Network-Oriented Short-Pulse Field Representations for Periodic Arrays using Time Domain Floquet Waves

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Abstract

Through use of Floquet wave (FW) representations, sequentially pulsed periodic plane arrays of dipoles form the building blocks for the efficient treatment of transient radiation from actual periodic array configurations. A network-oriented formulation is investigated here, which extends useful network and transmission line constructs from the frequency domain to the time domain (TD). TD-FW-based modal voltage and current sources on TD-FW transmission lines, and their TD voltage/current responses, are constructed from the vector field equations and, with proper normalization of the dispersive TD-FW eigenmodes, are evaluated in closed form for an infinite sequentially pulsed array in free space.

I. Introduction

Wide-band and short-pulse radiation from actual rectangular phased array antennas, infinite and truncated periodic structures, frequency selective surfaces and related applications is a topic of increasing interest. Prototype studies of short-pulse radiation by infinite and semi-infinite periodic arrays of dipoles form the basic building blocks for the efficient modeling of time-dependent radiation from, or scattering by, such periodic structures. Our planned research agenda so far has dealt with investigations of basic canonical time domain (TD) dipole-excited Green's functions for infinite [1] and truncated [2] periodic line arrays, and of infinite [3] and semi-infinite [4] periodic planar arrays. Through use of Poisson summation, the TD element-by-element radiations have been converted to superpositions of nontruncated [1] or truncated conical TD-Floquet waves (FW), truncation-induced TD-FW-modulated tip diffractions [2], and nontruncated [3] as well as truncated planar TD-FWs with truncation-induced TD-FW modulated edge diffractions [4], respectively. These problem-matched modelings pertaining to spatial periodicity furnish understanding of the TD-FW critical parameters and the corresponding phenomenologies. We have used exact constructions as well as asymptotics, directly in the TD, when possible [1]-[3], or via initial formulation in the frequency domain (FD) with subsequent inversion to the TD. Asymptotic inversion from the FD yields the *instantaneous* frequencies, with corresponding *instantaneous wavenumbers*, which parameterize the constituent TD-FW behavior.

In the present investigation, we take the first step toward constructing a TD *network-oriented* theory of pulsed radiation from, or scattering by, planar sequentially excited array configurations, with a view toward applying this methodology subsequently to arrays on or within layered dielectrics. Referring to previous results for infinite planar dipole arrays [4], we exhibit first some of the properties, such as orthogonality and completeness, pertaining to the TD-FW as basis sets, and deal directly with the electric and magnetic vector fields instead of the electric vector potential as in [4]. This is important for the study of wide-band periodic structures when the TD-FW basis is used to represent the equivalent electric and magnetic current distributions for actual patch arrays or other printed geometries, in conjunction with numerical techniques like the finite difference time domain (FDTD) method. In this connection, we show how to project transient fields onto a modal TD-FW basis; this may serve as the initial condition for the propagation of TD-FWs away from the array. The resulting analytic propagators may be used to characterize the behavior of several layers of periodic structures in terms of scattering or other problem-matched matrix network formulations directly in the TD. We also formalize the TD network problem in terms of TD transmission lines (TL), for TEM, TE and TM TD-FWs, with TD voltage and current source generators for excitation of the TD voltage and current response. To learn the TD rules, we proceed by Fourier inversion of the well established FD network methodology [5]. We note that in the special case of *nonphased* arrays the FW eigenmodes are nondispersive and reduce the TD field solutions to the closed forms for conventional waveguides [6]. Practical implementation of the analytic results for nonphased infinite arrays excited by arbitrarily shaped printed elements has been explored in [7], using an efficient hybrid combination of the FDTD method and the TD-FW field expansion. Transient radiation from a wideband array of bowtie antennas has been calculated in this manner by retaining only a few TD-FWs which are excited by localized current or voltage band-limited pulsed modal generators. However, the TD-FW treatment of *phased* arrays with

