FAST INTEGRAL METHODS FOR VOLUMETRIC STRUCTURES

K. Sertel and J. L. Volakis
Radiation Laboratory
Department of Electrical Engineering and Computer Science
The University of Michigan
Ann Arbor, MI 48109-2122

ABSTRACT

In this paper we discuss the development and implementation of volumetric integral equations for both dielectric and magnetically permeable materials using curvilinear hexahedral elements. Both piecewise constant and higher order basis functions will be examined in the context of volumetric multilevel fast multipole method implementation. Comparisons with corresponding finite element–boundary integral solutions will also be presented and compared in terms of accuracy and efficiency.

INTRODUCTION

Accurate solution methods for scattering and radiation problems involving composite materials have constituted a significant part of computational electromagnetics research. The Finite Element-Boundary Integral (FE–BI) method has been the preferred analysis tool for problems involving composite materials [1]. However, being a partial differential equation (PDE) method, the accuracy of the FE-BI solution is affected by numerical error propagation. For electrically large solution domains, using the integral equation approach can generate more accurate results [2]. In light of recent developments in fast algorithms for integral equation methods, such as the multilevel fast multipole method (MLFMM) [3, 4] and advances in computer technology, full-wave solutions involving electrically large geometries have become realizable.

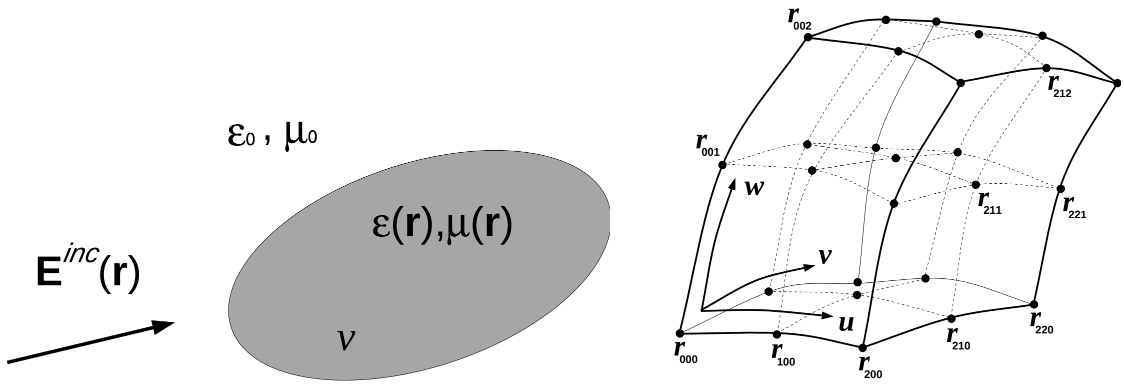
In this paper, we adapt and use the MLFMM to volume integral equations (VIE). An integral representation in terms of the electric field intensity inside the problem geometry is used. This VIE introduces differentiability requirements on the basis function to be utilized. This requirement can be relaxed if one resorts to an alternative volume-surface integral equation representation [5].

INTEGRAL EQUATION FORMULATION

For the setup shown in Fig. 1 (a), direct integration of Maxwell's equations results in a volume integral equation of the form

$$\begin{aligned} \mathbf{E}(\mathbf{r}) &= \mathbf{E}^{inc}(\mathbf{r}) + k_0^2 \int d\mathbf{r}' \mathbf{G}(\mathbf{r},\mathbf{r}') \cdot (\overline{\epsilon}_r - 1) \cdot \mathbf{E}(\mathbf{r}') \\ &+ \int d\mathbf{r}' \nabla' \times \mathbf{G}(\mathbf{r},\mathbf{r}') \cdot (\overline{\mu}_r^{-1} - 1) \cdot \nabla' \times \mathbf{E}(\mathbf{r}') \end{aligned} \quad (1)$$

where, $\mathbf{E}^{inc}(\mathbf{r})$ is the excitation generated by impressed sources.



