.5*. (b) Walsh functions for scaling 2 and 3 respectively.

The randomization on a fractal object is obtained using a Gaussian generator and a random parameter is involved in the construction of this object, as it is made with other methods [14]. The functional form of density Cantor bars defined by Eq. (3) allows us the study of intermediate states between deterministic and random fractals. The quasi self-similarity is achieved when some rectangular components in the density function have a random parameter with a any statistical distribution as shown next.

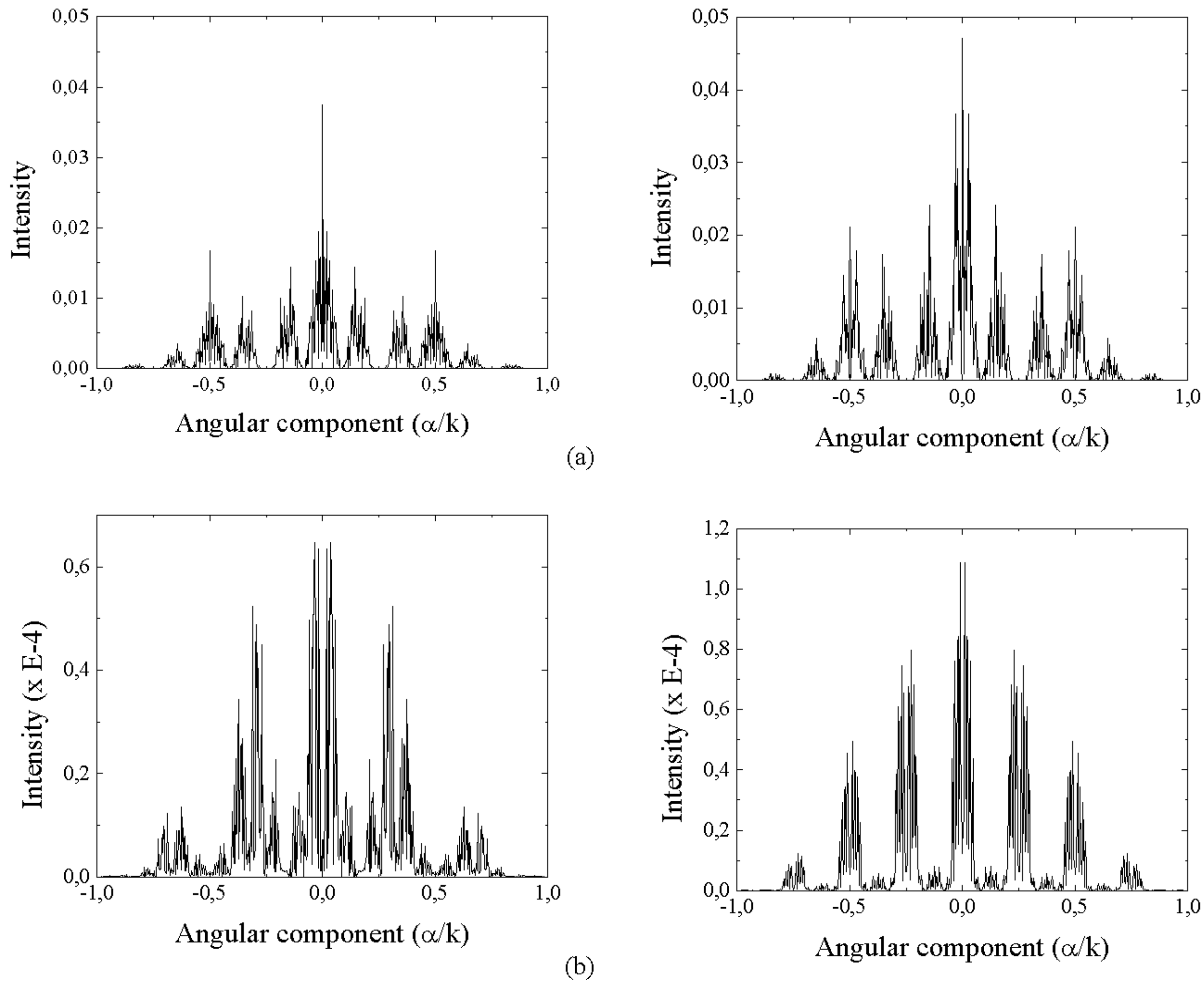


Fig. 2. Scattering from random gratings: (a) Cantor fractals, (b) generalized Walsh functions.