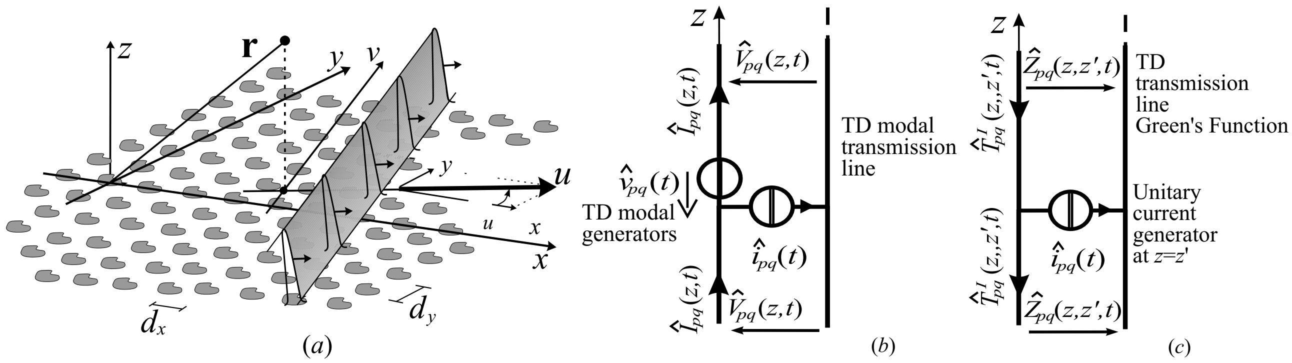

Fig. 1. Generic planar periodic array geometry of elementary radiators, and TD-TL schematizations of the FW-based modal fields and Green's functions. (a) Array geometry. d_x, d_y: interelement spacing along x and y, respectively; $\eta\omega/c = k\eta$: phase gradient of the excitation (i.e., the wavefront) along the direction $\mathbf{1}_u$(see [3]); $v_u^{(p)} = c/\eta$: phase speed along $\mathbf{1}_u$. (b) Equivalent TD transmission line for pqth FW, with modal voltage $\hat{V}_{pq}$ and current $\hat{I}_{pq}$, excited by modal voltage and current sources $\hat{v}_{pq}$ and $\hat{i}_{pq}$. (c) TD transmission line voltage and current Green's functions $\hat{Z}(z, z', t)$ and $T^I(z, z', t)$ excited by a unit *current* generator at $z = z\prime$.

their dispersive eigenmodes require convolutions between TD voltages/currents and TD-FW eigenmodes which complicate the analysis. These and related aspects are explored in the presentation that follows.

II. Statement of the Problem

We consider the generic infinite periodic array geometry shown in Fig.1a, with periodicities d_x and d_y along the x and y directions, respectively; the corresponding TD transmission line (TL) representations for the FW-based modal fields and Green's functions are schematized in Figs.1b and 1c, respectively. Concerning notation, a caret ˆ tags time-dependent quantities; bold face symbols define vector quantities; $\mathbf{1}_x$, $\mathbf{1}_y$ and $\mathbf{1}_z$ denote unit vectors along x, y, and z, respectively; the observation point is denoted by $\mathbf{r} = \boldsymbol{\rho} + z\mathbf{1}_z$, with $\boldsymbol{\rho} = x\mathbf{1}_x + y\mathbf{1}_y$. The FW-based modal FD and TD fields due to the array are related by the Fourier transform pair $f(\mathbf{r}, \omega) = \int_{-\infty}^{\infty} \hat{f}(\mathbf{r}, t)e^{-j\omega t}dt$, $\hat{f}(\mathbf{r}, t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(\mathbf{r}, \omega)e^{j\omega t}d\omega$, in which f can be either a scalar or a vector quantity. These Fourier -transform-related fields satisfy the respective FD and TD periodicity conditions

