Peculiarities in the vertical and horizontal electric field signatures in the presence of a sprite

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Abstract

Recently Pasko, Barrington-Leigh, and Inan have discussed models explaining the electromagnetic and optical behavior of sprites. Research presented herein extends Baginski's lightning transient model by including Pasko's self-consistent conductivity model for parameters consistent with sprite observations. Equations are solved simultaneously in differential form (Maxwell's equations and electron ionization). Cylindrical symmetry is assumed and fields simulated at radius $\sim$ 0 - 50 km, and altitude $\sim$ 40 - 80 km. Vertical and horizontal field signatures are presented and anomalous behavior discussed. The research differs from earlier models by including the low altitude conductivity in the solutions and over the entire region simulated.

1 INTRODUCTION

Since 1994 there has been considerably interest in investigating the sprite phenomenon. Much of the past research attempting to explain certain characteristics of the sprite has been based on numerical solutions of the governing equations. An excellent example of this was the use of modeling to identify the fundamental mechanism responsible for vertical stratification in sprites. Pasko et al., [1] and Raizer et al. [2] Inan et al. [3] among others used numerical models to simulate the upper atmospheric discharge phenomenology. Their work showed that vertical stratification is a result of a large number of streamers closely grouped. However, because of the complexity involved in solving for individual streamers over the region of the sprite, it becomes necessary to use bulk parameters for calculations of the electric field signatures, optical emissions, etc, over any significant region (e.g., Barrington-Leigh [4], Pasko [1], Inan [3], etc.,).

1.1 The Proposed Research Methodology

The research presented investigates the electric field signatures during a sprite event (100 coulomb event) and contrasts the results to a 1 coulomb lighting event [5]. A finite element simulation of the governing differential equations is developed based on the model of Baginski [6] incorporating the ionization model used by Pasko (dn_e/dt = $(vi - va) \cdot n_e$) [1]. All differential equations are solved simultaneously resulting in a accurate model of the electric field's temporal structure.

The primary focus of this study was to examine the waveform signatures not associated with lighting current moments (Barrington-Leigh) [4] but generally considered quasi-static fields even though modes of propagation are allowed (QE not EMP fields modeled). The sprite was modeled assuming cylindrical symmetry with a charge perturbation centered on the z axis at an altitude of 10 km. A spherical Gaussian profile was assumed for the spatial profile of the induced charge and the electric fields were characterized (simulated) at altitudes of $\sim 40 - 80$ km for radial distances of $\sim 40 - 80$ km. It was observed that the vertical electric field signatures tend to have the same fundamental characteristics throughout the region investigated with the amplitude and relative duration being a function of the position and magnitude of the charge perturbation and the point of observation of the electric field. A trend in the field structure observed in previous studies [7], [5] was that the vertical electric field tends to be compressed temporally and in magnitude as altitude is increased. No charge is assumed displaced and the electric field everywhere is assumed zero at the beginning of the simulations.

2 DISCUSSION OF THE MODEL

2.1 Maxwell's equations and electron density equation

The simulations described here do not account for the lightning return stroke currents $\mathbf{J}_s$ contribution to the transient. Lightning return stroke currents are usually considered the source of the propagating component (EMP or launched wave front) of the induced field and involve time scales $\sim$ 10-100 μ sec. This type of sferic analysis emphasizes the wave front's refraction and attenuation. The changes in electron density caused by this source will be at a maximum well away from the z axis (assuming CG lightning centered about z-axis and aligned with vertical axis (Barrington-Leigh) [4]. The focus of this study is on the late time electric field and high altitude charge density signatures for the late-time (quasi-static) component of the sprite. The omission of $\mathbf{J}_s$ in the simulations is a widely accepted assumption for late time thunderstorm electrodynamics [2] and will be used here.

The partial differential equations in the form used for the simulations are based on Maxwell's equation and the electron ionization equation [1]. The SI system of units is used for all equations where $\mathbf{E}$ denotes the electric field (V/m), V is the electric potential (V), $\mathbf{J}_s$ the source current density associated with return stroke current (A/m^2) , ϵ_0 the permittivity of free space (F/m), $\partial p_f/\partial t = \nabla \cdot \mathbf{J_s}$ source of forced charge density associated with deposition of return stroke current $\frac{C}{m^3 \cdot s})$ causing the sprite v_i and v_a are the ionization rates defined by Pasko, n_e the electron density C/m^3, μ_e the electron mobility (only used for altitudes greater than 60 km) $\frac{m^2}{V \cdot s}$ [1]. The equations are used are in the differential form shown here:

$$\sigma = q \cdot \mu_e \cdot n_e \tag{1}$$

$$\mathbf{E} = -(\nabla \mathrm{V} + \partial \mathbf{A}/\partial \mathbf{t}) \tag{2}$$

$$\nabla \times \mathbf{A} = \mu \mathbf{H} \tag{3}$$

$$\frac{dn_e}{dt} = (v_i - v_a) \cdot n_e \tag{4}$$

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \epsilon_{\mathbf{0}} \partial \mathbf{E}/\partial \mathbf{t} + \mathbf{J_s} \tag{5}$$

$$\nabla \times \mathbf{E} = -\mu_{\mathbf{0}} \partial \mathbf{H}/\partial \mathbf{t} \tag{6}$$

