MODELING OF "INSTANTANEOUS" ESTIMATION OF RADAR OBJECT BACK-SCATTERING MATRIX AND CO-ORDINATE PARAMETERS

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ABSTRACT

The results of math modeling of fluctuating reflectors back scattering matrix (BSM) correct ("instantaneous") estimation are described in the paper. The algorithm is based on complex quasi-continuous orthogonal signals. These signals are used to form orthogonal by polarization illumination [1-3]. There is investigated dependence of the received BSM components estimation errors on the kind of orthogonal signal uncertainty function. To carry out the investigation there has been written a program complex modeling the process of signal forming and reception in case of active polarization radar sounding non-stable objects.

THEORETICAL ASPECTS OF THE CORRECT BSM ESTIMATION

Realization of real media and objects of active radars BSM correct estimation methods meets some problems. First of all, it is connected with the fact, that in order to solve the problem, it is necessary to form non-polarized illumination, which orthogonal (by polarization) components utilize orthogonal (by time) scalar signals having the same frequency bands and the same duration. These demands follow from the analysis of relation between the vectors of the illuminated electromagnetic fields in case when takes place the interaction of the illuminated field \( e_0 \) and the radar object described by the BSM \( S \):

\[
e_0 = S \cdot e_0
\]

In general case the Jones vector \( e_0 \) can be written:

\[
e_0 = \begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix}
\]

where functions \( f_1(t) \), \( f_2(t) \) describe complex envelopes of orthogonal (by polarization) components of the illuminated field \( e_0 \). 2x2 BSM \( S \) in (1) is defined by four complex values \( S_{ij} \):

\[
S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}
\]

Solution of the equation (1) with the respect to \( S \) is possible when there are answered some conditions of functions \( f_1(t) \), \( f_2(t) \). Multiplying from the left (by Kronecker) both parts of the equation (1) by vector \( e_0 \) and averaging on the time interval of functions \( f_1(t) \), \( f_2(t) \), definition, one can get:

\[
J = \{ \{ e_0 \otimes e_0 \} \} = \{ S \cdot e_0 \otimes e_0 \}
\]

where symbol \( \otimes \) and brackets \( \{ \} \) mean operations of multiplying by Kronecker and averaging. If BSM parameters fluctuate in such a way that operator \( S \) is practically the same on the interval of averaging in (4), the equation (4) can be written as follows:

\[
J = \{ \{ e_0 \otimes e_0 \} \} = \{ S \cdot e_0 \otimes e_0 \} = S \cdot \{ e_0 \otimes e_0 \} = S \cdot M_0
\]

Equation (5) has a solution with the respect of operator \( S \) if matrix of coherency \( M_0 \) of the illuminated signal \( e_0 \) should be non-generated and consequently:

\[
det M_0 \neq 0
\]

Only in such case the operator \( J \) in (5) contains the whole information about the radar object BSM, because the operator \( S \) can be described as \( S = J \cdot M_0^{-1} \). The needed properties of the scalar signals \( f_1(t) \), \( f_2(t) \) comes from (6). These signals should not be correlated on the interval of their definition for all possible time and frequency shifts between them.

In case, when during the time interval \( T \) of illuminated vector signal \( e_0 \) existence the BSM \( S \) changes noticeably, operator
\( J \) in (4) is only an estimation of the matrix on the interval \( T \). It is necessary to keep the symmetry relation of the operator \( J \) according to the symmetry relation of BSM \( S \), as a math object. The symmetry relation contains the information about radio-physical properties of a radar object \([3,4]\). The necessity to keep the BSM symmetry relation leads to the equation:

\[
J_{21} = \{S_{21} \cdot f_1\} \oplus f'_1 + \{S_{22} \cdot f_2\} \oplus f'_2 = \{S_{12} \cdot f_1\} \oplus f'_1 + \{S_{11} \cdot f_2\} \oplus f'_2 = J_{12}
\]

where the symbol \( \oplus \) operation of time convolution and \( S_{ij} = S_{ji} \). There arises a question about equal rights of different pairs of orthogonal signals \( f_1(t), f_2(t) \) in the mentioned above problem, because orthogonality can be provided independently on the kind of these signals function of uncertainty, if all other conditions are equal.

**APPLIED ASPECTS OF ORTHOGONAL SIGNAL APPLICATION FOR CORRECT ESTIMATION OF A NON-STABLE OBJECT BACK SCATTERING MATRIX**

Influence of orthogonal signals isolation on accuracy of estimation of a stable object BSM has been investigated by the authors in \([3]\). In case, when the object is a stable radar object against non-stable back-ground (the sum of elementary moving independently reflectors in the frames of an element of resolution by range and angle co-ordinates), the scattering properties of such element of resolution can be described as follows:

\[
S = S_s + N_c
\]

where \( S_s \) is BSM of a stable radar object, \( N_c \) is BSM of back-ground. If the elementary reflectors, forming the back-ground, are absolutely independent and reciprocal, the back-ground BSM can be written:

\[
N_c = \begin{pmatrix}
n_{11}(t) & n_{12}(t) \\
n_{21}(t) & n_{22}(t)
\end{pmatrix}
\]

where \( n_{ij}(t) \) are complex random processes; due to the mentioned reciprocity the relation \( n_{12}(t) = n_{21}(t) \) takes place.