$$\mathbf{E}(\mathbf{r} + \mathbf{d}, \omega) = \mathbf{E}(\mathbf{r}, \omega)e^{-j\eta(\omega/c)(\mathbf{1}_u\cdot\mathbf{d})}, \qquad \hat{\mathbf{E}}(\mathbf{r} + \mathbf{d}, t) = \hat{\mathbf{E}}(\mathbf{r}, t - \eta(\mathbf{1}_u \cdot \mathbf{d})/c) \tag{1}$$

where $\mathbf{d} = d_x\mathbf{1}_x + d_y\mathbf{1}_y$, $k = \omega/c$ denotes the ambient wavenumber and c denotes the ambient wavespeed. In the FD, the composite linear phasing on the array is along the direction $\mathbf{1}_u$, perpendicular to $\mathbf{1}_v = \mathbf{1}_z \times \mathbf{1}_u$ (see Fig.1a), with *projected* phasings $\omega\eta_x/c$ and $\omega\eta_y/c$ along the x and y directions, respectively. In the TD, this translates into intercell excitation delayed by $\eta(\mathbf{1}_u \cdot \mathbf{d})/c$. The important nondimensional *single* parameter

$$\eta = \sqrt{\eta_x^2 + \eta_z^2} = c/v_u^{(p)}, \tag{2}$$

which is tied to the rotated coordinate system defined by u (see Fig.1),combines both phasings η_x and η_z. In (2), $v_u^{(p)} = c/\eta$ is the impressed phase speed along u. The TD cutoff condition $\eta = 1$ ($v_u^{(p)} = c$) separates two distinct wave dynamics. Here, we treat the case $\eta < 1$ which implies excitation phase speeds $v_u^{(p)} = c/\eta$ (and corresponding *projected* phase speeds c/η_x and c/η_z) larger than the ambient wavespeed c.

III. Frequency-Domain Modal Representation of the FW-based Fields and Their Sources: FD Modal Transmission Line Fields and Green's Functions

In the FD, the transverse (to z) field is expressed conventionally in terms of a complete orthogonal eigenvector basis set (here, the (p, q)-indexed FW modes) comprising both E (TM) mode functions $\mathbf{e}_{pq}^E(\boldsymbol{\rho}, \omega)$, $\mathbf{h}_{pq}^E(\boldsymbol{\rho}, \omega)$ and H (TE) mode functions $\mathbf{e}_{pq}^H(\boldsymbol{\rho}, \omega)$, $\mathbf{h}_{pq}^H(\boldsymbol{\rho}, \omega)$,

$$\mathbf{E}_t(\mathbf{r}, \omega) = \sum_{p,q} V_{pq}(z, \omega)\mathbf{e}_{pq}(\boldsymbol{\rho}, \omega), \qquad \mathbf{H}_t(\mathbf{r}, \omega) = \sum_{p,q} I_{pq}(z, \omega)\mathbf{h}_{pq}(\boldsymbol{\rho}, \omega) \tag{3}$$

where the summation extends over both E and H modes, and an $\exp(j\omega t)$ dependence is suppressed. The eigenfunctions $\mathbf{e}_{pq}$ and $\mathbf{h}_{pq}$ satisfy the z-independent equations in [5, eq.(10), p.188]. Excitations by modal transverse equivalent electric and magnetic current sources $\mathbf{J}_{te}(\mathbf{r},\omega)$ and $\mathbf{M}_{te}(\mathbf{r},\omega)$ are likewise represented in terms of this eigenbasis, with amplitudes given by the strengths i_{pq} and v_{pq} of current and voltage generators: $\mathbf{J}_{te}(\mathbf{r},\omega) = \sum_{p,q} i_{pq}(z,\omega)\mathbf{e}_{pq}(\boldsymbol{\rho},\omega)$, $\mathbf{M}_{te}(\mathbf{r},\omega) = \sum_{p,q} v_{pq}(z,\omega)\mathbf{h}_{pq}(\boldsymbol{\rho},\omega)$. The z components of the field can be obtained from these transverse components. Alternative options can be exercised for normalization of the eigenbasis. For our purposes, as seen in Sec.IV, we choose the orthogonal (but *not* orthonormal) set