By using the definition for $\mathbf{E}$ and equation 6 a wave equation is developed that is non-linear 7 with the gauge condition given by 8.

$$\nabla \times \nabla \times \mathbf{A} = -\mu_{\mathbf{0}} \sigma (\partial \mathbf{A}/\partial \mathbf{t}) - \mu \epsilon_{\mathbf{0}} \partial^{\mathbf{2}} \mathbf{A}/\partial \mathbf{t}^{\mathbf{2}} + \mu \mathbf{J_s} \tag{7}$$

$$\nabla \nabla \times \mathbf{A} = \sigma \nabla \mathrm{V} + \epsilon_{\mathbf{0}} \frac{\partial \nabla \mathrm{V}}{\partial \mathbf{t}} \tag{8}$$

The resulting partial differential equation is analytically solvable for only the simplest cases. To pursue this problem further either simplifying assumptions must be made or a numerical solution must be developed (the latter approach is used here). The continuity equation as implemented in this study is derived by taking the divergence of the Maxwell current density (5) and assuming that the $\partial \mathbf{A}/\partial t = 0$ and therefore $-\mu \partial \mathbf{H}/\partial t = -\mu \nabla \times \frac{\partial \mathbf{A}}{\partial t} = 0$ resulting in the following equation:

$$\nabla \cdot (\sigma \nabla \mathbf{V} + \epsilon_0 \frac{\nabla \partial V}{\partial t} + \partial \rho_f/\partial t) = 0 \tag{9}$$

[1] space does not allow the complete development of the mobility, electron density and ionization equations used in the research. For additional information please see [1]

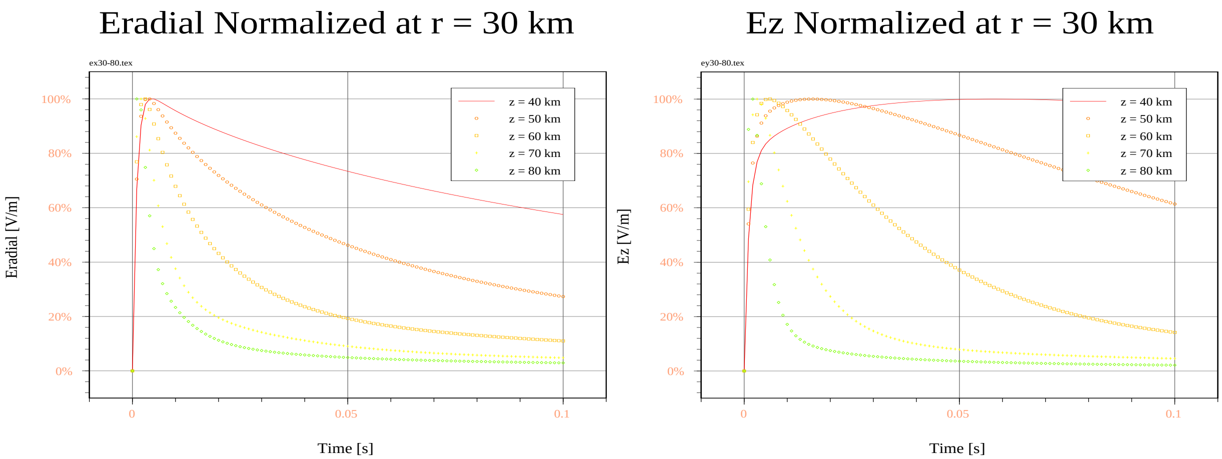

Figure 1: Comparison of vertical and horizontal electric fields at radial distances of r = 30 km

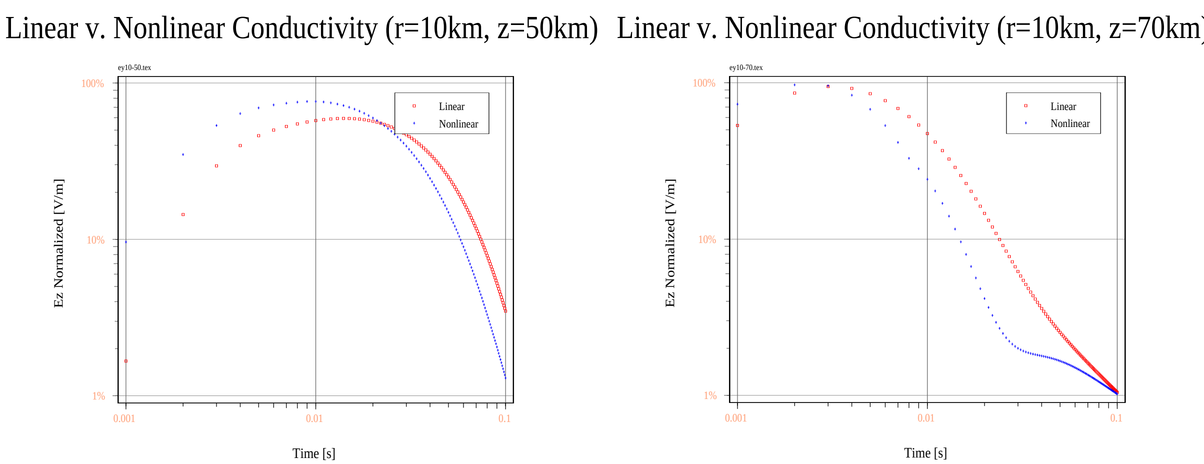

Figure 2: Comparison of vertical radial distances of r = 10 km z = 50 km, 70 km, Q_{total} = 1 and 100 Coulomb events

