SMALL AND LARGE SCALE SPATIAL DISTRIBUTION OF PRECIPITATION

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ABSTRACT

The paper presents the most recent contributions of the italian research to the field of the spatial distribution of rainfall intensity, with particular regard to the impact of the rain structure on the design of the modern transmission systems (both TLC and broadcasting). Considering the foreseen utilization of advanced techniques such as on-board resurce sharing, space or route diversity, adaptative antenna footprint etc. the problem of describing rain over a wide range of spatial scales has become nowadays of primary importance and models in this field are receiving particular attention.

INTRODUCTION

The subject of rain attenuation and of the spatial distribution of rain is central in the field of cm and mm waves propagation since more than 30 years [1, 2, 3]. Still nowadays, with the advent of modern TLC and broadcasting system utilizing these wavelengths, the radio-transmission systems designers need reliable physically-based models of the spatial distribution of rain over a wide range of spatial scales, allowing the evaluation of the margins necessary for both the cases of single and multiple radio links. Naturally this issue has become the subject of extensive studies by many research centers in the world and, in particular, in Italy by the Politecnico di Milano and the Fondazione Ugo Bordoni; based on a large collection of meteo-radar and rain-gauge data available in Italy, this paper overviews the milestones and the most significant advances in this field.

A STRUCTURED MODELLING APPROACH TO THE RAIN SPATIAL DISTRIBUTION.

Moving from the simplest to the most sophisticated systems encountered the history of the microwave- cm- or mm-wave transmission, the evolution of the radio transmission sow:

- single links (terrestrial or spatial; this is the case of the traditional point-to-point transmission in ku band);
- dual or triple link diversity systems as in the case of feeder-links or HUBs, requiring maximum protection especially in ka bands;
- multiple link as in LMDS (Local Multipoint Distribution Ststems, mainly in ka or V bands)
- Adaptive systems covering wide areas as TV-sat or medium- or large-scale point-to-multipoint systems (satellite constellations, drones, balloons etc. foreseen in V and perhaps in W bands).

The prediction needs for all these types of systems require a coordinated set of models for the space distribution of rain, characterized by a sound physical basis and applicable over a wide range of distances. By the Politecnico di Milano and the Fondazione Ugo Bordoni, having at disposal many raingauge and meteorological radar data, some models of this kind were devised and widely tested using the OLYMPUS and ITALSAT slant path attenuation data with the support of multifrequency radiometers data for calibration and extrapolation purposes; in increasing order of complexity these models are:

- LOWERED EXCELL model for the prediction of the exceedance probability of rain attenuation in single or multiple links and *limited extension* area services (single cells) [4, 5];
- RAINCELL SIMULATOR for the realization of *wide area* realistic rain maps to be used for simulation of complex systems like LMDS, adaptative antenna systems etc. also in presence of rain-cell clusters (multicells) [4];
- MULTIDIMENSIONAL gaussian model for for the prediction of the performance evaluation of systems utilizing the *common resource principle* (e.g. lower-frequency or high-gain back-up beams) [6].

LOWERED EXPONENTIAL CELLS MODEL
(EXCELL, a new extended version applicable down to zero rain intensity for use in low margin systems)

In this model rain is modeled by a population of circularly symmetrical raincells obeying to a “lowered” exponential law, as shown in Fig. 1.

For each raincell, the point rain-rate R at a distance ρ [km] from the center is given by the following expression:

$$R(\rho) = \left(R_M + R_{low}\right) e^{-\frac{\rho}{\rho_0}} - R_{low} \quad [\text{mm/h}] \tag{1}$$

where R_M is the peak rainfall intensity (at the center of the cell), ρ_0 defines the cell size, and R_{low}, a parameter assuming the same value for all the cells of the same population, is a small "raincell lowering parameter" introduced so as to ensure that the surface of the rainy areas, and the statistical cumulative distribution function (CDF) of the rainfall intensity $P(R)$, do not diverge for R approaching zero. The cell descriptors ρ_0 and R_M characterize different types of raincells of the same population. The Cumulative distribution function (CDF) of rainfall intensity $P(R)$ depends naturally on the single-cell characteristics ρ_0, R_M but also on the spatial density of the cells $N^*(\rho_0, R_M)$ expressing how many cells of given characteristics exist, in average, in the unit area.

