

# DEVELOPMENT OF A MODEL FOR REALISTIC PORTRAYAL OF RANDOM TIME VARIATIONS ON MOBILE RADIO CHANNELS

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## ABSTRACT

Classical models for fading radio channels are based upon the assumption that the channel process is random, wide sense stationary, and infinite in extent. However, consideration of physical phenomena that influence many channels over which digital communications is currently of interest reveals that such phenomena change as, for example, either the location of one or both of the terminals changes, or the surrounding environment changes. It is therefore considered reasonable that channel models should also change by similar increments and over similar intervals. This paper outlines the method by which a dynamic model is being developed to have the desired characteristics.

## INTRODUCTION

Consideration of mobile radio channels [1] allows one to envisage a scenario wherein a set of plane waves impinges at the antenna of a moving receiver, but persists only for a limited distance along the receiver's trajectory. At the end of such a distance, herein referred to as a consistency length [2], there are changes in some of the parameters of the wave set. These could be changes in angles of arrival of the waves, or a change in their number as new waves are received, and others become weak or obstructed. In addition to this, as a result of motion of the receiver and objects in the environment surrounding it, the impinging plane waves as well as other waves with non-planar wavefronts experience Doppler shifts, leading to an overall Doppler spread of the received signal. Since the location and speed of reflectors and scatterers is frequently changing and unpredictable, a receiver in a narrowband transmission system yields a random output that exhibits fast fading due to multipath propagation and Doppler spreading. As well, such an output can exhibit slower variations in its mean as a result of changes in the parameters of the impinging wave set.

In this paper an approach to the development of a new model that better reflects the influence of the above-described changes on microcellular mobile radio channels is reported. The approach involves the introduction of a mathematical model for the channel within a single consistency length using a state space representation. It is planned to implement this model based on the estimation of its parameters from noisy propagation measurement data. Ultimately, the single consistency length model will be integrated into a perturbation model that is valid over multiple consistency lengths. The presentation at the General Assembly will focus on the analysis of measured data for different propagation scenarios, which will lead to the derivation of a set of model parameters.

## THE CHANNEL MODEL

If a CW signal  $\cos(\omega_c t + \theta)$  is transmitted in a cluttered environment, after accounting for attenuations and phase shifts on  $N$  different propagation paths between the transmitter and receiver, the received signal can be written as

$$R(t) = \sum_{i=1}^N V_i \cos(\omega_c t + \theta + \phi_i - kd_i^t - k(x_i \cos\alpha_i \cos\beta_i + y_i \sin\alpha_i \cos\beta_i + z_i \sin\beta_i)), \quad (1)$$

in which the approach in [3] has been adopted, with several changes. The location of the receiver has been moved to  $(0,0,0)$ , that of the transmitter has been moved to  $(x_0, y_0, z_0)$ , and that of a single obstruction on the  $i^{\text{th}}$  radio path between the transmitter and the receiver has been denoted  $(x_i, y_i, z_i)$ . Also the direct path has been excluded from

consideration for the purposes of this paper. Finally, the history of the waves prior to their interaction with the obstacle, as well as the effects of that interaction (reflection, scattering, or diffraction) have been implicitly considered. This relates the phases of all waves to their propagation from a single source location before their interaction with the obstruction and could eventually be important in the explanation of observations from the results of analysing measured data. Only one obstruction is considered on every path. In (1),  $V_i$  is the amplitude of the  $i^{\text{th}}$  wave and includes the effects of spreading loss and the loss upon interaction with the obstacle. The term  $\phi_i$  is the phase shift undergone as a result of the same interaction,  $k = \frac{2\pi}{\lambda}$ , and  $d_i^t$  is the distance between the transmitter and the obstacle. The azimuthal angle of arrival (aoa) at the receive antenna of the wave received via the obstacle is denoted  $\alpha_i$ , and the elevation aoa at the receiver for the same wave is represented by  $\beta_i$ . In the following equations,  $(\theta + \phi_i - kd_i^t)$  will be represented as  $\xi_i$  and the difference of the final term in the cosine argument from  $\xi_i$  is denoted as  $\Psi_i$ . The final term in the cosine argument represents the phase shift over the distance  $d_i^r$  from the obstacle to the receiver.