(a) (b)

Figure 1: (a) Problem geometry (b) Parametric conformal hexahedral finite elements

Numerical discretization of (1) via the Method of Moments (MoM) implies a basis function expansion for the unknown function $\mathbf{E}(\mathbf{r})$. However, for magnetic materials these basis functions must be chosen to be of higher order so that the curl operator in the second integral in (1) can be properly evaluated and tested. For non-magnetic materials, (1) simplifies to

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^{inc}(\mathbf{r}) + k_0^2 \int d\mathbf{r}' \mathbf{G}(\mathbf{r}, \mathbf{r}') \cdot (\overline{\epsilon}_r - 1) \cdot \mathbf{E}(\mathbf{r}'). \tag{2}$$

By duality, for purely permeable materials, an integral equation for the magnetic field intensity takes the form

$$\mathbf{H}(\mathbf{r}) = \mathbf{H}^{inc}(\mathbf{r}) + k_0^2 \int d\mathbf{r}' \mathbf{G}(\mathbf{r}, \mathbf{r}') \cdot (\overline{\mu}_r - 1) \cdot \mathbf{H}(\mathbf{r}'), \tag{3}$$

and a corresponding integral equation for the electric field intensity is

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^{inc}(\mathbf{r}) + \int d\mathbf{r}' \nabla' \times \mathbf{G}(\mathbf{r}, \mathbf{r}') \cdot (\overline{\mu}_r^{-1} - 1) \cdot \nabla' \times \mathbf{E}(\mathbf{r}'). \tag{4}$$

Of course, $\mathbf{E}(\mathbf{r})$ can be obtained from (3) using Maxwell's equations. Thus, one can solve (3) and (4) and then correlate the results to verify each other's accuracy. Of particular interest is to verify (4) using the more standard integral equation (3).

Having established an accurate solution of (4), we can then combine it with (2) to formulate general material structures. In the solution process, we use second order parametric hexahedral finite elements and conformal basis functions to ensure the fidelity of the geometric model of the problem [2] (see Fig. 1 (b)).

MLFMM FOR VOLUME INTEGRAL EQUATIONS

The multilevel version of the original FMM algorithm [3, 4] has already been applied in the context of surface integral equations (SIE). For volumetric formulations, matrix systems size grows even more rapidly compared to SIEs. Here, the MLFMM is adapted and applied to VIE formulations. The core of MLFMM algorithm (multilevel grouping of basis functions and diagonal forms of the translation operators) can be used in the VIE context as before. Only the new signature functions for volumetric basis functions need to be introduced for a given multilevel grouping.

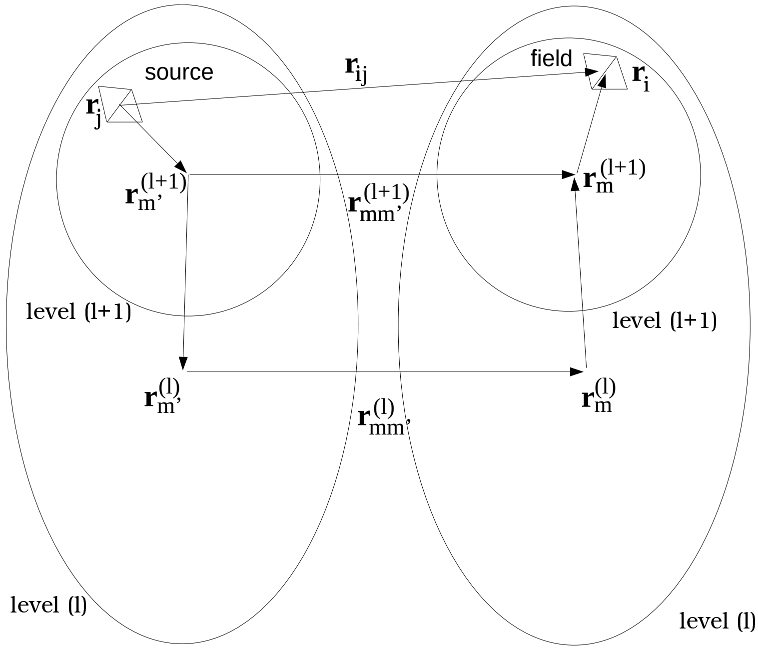


Figure 2: Two level FMM construct for a general multilevel case.

A brief summary of the MLFMM methodology [3, 4, 6, 7] is given below below. For illustration, consider the two level vector construct shown in Fig. 2. Using the decomposition

$$\mathbf{r}_{ij} = \mathbf{r}^{(l+1)}_{im} + \mathbf{r}^{(l)}_{mm} + \mathbf{r}^{(l)}_{mm'} - \mathbf{r}^{(l)}_{m'm'} - \mathbf{r}^{(l+1)}_{jm'} \tag{5}$$