Here we use R_i and S_j as random parameters with Gaussian distribution between –1 and 1, being $Z_o = 0.4$. Fig. 2(a) shows the corresponding patterns for two periodical components, that contributes to orders 3 and 4, with randomness. For left figure, the periodical components of order 3 and 4 possess a degree of randomness and a deviation of the deterministic fractal behavior in the intensity pattern can be observed. In right figure, the component of order 3 had been filtered and the intensity distribution has less fractality [10] and the randomness becomes more important. A similar behavior can be demonstrated for deterministic fractals through the self-correlation function [6, 10]. Something similar it happens for Walsh gratings with different of Figs. 2(b) with randomness in some periodic components for the two scaling factor of Fig. 1. These results for the scattered fields have importance because the different structures and distribution in the intensity patterns outlines the possibility of using the properties in the electromagnetic processing information.

CONCLUSIONS

The link between fractal gratings with a quasi-randomization in their structure and the corresponding intensity pattern of far field is outlined. The quasi randomization is introduced through the randomization in each periodical function and all these results are important to establish a relation between the geometric-statistics properties of the object and the scattered fields by it, to study defects in fractal gratings and for applications in optical and electromagnetic information processing.

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REFERENCES

1. A. Hessel and A. A. Oliner, "A new theory of Woods anomalies on optical gratings", *Appl. Opt.* , Vol. 4, 1965, pp. 1275-1297.
2. R. A. Depine, "A simple approach to light scattering from absorbing micro-rough surfaces", *Optik*, Vol. 82, 1989, pp. 5-8.
3. R. A. Depine, "Scattering of light from one-dimensional random rough surfaces: a new antispecular effect in oblique incidence", *J. Opt. Soc. Am. A*, Vol. 10, No. 5, 1993, pp. 920-927.
4. M. Lehman and E. E. Sicre, "Scattering from a Medium with Fractal Refraction Index", in 17th Congress of the International Commission for Optics: Optics for Science and New Technology, Joon-Sung Chang, Jai-Hyung Lee, Soo-Young Lee, Chang-Hee, Eds., *Proc. SPIE 2778*, Aug. 1996, pp. 365-366.
5. S. Zhang and T. Tamir, "Spatial modifications of Gaussian beams diffracted by reflection gratings", *J. Opt. Soc. Am. A6*, 1989, pp. 1368-1379.
6. Y. Sakurada, J. Uozumi and T. Asakura, "Fresnel diffraction by one-dimensional regular fractals", *Pure and Appl. Opt. 1*, 1992, pp. 29-40.
7. C. Allain and M. Cloitre, "Optical diffraction on fractals", Phys. Rev., Vol. 33, No. 5, pp. 3566-3569.
8. D. L. Jaggard, "On Fractal Electrodynamics", in *Recent Advances in Electromagnetic Theory*, H. N. Kritikos and D. L. Jaggard, Eds., Springer-Verlag, New York, 1990.
9. D. L. Jaggard and T. Spielman, "Triadic Cantor target diffraction", Microw. And Opt. Tech. Lett., Vol. 5, No. 9, pp. 460-466, 1992.
10. M. Lehman, "Fractal Diffraction Gratings build through Rectangular Domains", *Optics Communications* **195**(1-4), pp. 11-26 (2001).
11. M. Lehman, D. Patrignani, L. De Pasquale and J. L. Pombo, "Properties of in-order self-similarity function in the Fresnel region for the Sierpinski carpet grating", 1997, *Proc. SPIE 3159*, pp. 261-268.
12. C. Colautti, B. Ruiz, E. E. Sicre and M. Garavaglia, "Walsh functions: analysis of their properties under Fresnel diffraction", *J. Mod. Opt.*, Vol. 34, 1987, pp. 1385-1391.
13. L. D. Landau and E. M. Lifshitz, "Electrodynamics of Continuous Media", 1981, Addison Wesley.
14. M. Lehman, "Diffraction by a fractal transmittance obtained as superposition of periodical functions", *Fractals* **6**(4), pp. 313-326 (1998).
15. Y. Kim and D. L. Jaggard, "The Fractal Random Array", Proc. of the IEEE, Vol. 74, No. 9, pp. 1278-1280, 1986.