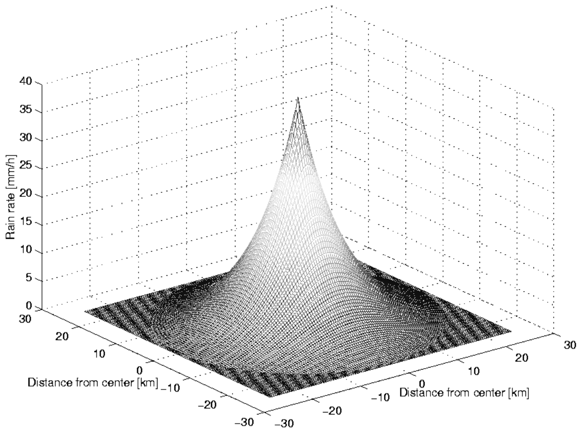

Fig.1. Lowered exponential cell model

For each raincell a surface, called *rain kernel* area $S_0(R)$, defined as the area in which the rain-rate exceeds a certain prefixed value, must be calculated; from (1) we easily obtain:

$$S_0(R, \rho_0, R_M) = \pi \rho_0^{\,2} \ln\left(\frac{R_M + R_{low}}{R + R_{low}}\right) \quad [\text{km}^2] \tag{2}$$

This quantity constitutes the individual contribution of each raincell to the overall rain statistics $P(R)$ according to the following integral:

$$\mathrm{P(R)} = \int_R^\infty \int_0^\infty S_0(R, \rho_0, R_M)\, N^*(\rho_0, R_M)\, d\rho_0 dR_M \tag{3}$$

where N^* is the spatial raincell density (number of cells per km^2 and per unit range of ρ_0 and R_M) which obeys to the mathematical expression:

$$N^*(\rho_0, R_M) = N_0^{\,*}(R_M) \frac{1}{\bar{\rho}_0} e^{-\left(\frac{\rho_0}{\bar{\rho}_0(R_M)}\right)} \quad [\text{km}^{-2}] \tag{4}$$

where $\bar{\rho}_0(R_M)$, the average raincell radius, is given by:

$$\bar{\rho}_0(R_M) = 1.7\left[\left(\frac{R_M}{6}\right)^{-10} + \left(\frac{R_M}{6}\right)^{-0.26}\right]; \quad R_M > 5 \quad [\text{km}] \tag{5}$$

By substituting (4) and (5) in (3) and integrating with respect to $\rho_0 / \bar{\rho}_0$, it can be shown that $N^*(R_M)$ can be written as a function of the rain cumulative distribution function, and more precisely:

$$N_0^{\,*}(R_M) = -\frac{1}{4\pi R_M \bar{\rho}_0^2(R_M)} \left.\frac{d^3 P(R)}{d(\ln R)^3}\right|_{R=R_M} \tag{6}$$

The definition of kernel can be easily extended to other parameters concerning propagation, e.g. a single link attenuation or a combination of attenuations in multiple links (e.g. the maximum or the minimum attenuation out of a group of n links): the new definition becomes now "the surface $S(A)$ constituted by the points in which the system undergoes an attenuation greater than A"; if we substitute the attenuation kernel area, $S(A)$ in place of the rain kernel area $S_0(R)$, (3) above still applies and gives the attenuation cumulative distribution function, $P(A)$. Therefore, we can derive the spatial raincell density $N^*(R_M,\rho_o)$ from the local $P(R)$, and then utilize this quantity to find $P(A)$. The main limitation of the model is that one single cell a time can be present.

Fig. 2 gives three examples of kernels: the first one is the kernel relative to a single link; the second one (shadowed surface) is the kernel relative to a couple of attenuations in a space diversity system (the considered attenuation here is the minimum of the 2 attenuations). It gives, when the cell peak falls in it, the occurrence of the diversity-system undergoing failure (both attenuations exceeding a threshold A); finally the third one is the kernel relative to a quadruplet of attenuations in an hypothetical satellite-based positioning system at cm wavelengths (the considered attenuation here is the maximum of 4 attenuations, rain-induced plus free-space). It gives, when the system falls in it, the occurrence of the system failure (at least one attenuation exceeding a threshold A, i.e. one of the four necessary signals missing).

For $P(R)$ we propose a *log-power low* model, which is easily differentiable with respect to $\ln(R)$ such as:

$$P(R) = P_0\left[\ln\left(\frac{R_{asymp}+R_{low}}{R+R_{low}}\right)\right]^n \tag{7}$$

where P_0, R_{asymp} and R_{low} and n can be adjusted so that (7) fits the local long-term rain statistics. Some examples are given in Table 1 which considers the “old” ITU-R rain zones. Fig. 3 shows, for the “old” *k* zone, the CD obtained using the parameters of table 1 superimposed to the ITU-R values; it can be noted that the distribution can be closed down to the zero value of R: to this purpose the customary “rainy time” condition (in Italy about 5%) can be directly forced; alternatively the total fallen water column (in one year or shorter period) can be forced.