allows us to expand the scalar Green's function as

$$\frac{e^{ikr_{ij}}}{r_{ij}} = \frac{ik}{4\pi}\int d^2\hat{k} e^{i\mathbf{k}\cdot\mathbf{r}^{(l+1)}_{im}} e^{i\mathbf{k}\cdot\mathbf{r}^{(l)}_{mm}} \alpha_L(kr_{mm'}, \hat{k}\cdot\hat{r}_{mm'}) e^{-i\mathbf{k}\cdot\mathbf{r}^{(l)}_{m'm'}} e^{-i\mathbf{k}\cdot\mathbf{r}^{(l+1)}_{jm'}}. \tag{6}$$

where the superscripts $(l+1)$ and (l) denote the grouping levels with (l) being the coarser level and $(l+1)$ the finer. With this expansion, the far zone impedance matrix elements take the form

$$Z_{ij} = \sum_{\hat{k}^{(l)}} w_{\hat{k}^{(l)}} \mathbf{V}_{fim}(\hat{k}^{(l+1)}) e^{i\mathbf{k}\cdot\mathbf{r}^{(l)}_{mm}} \alpha_L(k^{(l)} r_{mm'}, \hat{k}^{(l)}\cdot\hat{r}_{mm'}) e^{-i\mathbf{k}\cdot\mathbf{r}^{(l)}_{m'm}} \mathbf{V}^*_{sjm'}(\hat{k}^{(l+1)}). \tag{7}$$

However, since the discrete values of the signature functions $\mathbf{V}_{fim}(\hat{k}^{(l+1)})$ and $\mathbf{V}^*_{sjm'}(\hat{k}^{(l+1)})$ are sampled in the k-space and for level $(l+1)$, their values at level (l) must be computed by interpolation. In doing so, though, the interpolation operations must not exacerbate the overall complexity of the MLFMM. Several interpolation methods have been reported, the simplest being a low-order polynomial interpolation. However, as the problem size increases, the polynomial interpolation error grows, due to the high harmonic content in the aggregated signature functions. Fast filtering methods for bandlimited functions on a sphere have been reported in [8, 9]. Another interpolation method is presented in [6] and has been improved in [7].

Either the 3 Cartesian components or the 2 spherical components of $\mathbf{V}(\hat{k})$ can be used in the MLFMM matrix-vector product since $\mathbf{V}(\hat{k})$ is normal to $\hat{k}$. The latter approach is more attractive since it reduces the memory and CPU time by a factor of 2/3. However, the spherical components of $\mathbf{V}(\hat{k})$ are not bandlimited in the spherical harmonic representation. In this paper, a simple polynomial interpolation is used in both θ and ϕ directions.

At the conference we will present various results dealing with material simulations and the use of MLFMM in the solution of the proposed volume integral equations.

References

[1] G. E. Antilla and N. G. Alexopoulos, "Scattering from Complex 3D Geometries by a Curvilinear Hybrid Finite Element-Integral Equation Approach", *J. Opt. Soc. Am. A*, vol. 11, no. 4, pp. 1445–1457, 1994

[2] K. Sertel and J. L. Volakis, "Method of Moments Solution of Volume Integral Equations Using Parametric Geometry Modeling", *Radio Sci.*, vol. 37, no. 1, pp. 1–7, 2002

[3] R. Coifman, V. Rokhlin, and S. Wandzura, "The Fast Multipole Method for the Wave Equation: A Pedestrian Prescription", *IEEE Antennas and Propagation Magazine*, vol. 35, pp. 7–12, June 1993

[4] J. M. Song, C. C. Lu, and W. C. Chew, "Multilevel Fast Multipole Algorithm for Electromagnetic Scattering by Large Complex Objects", *IEEE Trans. Antennas Propagat.*, vol. AP-45, no. 10, pp. 1488–1493, 1997.

[5] J. L. Volakis, "Alternative Field Representations and Integral Equations for Modeling Inhomogeneous Dielectrics", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-40, pp. 604–608, 1992.

[6] C. Lu and W. C. Chew, "A Multilevel Algotirhm for Solving a Boundary Integral Equation of Wave Scattering", *Microwave Opt. Tech. Lett.*, vol. 7, no. 10, pp. 466–470 July 1994

[7] E. Darve, "The Fast Multipole Method: Numerical Implementation", *J. Comput. Physics*, vol. 160, pp. 195–240, 2000

[8] R. Jacob-Chien and B. K. Alpert, "A Fast Spherical Filter with Uniform Resolution", *J. Comput. Physics*, vol. 136, pp. 580–584, 1997

[9] N. Yarvin and V. Rokhlin, "A Generalized One-Dimensional Fast Multipole Method with Application to Filtering of Spherical Harmonics", *J. Comput. Physics*, vol. 147, pp. 594–609, 1998