NON-SPECULAR EFFECTS IN FRACTAL CANTOR GRATINGS

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ABSTRACT

In this paper, we use the impedance method to study the angular change, between the incident and reflected beams, from grating characterized by a Cantor density. This density is obtained from a multiplicative superposition of periodical functions. The incident beam have a supergaussian profile, and multiple angular changes must be taken into account. The results can be also extended to estimate the longitudinal change and have applications in the microoptics area.

INTRODUCTION

Optical beams undergo a lateral (Goos-Hänchen [1]) displacement at a dielectric interface if incidence is under a total-reflection regime. Furthermore, it has been demonstrated that, when an electromagnetic beam is reflected by a planar interface, some characteristics of the reflected beam are not determined by the laws of geometrical optics. Such effects are known as non-specular [2] and the longitudinal and angular shifts became the more importants [3]. Also, whereas the Goos-Hänchen shift at a single dielectric interface is of the order of a few wavelenghts, larger displacements can occur for beams reflected by layers with two or more interfaces . Recently, new calculation methods have been published for the prediction of the non-specular effects and an entropic foundation was developed [4].

For laser applications, graded reflectivity mirrors with super-gaussian profile give a better filling of the active material, generating diffraction-limited beams and modes with higher energies than gaussian reflectivity mirrors. Such property has received the attention of several authors [5]. Also, theoretical studies of the propagation of non-gaussian beams and the characterization from the experimental point of view are subjects recently developed.

In this paper, the angular and lateral shifts by reflection for monochromatic super-gaussian beams (obtained as angular superposition of planar components with wave-vector k) is computed numerically, as a function of super-gaussinity index and for the relative maximum of the intensity pattern. The gaussian profile is a particular case and the calculation can be developed for both polarizations.

The study of the interaction between electromagnetic waves and objects with different geometric forms is very important for establishing a relationship between such objects and the intensity distribution of the scattered fields. Our interest is to study this problem using fractal gratings, which possess a fine structure with complex geometric forms [6].

We are interested in characterizing complex media and multi-scale surfaces, such as those occurring with deterministic fractal objects. These are representative examples for the study of general properties for fractal electrodynamics. There are several works on this subject, which examine the scattering of electromagnetic waves by fractal surfaces from a scalar or vectorial point of view. But in this work, we use the impedance approximation.

The results of the calculated scattering are shown using a density Cantor function, for different dimensions, as a perturbation to constant impedance on the flat surface. These Cantor distributions are obtained as a product superposition of scaled periodical functions and can be combined in different ways according with the dimension or lacunarity of the corresponding Cantor set.

For certain kind of surfaces, where we can use the impedance formalism, the non-specular effects have very interesting results. In periodic grating, when an electromagnetic beam is incident on it, the non-specular effects can be a method to determine the order of the corrugation. Here, we are interested in establish a relation between the fractal grating and the non-specular effects obtained in the reflection from such gratings.

The angular displacements for different vacuum-material medium interfaces make also possible to derive directly a relationship between the angular widening of the incident beam and the correspoding width of the reflected beam.

The angular shift under reflection by a super-Gaussian beam is investigated, shown that the spectral distribution of these beams conduct to multiple non-specular effects when the paraxial approximation is applied to each maximum. The results obtained shown different displacements as a function of the central and secondary peaks, and depending on the supergaussianity index. These results allows to obtain the characteristics of the structure if the angular changes for each peaks should be measured. We believe that it can be a method to obtain the characteristics of a structure through the

characterization from the non-specular effects. The existence of several secondary maxima in super-gaussian beams would allow to obtain information from each of these maxima.
It is our objective to apply, in the future, these studies to compare different geometries for obtaining the influence of such structures on the non-specular changes here studied and so to characterize them by means of super-gaussian beams. The simple method used in the calculation of the input impedance allows such extension to many layers [7] with corrugations, applying a simple iterative method, and reducing the problem to a simple surface. Non-linear or anisotropic materials can be studied, and even, the entropy function can be related with all changes that are carried out in these structures.