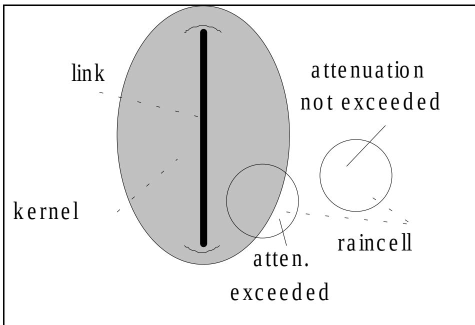	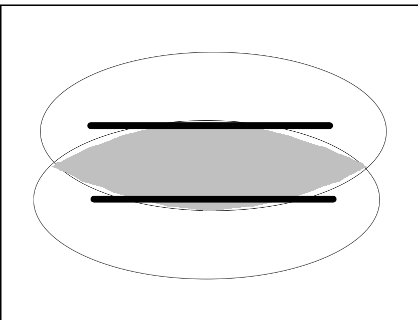	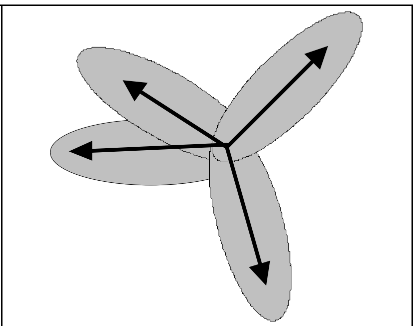
Fig. 2a. Single-link kernel: the shadowed area represents the single-link kernel; when the cell peak falls in it the link experiences an attenuation greater than a given thresold.	Fig. 2b. Composite kernel for diversity system: the shadowed area represents the dual-link kernel; when the cell peak falls in it both the links undergo an attenuation greater than threshold and the diversity countermeasure fails.	Fig. 2c. Composite kernel for a localization system: the picture represents the case of 4 satellites in geometrical visibility; when a cell falls within the grey area at least one satellite is lost and the localization fails

Fig. 2. Examples of kernels associated to various kind od systems

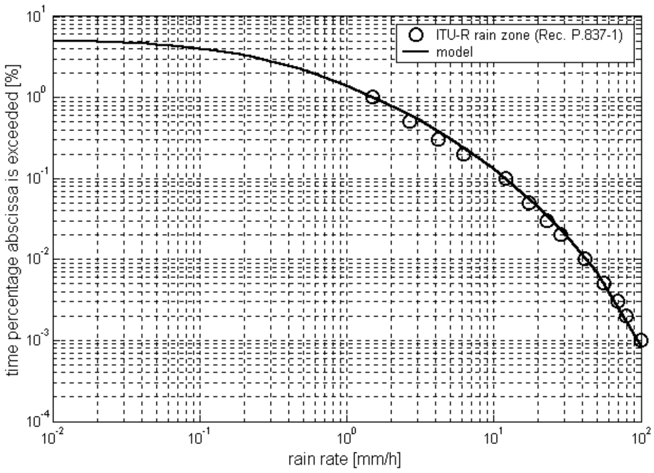

Fig. 3. Cumulative Distribution of rain for the the “old” ITU K rain zone: given points and model

Table 1. Parameters of the log-power low model for the rain CD characterizing various "old" ITU-R rain zones

Rain zone	P_o	R_{asymp}	R_{low}	n
E	0.00014	312	0.1562	5.19
G	0.00009334	260	1.048	6.381
H	0.00012	332	0.3173	5.498
J	0.0001	220	7.803	8.91
K	0.00016	400	0.2339	5.17

RAINCELL SIMULATOR

Differently from the cases considered in Fig. 2, some problems can not easily be represented by the intersection or union of simple kernels; this happens for example when it is not possible to fix a single attenuation threshold separating univoquely the "working" from the "non working" states; this is the case of LMDS systems where the possible interference from estraneous radio-bases plays the major role in the event of strongly unbalanced couplings (caused by the non uniform distribution of rain) between the users and the numerous radio-bases acting on the territory. Another important case is the one of the antenna directivity control (future TV-sat in ka band [4]) or the burst-length control (DAVID experiment) in the transmission from a satellite. In these cases a limited amount of power can be shared among many users; however, because the power allocation is adaptively rearranged at any instant, the possibility a single user to face a given level of attenuation depends on the situation of all the others and a definite kernel can not be identified.

In these cases the problem can be solved by resorting to a simulation in which a set of cells presenting a well definite spatial density are "thrown at random" on a plane where the system is placed. In any position the actual signal-to-noise C/N or signal-to-inteference C/I ratios can be evaluated and, in case, optimized either by minimizing the number of terminals in outage or reducing the transmission rate; after a sufficient number of trials many samples can be collected and statistically classified. It is worth noting that this way of proceeding allows to take into consideration the case of cell clustering or climatologically different sub-areas; this objective is simply achieved by properly acting on the abovementioned parameters of the cell families. Fig. 4 shows a sintetic distribution of cells utilized to optimize the antenna footprint of a TV-sat in ka band: in this case the cell density in any part of the served area was chosen so as to preserve the total amount of fallen rain in a period of 6 hours as determined by using ECMWF data.