3 Discussion of the simulations

Figures 1 through 3 show vertical and horizontal field simulations that are normalized for the purpose of contrasting the effect of ionization caused by the sprite. The 12 waveforms selected are representative of schema from a much larger data set consisting of hundreds of simulations. 1 contrasts the normalized vertical and horizontal electric field components at altitudes of 40 û 80 km for a radial distance of 30 km. Figures 2, 3 compare the vertical electric field waveforms for altitudes of 50 km and 70 km and radial distances of 10 km and 30 km. A small scale lightning event (Q $\sim$ 1 C) is used in the comparison since the charge is insufficient to cause any significant shift in the ambient electron density profile or conductivity and these simulations serve primarily as a method of exposing the primary effect of the sprite on the electric field signatures. All simulations are shown at altitudes where sprites have been detected and categorized via optical photometry. Blue jets, red sprites, and elves have been identified at altitudes of z = 40-95 km having horizontal dimensions that are in excess of 50 km. The decreased duration of the field signatures during a sprite is expected since the large electric fields significantly increase the transient electron density and local conductivity ($\sigma = q\mu_e n_e$ for z > 40 km. In all cases the simulated, the horizontal component of the electric field showed a typical exponential decay at a rate corresponding to the dielectric relaxation time of the atmosphere at the point of observation ($\tau = \epsilon/\sigma$) followed by behavior that has been referred to as “long tails". A second trend is that the vertical electric fields in all cases presented here show a significant delay from the time the charge is present in the region to the onset of the peak electric field (“peaking” behavior) . The mechanism governing this behavior has been discussed in detail by Baginski and is a result of the gradient in the atmosphereÆs conductivity (surge impedance of global circuit) [8] complicating the response.

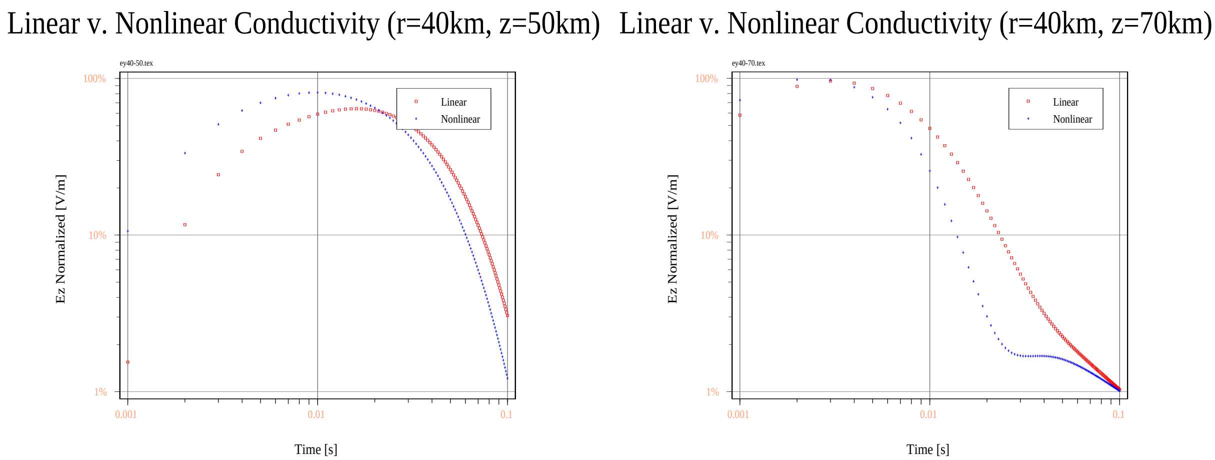

Figure 3: Comparison of vertical radial distances of r = 40 km, z = 50 km, 70 km Q_{total} = 1 and 100 Coulomb events

The tendency of the peak electric field magnitude to decrease with altitude is consistent with what conventional wisdom would suggest. Generally, the further the point of observation is from the source of the charge or current causing the electric field the smaller the magnitude of the field. This behavior is also further enhanced by the conductivity's damping of any electrodynamic transients citebaghaloli88.

4 Conclusions

The primary purpose of this study has been to investigate the effects of a sprite on the electric fields signatures In all simulations two features were evident: The greater the level of electron ionization the faster the decay following the peak field. The vertical electric field exhibits a significant a time delay prior to the onset of its peak value where as the horizontal field shows relatively immediate decay. This time delay occurs regardless of the level of ionization. Similar behavior has been observed in prior investigations involving both simulated and measured behavior [7],[6] and,[9].

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