$$\mathbf{e}^E_{pq}(\boldsymbol{\rho},\omega) = \frac{j\mathbf{k}_{t,pq}e^{-j\mathbf{k}_{t,pq}\cdot\boldsymbol{\rho}}}{\sqrt{d_x d_y}}, \qquad \mathbf{e}^H_{pq}(\boldsymbol{\rho}) = \mathbf{e}^E_{pq}(\boldsymbol{\rho}) \times \mathbf{1}_z, \qquad \mathbf{h}_{pq} = \mathbf{1}_z \times \mathbf{e}_{pq} \tag{4}$$

for both E and H modes. Here, $\mathbf{k}_{t,pq} = \eta(\omega/c)\mathbf{1}_{u_1} + \boldsymbol{\alpha}_{pq}$ is the transverse Floquet wavenumber with amplitude $k_{t,pq} = |\mathbf{k}_{t,pq}| = \left[(\eta\frac{\omega}{c} + \mathbf{1}_{u_1}\cdot\boldsymbol{\alpha}_{pq})^2 + (\mathbf{1}_{u_2}\cdot\boldsymbol{\alpha}_{pq})^2\right]^{1/2}$ and frequency-independent component $\boldsymbol{\alpha}_{pq} = \alpha_{x,p}\mathbf{1}_x + \alpha_{y,q}\mathbf{1}_y$, with $\alpha_{x,p} = 2\pi p/d_x$, $\alpha_{y,q} = 2\pi q/d_y$, $\alpha_{pq} = |\boldsymbol{\alpha}_{pq}| = \sqrt{\alpha^2_{x,q} + \alpha^2_{y,p}}$. The eigenfunctions in (4) satisfy the orthogonality conditions $< \mathbf{e}_{pq}; \cdot\mathbf{e}_{p'q'} >= k^2_{t,pq}\delta_{pp'}\delta_{qq'}$ (which defines the normalization), and $< \mathbf{e}^E_{pq}; \cdot\mathbf{e}^H_{p'q'} >= 0$; the inner product is defined as $< \mathbf{e}_{pq}; \cdot\mathbf{e}_{p'q'} >= \int_0^{d_x}\int_0^{d_y} \mathbf{e}_{pq}\cdot\mathbf{e}^*_{p'q'}dxdy$ where * denotes the complex conjugate. The generator strengths $i_{pq}(\omega)$ and $v_{pq}(\omega)$ (see Figs.1b,c) exciting the various E and H modes are found by projecting the total equivalent electric and magnetic currents onto each FW mode: $i_{pq}(z,\omega) = k^{-2}_{t,pq} < \mathbf{J}_{te}(\mathbf{r},\omega); \mathbf{e}_{pq}(\boldsymbol{\rho},\omega) >$, $v_{pq}(z,\omega) = k^{-2}_{t,pq} < \mathbf{M}_{te}(\mathbf{r},\omega); \mathbf{h}_{pq}(\boldsymbol{\rho},\omega) >$ (see [5, p.189]). For both E and H modes, the TL voltages $V_{pq}(z,\omega)$ in (3) are obtained by superposing contributions from appropriate *point* voltage and current generators distributed along z': $V_{pq}(z,\omega) = -\int dz' T^V_{pq}(z,z',\omega)v_{pq}(z',\omega) - \int dz' Z_{pq}(z,z',\omega)i_{pq}(z',\omega)$, in which we have introduced the TL *Green's functions*