MULTIDIMENSIONAL GAUSSAN MODEL

As shown in Fig. 4, the spatial cell density is not uniform in nature: indeed the presence of the meteorological fronts introduces a correlation of rain intensity up to many hundreds of kms [6].

This enhanced correlation with respect to the uniform case can be very clearly put in evidence also by the Fig. 5 which shows the ratio between the actual joint exceedance probability and the value this parameter would assume in the case of statistical independence (statistical dependence index). As shown, this ratio reaches the unit value, i.e. the value of the statistical independence, only at very large distances (>800÷1000 km).

Mathematically the progressive decorrelation can be taken into account by introducing the concept of multivariate log-normal distribution for the rain (or rain attenuation) in multiple sites [7, 8]; this model allows easily to take into account the progressive decorrelation with increasing distance by introducing a covariances matrix whose terms decrease with the distance according to a law fit on the experimental data. This conceptually simple method needs however a refinement to account for the frequent cases in which in some location it does not rain at all; in other words it is a model conditioned to the rain exceeding zero. This problem is tackled by expressing the total probability as product of two separate probabilities: the first one gives the probability that rain R_i in the i-th location exceeds zero (*rainy time*) and can be expressed by the probability $P_{conditioning}(t_i)$ that an auxiliary random variable t_i, independent of R_i, exceeds a threshold t^*_i; t_i is a multivariate joint-normal variable with zero-average and unit-power and its covariance matrix is forced to reflect the decorrelation of the rainy time with increasing distance; t^*_i is fixed so that, when exceeded by t_i , the weather is rainy (e.g. for rain time = 5% we have $\int_{t_i^*}^{\infty}\left(1/\sqrt{2\pi}\right)\exp\left(-0.5t_i^2\right)dt = 0.05 \;\Rightarrow\; t^*_i$=1.645).

The second probability $P_{conditional}$ $(\ln(R_i))$ (called "*conditional probability*") is still assumed as joint normal, characterized by averages, standard deviations and covariance matrix, fit on data.

We have then:

$$\mathrm{P_{tot}}(\mathrm{A_i}) = \mathrm{P_{conditioning}}(\mathrm{t_i})\mathrm{P_{conditional}}(\ln \mathrm{A_i}) \tag{8}$$

The process is fully described in the companion paper [8].

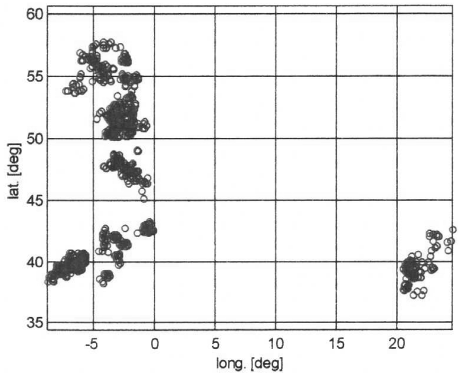

Fig. 4. Rain-cells thrown randomly on a wide area (Europe) respecting the greater concentration within the meteorological fronts

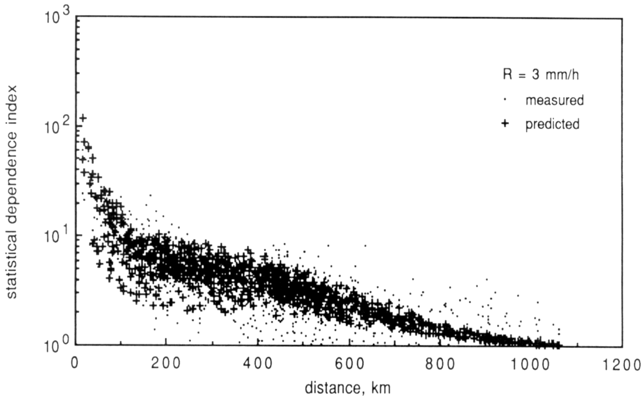

Fig. 5. Statistical dependence index (i.e joint probability that rain exceeds 3 mm/h normalized to to the statistical independence value) as a function of the distance (courtesy of Fondazione Bordoni).

CONCLUSIONS

The need of circumvent the rain attenuation effects has put the problem of devising a set of methods allowing to predict the reduction of availability and/or quality of service (QOS) in complex systems of the future like space-diversity systems, dense terrestrial networks or common-resource satellite-based TLC, broadcasting systems etc.. The paper gives some models devised by the Politecnico di Milano and Fondazione Bordoni, utilizing data collected in Italy.

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