MATHEMATICAL BASIS AND RESULTS OBTAINED

The impedance function Z is included into the relation between tangential components of electromagnetic fields ($\mathbf{E}_{//}$ and $\mathbf{H}_{//}$) at the interface between two media [8]. We must to define the components of the wave vector as $\alpha=k\ sin\theta$ and $\beta=k\ cos\theta$ for the intensity distribution. Also, there is a central component α_0 for the incident beam.
We use, for example TM polarization and, for this case, the integral equation for the calculation of the total fields is:

$$\tilde{H}_r(\alpha)=\frac{k\,Z_0-\beta(\alpha)}{k\,Z_0+\beta(\alpha)}\,\tilde{H}_i(\alpha)+k\,Z_0\int_{-\infty}^{+\infty}\tilde{Z}[\alpha'-\alpha]\tilde{H}_i(\alpha')\,d\alpha'-k\,Z_0\int_{-\infty}^{+\infty}\tilde{Z}[\alpha'-\alpha]\tilde{H}_r(\alpha')\,d\alpha' \qquad (1)$$

where $\tilde{Z}[\alpha'-\alpha]$ is the Fourier transform of the perturbation function and $\beta=\sqrt{k^2-\alpha^2}$. The Eqs. (4) are Freedholm integral equations of second kind, similar to those found for reflection from a metallic surface [9]. However, in this case we directly deduce them from the Fresnel coefficients.

The incident field that we will use for Eq. (1) have a supergaussian profile given by $\exp\left[-\left(\frac{x}{\sigma}\right)^{2a}\right]$. We can see that the case *a=1* represent a gaussian beam. Some cases are shown in Fig. 1 and Fig. 2 is a plot of the corresponding Fourier transform for *a=5*.

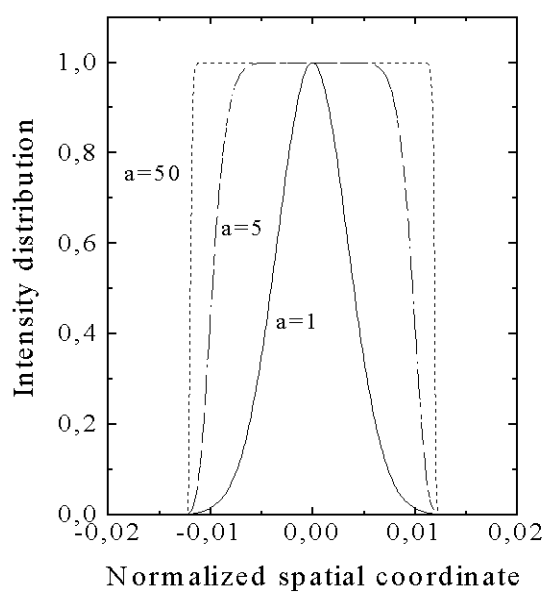


Fig. 1. Spatial distribution for super-Gaussian beams, as a function of the supergaussianity index.

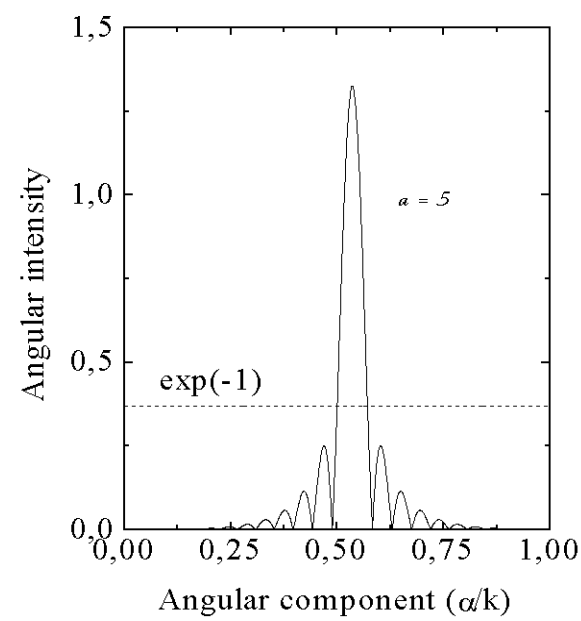


Fig. 2. The correspondig profile of the Fourier transform.

The density Cantor function *C(x)* is defined in a way that allows us to represent gratings with different dimension and lacunarity [9]:

$$C(x) = \prod_{l=1}^{N} \left\{ \sum_{i=1}^{n_1(l)} R_i \, rect\left[\frac{x - x_i}{\Delta_1}\right] \right\}^{g_l} \left\{ \sum_{j=1}^{n_2(l)} S_j \, rect\left[\frac{x - x_j}{\Delta_2}\right] \right\}^{h_l} \tag{2}$$

R_i and S_j being constant values into the interval defined for each rectangular function and centered at x_i and x_j respectively. Here, g_l and h_l can take the values within the set {0,1} and Δ_1 , Δ_2 are the corresponding periods of the rectangular scaled functions. A transmission grating with a Cantor distribution can be obtained if all exponents are equal to 1. In this way, different Cantor sets can be obtained as a product of periodical functions, which represents a generalization of previous results.