In the CRC system used for the measurement of data to be analysed in this work, the received signal is applied to a quadrature downconverter with stable local oscillators to give a complex envelope equivalent low pass signal,  $r = I + jQ$ , which for a specific receiver location, “s” is non-time-varying. If there is a local oscillator phase shift,  $\Delta\theta$ , with respect to the phase of the received signal carrier, after low pass filtering, both  $I$  and  $Q$  are functions of  $\Delta\theta$  and include sums involving both  $\cos \Psi_i$  and  $\sin \Psi_i$ . In real-world situations,  $\Delta\theta$  is a function of “s”, and of time as a result of motion surrounding the receiver. However, herein,  $\Delta\theta$  will be assumed to be both fixed, and equal to 0. Under these conditions,

$$r(s) = \sum_{i=1}^N r_i(s) \cos \Psi_i(s) - j \sum_{i=1}^N r_i(s) \sin \Psi_i(s) = \sum_{i=1}^N r_i(s) e^{-j\Psi_i(s)}. \quad (2)$$

Consider now the spatial correlation of this signal over a consistency length, wherein it will be recalled that the amplitudes and aoas of the multipath waves remain constant. Under a small translation,  $\Delta s$ , in receiver location along the vehicle’s trajectory with angle  $\gamma$  as in [3], with respect to the x-z plane, this ensures that  $\xi_i(s + \Delta s) = \xi_i(s) = \xi_i$  and  $r_i(s + \Delta s) = r_i(s) = r_i$ . Thus, if the change  $\Delta s$  is brought about by motion of the receiver with a speed  $v$  over a time duration  $\tau$ , the spatial autocorrelation of the sampled low pass received signal can be written as

$$R_r(\Delta s) = E \left\{ \sum_{i=1}^N \sum_{j=1}^M r_i r_j e^{+j(\Psi_j(s) - \Psi_i(s))} e^{-jkv\tau \cos \beta_j \cos(\gamma - \alpha_j)} \right\}, \quad (3)$$

where it has been assumed that the mean of the multipath sum is zero because of the absence of a direct wave (or any specular indirect components). Now, the first exponent in (3) is the difference of phase angles that are related to the distance along two different paths from the transmitter to the receiver, as well as two different phase shifts incurred on interaction with two different obstructions, resulting in two different sets of aoas at the receiver. The second exponent depends on a single set of aoas, and the vehicle’s heading. Because of this difference in dependencies, it is considered reasonable to assume that these exponents are independent random variables. Then, the expectation can be separated into two expectations, the first of which is equal to 0, if  $i \neq j$ , and since  $\Delta s = v\tau$ , it is possible [4] to write (3) as

$$R_r(\tau) = P_{ave} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} e^{-jkv\tau \cos \beta \cos(\gamma - \alpha)} f_{\alpha}(\alpha) f_{\beta}(\beta) d\alpha d\beta, \quad (4)$$

where  $P_{ave} = E \left\{ \sum_{i=1}^N r_i^2 \right\}$ , and  $f_{\beta}(\beta)$ ,  $f_{\alpha}(\alpha)$  represent probability density functions.

If, for exemplary purposes, the azimuthal aoa distribution is assumed to be uniform on  $[-\pi, \pi)$ , and the elevation angle distribution is assumed to be as in (25) of [3], the development in [3] can be followed to give a power spectrum  $S(f, \nu, \lambda, \beta_{\max})$  as in (26) of [3]. Under these conditions,  $S$  is real because of the symmetry in the integrand of (4) resulting from the assumed probability density functions for the aoas. In general, however, the expression is more complicated.

Since the spectrum  $S(f, \nu, \lambda, \beta_{\max})$  is band-limited, Weiner's factorisation theorem does not apply. However, one can approximate this spectrum by a proper rational function of any order. Here, a second-order transfer function is considered, so that

$$S(f, \nu, \lambda, \beta_{\max}) = H(j\omega)H(-j\omega), \text{ where } H(j\omega) = \frac{c}{-\omega^2 + 2j\omega\zeta\omega + \omega^2} \text{ and the parameters } \{\omega, \zeta, c\} \text{ are}$$

chosen to represent  $S(f=0)$ ,  $S(f = \frac{\nu}{\lambda} \cos \beta_{\max})$ , and  $c = \omega^2 \sqrt{S(f=0)}$  respectively, leading to a state-space representation for the channel.