$$\left.\begin{array}{l} Z_{pq}(z,z',\omega) \\ T^V_{pq}(z,z',\omega) \end{array}\right\} = \frac{1}{2}\left\{\begin{array}{l} Z_{0,pq}(\omega) \\ \mathrm{sgn}(z-z') \end{array}\right\} e^{-jk_{z,pq}(\omega)|z-z'|} \tag{5}$$

with $k_{z,pq}(\omega) = \sqrt{\left(\frac{\omega}{c}\right)^2 - k^2_{t,pq}}$, and $\Im m k_{z,pq}(\omega) < 0$ and $\Re e k_{z,pq}(\omega) \geq 0$ or ≤ 0 for $\omega > 0$ or $\omega < 0$, respectively [5, pp.207], [1], [3]. The characteristic impedances are given by $Z^E_{0,pq}(\omega) = k_{z,pq}(\omega)/(\omega\epsilon)$, $Z^H_{0,pq}(\omega) = \omega\mu/k_{z,pq}(\omega)$, for E and H modes, respectively. Analogous considerations apply to the currents $I_{pq}(z,\omega)$ in (3).

IV. Time-Domain Modal Representation of the FW-based Fields and Their Sources: TD Modal Transmission Line Fields and Green's Functions

The TD-TL representations are here obtained by Fourier-transforming their FD counterparts. Thus, the transverse (to z) TD field is expressed in terms of a time-dependent complete orthogonal eigenvector set comprising both E (TM) and H (TE) mode functions $\hat{\mathbf{e}}_{pq}(\boldsymbol{\rho},t)$ and $\hat{\mathbf{h}}_{pq}(\boldsymbol{\rho},t)$,

$$\hat{\mathbf{E}}_t(\mathbf{r},t) = \sum_{p,q}\hat{V}_{pq}(z,t)\otimes\hat{\mathbf{e}}_{pq}(\boldsymbol{\rho},t), \qquad \hat{\mathbf{H}}_t(\mathbf{r},t) = \sum_{p,q}\hat{I}_{pq}(z,t)\otimes\hat{\mathbf{h}}_{pq}(\boldsymbol{\rho},t), \tag{6}$$

where $\otimes$ denotes time convolution and the summation extends over both E and H modes. As in the FD case, excitations by TD modal sources $\hat{\mathbf{J}}_{te}(\mathbf{r},t)$ and $\hat{\mathbf{M}}_{te}(\mathbf{r},t)$ are likewise represented in terms of this eigenbasis, with amplitudes given by the strengths $\hat{i}_{pq}$ and $\hat{v}_{pq}$ of current and voltage generators: $\hat{\mathbf{J}}_{te}(\mathbf{r},t) = \sum_{p,q}\hat{i}_{pq}(z,t)\otimes\hat{\mathbf{e}}_{pq}(\boldsymbol{\rho},t)$, $\hat{\mathbf{M}}_{te}(\mathbf{r},t) = \sum_{p,q}\hat{v}_{pq}(z,t)\otimes\hat{\mathbf{h}}_{pq}(\boldsymbol{\rho},t)$. The eigenfunctions in (4) are readily transformed into the TD in view of the chosen normalization, leading to the closed form

$$\hat{\mathbf{e}}^E_{pq}(\boldsymbol{\rho},t) = \frac{e^{-j\boldsymbol{\alpha}_{pq}\cdot\boldsymbol{\rho}}}{\sqrt{d_x d_y}}\left\{\frac{\eta}{c}\mathbf{1}_u\delta'(\tau) + j\boldsymbol{\alpha}_{pq}\delta(\tau)\right\}, \qquad \tau = t - \eta\mathbf{1}_u\cdot\boldsymbol{\rho}/c, \tag{7}$$

together with $\hat{\mathbf{e}}^H_{pq}(\boldsymbol{\rho},t) = \hat{\mathbf{e}}^E_{pq}(\boldsymbol{\rho},t)\times\mathbf{1}_z$. Thus, the eigenfunctions are pulsed FW-modulated slant-stacked plane wavefronts. The magnetic mode functions are given by $\hat{\mathbf{h}}_{pq} = \mathbf{1}_z\times\hat{\mathbf{e}}_{pq}$. Expression (7) is particularly convenient