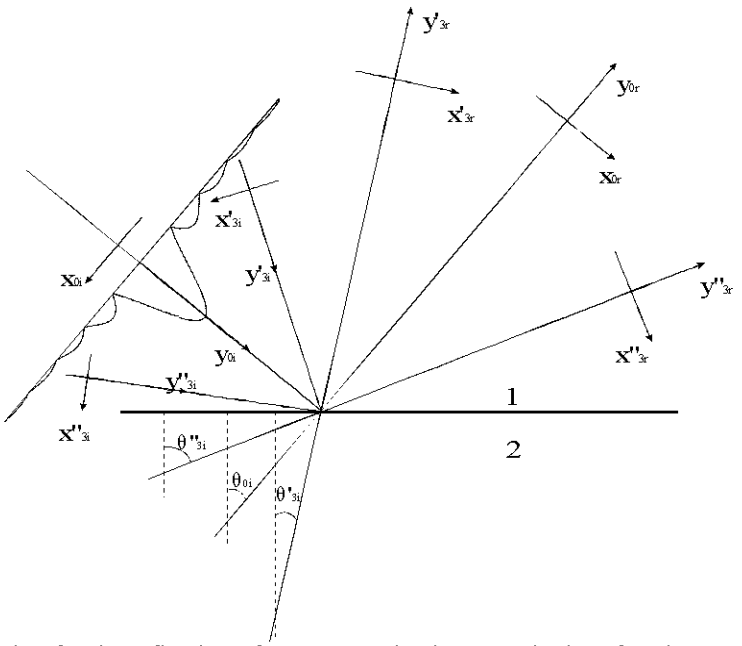
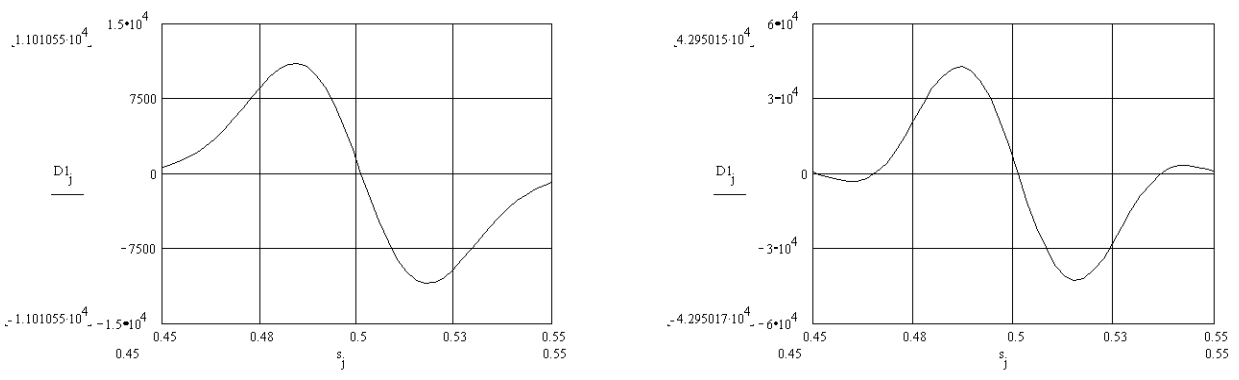


Fig. 3. Spectral representation for the reflection of super-gaussian beam at the interface between two media (1 and 2). It is shown the multiple coordinate transformations $(x_i,y_i)\rightarrow(x_r,y_r)$.



a=1 *a=5*

Fig. 4. Angular change for gaussian and supergaussian beams.

Fig. 3 show the spectral distribution of supergaussian beam and the reflection of the mean maximum and the secondary maxima. We must to use different coordinate systems for each maximum and the angular change can be calculated through:

$$\frac{d}{d\alpha}\left[R(\alpha)\,H_i(\alpha-\alpha_0)\right]=0$$

being $R(\alpha)$ the reflection coefficient.
The results obtained for two different cases are plotted in Fig. 4. The gaussian case is for the supergaussianity index $a=1$. For both cases we use the incident angle at 30 degree, the value 0.5 in the graphics. In both cases we can see the angular displacement of the reflected beam from the fractal gratings

CONCLUSIONS

We present the angular change in the reflected beam, with a supergaussian profile, from a Cantor fractal grating. This result represent an extension for the case of periodic gratings. Also, a more detailed study is necessary to analyze the change for each secondary maximum. All the results have application in integrated optics and micro-optics.

ACKNOWLEDGMENTS

This work was supported by Software Integral para Laboratorio (Sofilab) SACV, México DF, México, through the Research Project NeuroSofilab (Ref. SOF-971021-I75/2001-1) from Secretaría de Hacienda and Consejo Nacional de Ciencia y Tecnología (CONACyT, México).

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