In the process of modeling there were used two pairs of orthogonal signals. One of the pairs (LFM) was formed by two synchrony in time radio-pulses with duration \( \tau_0 \), up and down laws of modulation and frequency deviation \( \mp \Omega / 2 \) relatively the central frequency \( \omega_0 \). Another pair of signals was formed by two orthogonal binary m-sequences. It was represented by two synchrony in time radio-pulses with duration \( \tau_s \) and central frequency \( \omega_0 \). Phase of oscillations (inside the pulses) changed by 180° according to the law defined by the m-sequences (PCM). Bases of both pairs was chosen the same and equal \( N = \tau_0 \cdot \Omega = 126 \).

According to (8), (9), the equal elements \( S_{ij}, S_{ji} \) in (7) were formed as a sum \( S_{ij} = S_{ji} = 1 + n(t) \). The complex random process \( n(t) \) was formed out of the white noise with dispersion \( \sigma = 1 \) by processing it with a filter of law frequencies and exponential replica. It allows to change efficiently its auto-correlation properties by changing the filter frequency band. In case of chosen parameters the relation between the stable and disturbing components of elements \( S_{ij}, S_{ji} \) of BSM of the modeled fluctuating object are equal to unity. It means that reflected power of the stable object and the back-ground is chosen the same. In Fig.1 and Fig.2 there are shown correlation and spectral characteristics of the chosen pairs (LFM and PCM) of orthogonal signals.

Fig. 3-5 show replica functions \( J_{12}(t) = J_{21}(t) \), formed according to (7) for different relation between correlation intervals of process \( n(t) \) and duration \( \tau_0 \) of orthogonal pairs of LFM and PCM signals. Fig.3 shows the situation when correlation interval of the reflections from the back-ground is much smaller than duration of the signals. In such a case the replicas \( J_{12} \) and \( J_{21} \) qualitatively are the same, the difference depends only on the level of mutual correlation of each of orthogonal pairs (for the chosen base \( N = 126 \) the isolation is -15 dB).

![Fig.1. Spectrum of magnitudes and convolution products of LFM signals pair.](image-url)
Fig. 2. Spectrum of magnitudes and convolution products of PCM signals pair.

Fig. 3. Replicas $J_{12}$ and $J_{21}$ for LFM and PCM signal pair for the case of $\tau_s \ll \tau_0$.

Fig. 4 shows the case when correlation interval of the reflections from the back-ground is comparable with the duration $\tau_0$ of the signals used. In such a case the difference of replicas $J_{12}$ and $J_{21}$ for LFM signals is much larger than for PCM signals. In the vicinity of the main lobe of the replica functions of the filters in the channels $J_{12}$ and $J_{21}$ appear false pikes. The difference of these pikes defines asymmetry of the BSM estimation. In the vicinity of the radar resolution strobe, corresponding a stable object, the difference of $J_{12}$ and $J_{21}$ in case of PCM signals is much smaller. Replicas $J_{12}$ and $J_{21}$ in case of LFM signals are practically independent in these strobes. As the qualitative difference of LFM and PCM signals depends on their functions of uncertainty, it is possible to say that the influence of back-scattering non-stability on the accuracy of BSM estimation is minimal in case of orthogonal signals with needle function of uncertainty.

Fig. 4. Replicas $J_{12}$ and $J_{21}$ for LFM and PCM signal pair for the case of $\tau_s \approx \tau_0$. 
Fig. 5 shows the case when correlation interval of the reflections from the back-ground is much larger than duration of the signals. In such a case the difference of replicas $J_{12}$ and $J_{21}$ qualitatively is the same as in the case shown in Fig.3.

![Diagram of replicas $J_{12}$ and $J_{21}$ for LFM and PCM signal pair for the case of $\tau_c \gg \tau_0$](image)

Let we take a good look to BSM invariant estimation for the case $\tau_c \approx \tau_0$ and using of LFM signal pair. Using such signals at receiving offers erroneous conclusion about the radar object non-reciprocity due non-equality $J_{12}$ and $J_{21}$ in the area of the main lob and the first side lob (see Fig.4, top). On the other hand, PCM pair (see Fig.4, top) offers right conclusion about this radar object reciprocity.

**CONCLUSIONS**

The materials of the investigation and the results of modeling allows to line out the ways of solving the problem connected with the synthesis of optimal signals and algorithms of polarization radar functioning. The demand of minimal level of mutual correlation of the utilized signals [1-3] is necessary but not enough. The best orthogonal signals in case of non-stable objects are noise-like signals with needle function of uncertainty.

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**REFERENCES**