since the Dirac delta functions reduce the convolutions in (6) to closed-form sampling of the voltages and currents (and their derivatives) at the retarded time $\tau = t - \eta \mathbf{1}_u \cdot \boldsymbol{\rho}/c$ (see [1], [3] for a physical interpretation in terms of a moving coordinate system). Other normalizations, in particular the standard *orthonormal* choice which has a factor $k_{t,pq}$ in the denominator of (4), do not simplify the convolution and therefore require numerical techniques for evaluation.
For both E and H modes, the TD-TL voltages $\hat{V}_{pq}(z,t)$ in (6) are obtained by superposing contributions from appropriate *point* voltage and current generators distributed along z':

$$\hat{V}_{pq}(z,t) = -\int dz' \hat{T}^{V}_{pq}(z,z',t) \otimes \hat{v}_{pq}(z',t) - \int dz' \hat{Z}_{pq}(z,z',t) \otimes \hat{i}_{pq}(z',t). \tag{8}$$

Solutions for the TD-TL Green's functions $\hat{Z}_{pq}(z,z',t)$ and $\hat{T}^{V}_{pq}(z,z',t)$, excited by delta function current and voltage generators at z' (see Figs.1b,c), are found via Fourier inversion from the FD solutions in (5), which can be evaluated in closed forms in terms of Hankel functions and Incomplete Lipschitz-Hankel Integrals (see [7] for the nonphased array case $\eta = 0$, and [6],[8],[9] for use of Incomplete Lipschitz-Hankel Integrals). The TD modal voltage and current generator strengths $\hat{v}_{pq}(t)$ and $\hat{i}_{pq}(t)$ are found as before by projecting the total equivalent electric and magnetic TD currents onto each FW mode: $\hat{v}_{pq}(z,t) =< \hat{\mathbf{M}}_{te}(\mathbf{r},t); \hat{\mathbf{e}}^{\dagger}_{pq}(\boldsymbol{\rho},t) >_t$, $\hat{i}_{pq}(z,t) =< \hat{\mathbf{J}}_{te}(\mathbf{r},t); \hat{\mathbf{e}}^{\dagger}_{pq}(\boldsymbol{\rho},t) >_t$, in which the subscript $_t$ denotes time convolution in the inner product. The eigenfunction $\hat{\mathbf{e}}^{\dagger}_{pq}(\boldsymbol{\rho},t)$ is defined as the Fourier transform of $k^{-2}_{t,pq}(\omega)\mathbf{e}_{pq}(\boldsymbol{\rho},\omega)$ which can also be evaluated in simple closed form. The proposed TD-TL formulation for phased (i.e., sequentially excited) periodic structures reduces to the simpler nonphased case (simultaneous excitation) treated in [7] by setting $\eta = 0$.

V. **Conclusions**

In this investigation, we have taken the first step toward constructing a time domain network theory for radiation from, or scattering by, pulsed sequentially excited infinite planar periodic structures. To learn the rules, we have begun with the free-space environment, and have proceeded by Fourier-inverting the well-known network/transmission line (TL) machinery from the frequency domain. We have defined FW-based TD modal sources (generators) for excitation of modal voltage and current responses on modal TD-TLs. We have noted that the resulting TD-TL Green's functions can be evaluated in closed form. The proposed field representation is particularly efficient since, as already observed in [1]-[4], [7], only a few FW modes are necessary to describe the pulsed propagation at any space-time point away from the array. The TD analytic results obtained so far now need to be examined for their *physical interpretation*, i.e., how the TD response is parameterized by the FD constituents, like the frequency-dependent modal characteristic impedance, etc. Extension to radiation by periodic arrays on/within dielectric layers is a future objective.

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