### STATE SPACE REPRESENTATION OF THE CHANNEL

Following [5,6],  $S(f, \nu, \lambda, \beta_{\max})$  can be approximated by a rational transfer function and therefore represented through stochastic differential equations. Using second order equations, the Controllable Canonical State Space Form of the in-phase  $H_I(j\omega)$  and quadrature  $H_Q(j\omega)$  components of  $H(j\omega)$  can be written as

$$\begin{aligned} \dot{X}_I(t) &= A(\tau)X_I(t) + f_I(t) + B(\tau)\dot{w}_I(t), X_I(0) \approx N(0; \Sigma_0) \\ \dot{X}_Q(t) &= A(\tau)X_Q(t) + f_Q(t) + B(\tau)\dot{w}_Q(t), X_Q(0) \approx N(0; \Sigma_0) \end{aligned} \quad (5)$$

in which

$$A = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix}, B = \begin{bmatrix} 0 \\ c \end{bmatrix}, \dot{w}_I(t), \dot{w}_Q(t) \text{ represent white noise,}$$

and

$X_I \in \mathfrak{R}^n, X_Q \in \mathfrak{R}^n, \dot{w}_I \in \mathfrak{R}^n, \dot{w}_Q \in \mathfrak{R}^n, n = 2$ . The terms  $f_I(t), f_Q(t)$  represent deterministic components, and when they are non-zero, the envelope  $r(t) = \sqrt{I^2(t) + Q^2(t)}$  has a Rician distribution. Otherwise, the model applies to situations in which the envelope is Rayleigh and  $I(t)$  and  $Q(t)$  are zero-mean Gaussian with equal variance.

Using (5), spectra derived from measurements can be represented using only three parameters  $\{\omega, \zeta, c\}$ . In addition, the time domain representation of the random channel that yields the received signal can be represented using general linear stochastic differential equations of the form

$$\begin{aligned} \dot{X}(t) &= A(\tau)X(t) + f(t) + B(\tau)\dot{w}(t) \\ \hat{R}(t) &= C(t)(X(t) + D(t)\eta(t)), \end{aligned} \quad (6)$$

where,  $\hat{R}(t)$  is a model-generated approximation of  $R(t)$ , and

$C(t) \equiv [\cos \omega_c t \ 0 \ -\sin \omega_c t \ 0]$ ,  $D(t) \equiv [\cos \omega_c t \ -\sin \omega_c t]$ , the terms of  $\eta(t) = [\eta_I(t) \ \eta_Q(t)]$  are iid Gaussian noises with density  $N(0; \sigma_\eta^2)$ , and  $A, B$  represent the dependence of  $\{\omega, \zeta, c\}$  on the spatial variable. In instances

where measured data are too noisy to estimate a reliable power spectrum, the parameters  $\{A, B, C, D\}$  can be estimated from measured signals using the Expectation Maximisation algorithm discussed in [7], then applied to generate power spectrum models from which  $\{\omega, \zeta, c\}$  can be determined.

## MODELLING OVER MULTIPLE CONSISTENCY LENGTHS

Throughout single consistency lengths, the channel parameters required to describe the state space model in terms of  $(A, f_l, B, f_o)$  should not change. However, over multiple consistency lengths, significant changes in these are to be expected. An approach that accounts for continuously changing model parameters over multiple consistency lengths is to describe the changes in terms of a nominal dynamical channel model and its perturbation resulting from the uncertainty in model parameters, which are known to take on values within specific ranges. For example, a nominal maximum elevation angle  $\beta_{\max}$  can be estimated from spectra derived directly from measurements or generated from model parameters estimated from noisy data, as discussed in the foregoing section. A family of model spectra resulting from variations in this can then be derived and used in turn to generate continuously varying time domain channel models. Such a family can be generated as exemplified in the following.

Suppose the receiver velocity and the carrier frequency are fixed. Then reference to (26) in [3] shows that the shape of  $S(f, v, \lambda, \beta_{\max})$  is defined by  $\beta_{\max}$ . A nominal value for this parameter can be estimated from measurements, which, together with knowledge of  $v$ , can be used to construct a nominal transfer function  $H_n(j\omega)$  and the corresponding spectrum and state space model. Changes in the spectrum as a result of variations in  $\beta_{\max}$  can then be modelled by determining the radius of uncertainty of the nominal transfer function with respect to the true transfer function [6]. Thus, the family of transfer functions that represent different consistency lengths would be centred on  $H_p(j\omega)$  and deviate from it by a radius of uncertainty described by the weighting function  $W(j\omega)$ , such that

$$H_p(j\omega) = [1 + \Delta(j\omega)W(j\omega)]H_n(j\omega). \quad (7)$$

where  $W(j\omega)$  is a variable stable transfer function, and  $\Delta(j\omega)$  is a phase uncertainty that satisfies  $\sup_{\omega \geq 0} |\Delta(j\omega)| \leq 1$ . Based on the perturbed transfer function in (7), one can generate a robust state space channel model similar to that which can be described using the parameters in (5).

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