# PROPAGATION CHARACTERISTICS OF VLF WAVE TRANSMITTED AT GEOMAGNETIC CONJUGATE POINT IN MID-LATITUDE


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## ABSTRACT

Numerical simulations of the VLF wave propagation from the magnetosphere through the ionosphere to the ground are performed by the full-wave analysis. It is found that the ground-based direction finding methods are effective in the local area where the signal exceeds the noise. The simulations are meaningful in order to interpret the direction finding results which contain the systematic errors.


## INTRODUCTION

The artificial VLF signals transmitted from Kabarovsk (39°N) and received at Ceduna (40°S) have been analyzed by the ground-based direction finding techniques. The direction finding results show the periodic variation in the location of the ionospheric exit region of the VLF wave propagating through the magnetosphere. It is suggested, however, that the direction finding methods have systematic error in the estimates caused by the Earth-ionosphere waveguide effect, and the error is critical in some conditions. Therefore, it is very important to evaluate the systematic errors by the numerical simulations in order to interpret the DF results properly.

In this paper, the numerical simulation of the VLF wave propagation from the magnetospheric duct through the ionosphere to the ground surface is performed by the full-wave analysis, and the ground-based direction finding methods are examined with the simulated wave field to evaluate the effectiveness of the estimation.

## Full-WAVE THEORY AND MODEL

The full-wave analysis is employed to demonstrate the VLF wave propagation. The ionosphere is approximated to horizontally stratified media of anisotropic homogeneous cold plasma. The propagation of a plane wave in the layer is governed by the following equation:

$$\frac{\partial}{\partial z}\boldsymbol{e}(k_x, k_y, z) = -ik_0\mathbf{T}\boldsymbol{e}(k_x, k_y, z) \tag{1}$$

The Equation (1) is derived from the constitutive relation of the cold plasma medium. $\boldsymbol{e} = [E_x, -E_y, Z_0H_x, Z_0H_y]^T$ is the column vector of the horizontal field components of the plane wave, and $\mathbf{T}$ is a $4 \times 4$ matrix determined from the ionospheric properties and the wave vector. The downgoing magnetospheric wave in the whistler mode is incident into the highest layer. The lowest layer is the ground where the wave propagates downward. The solution of (1) is obtained by the full-wave calculation according to the multi-layered method, which includes scaling and orthogonalization process of specific waves to avoid the numerical swampings [1].

Given the distibution of downgoing wave at the incident altitude, the two-dimensional profile of the wave field at an arbitrary altitude can be carried out by Fourier synthesis of the full-wave solutions of the elementary plane waves as

$$\boldsymbol{e}(x, y, z) = \iint \boldsymbol{e}(k_x, k_y, z) e^{i(k_x x + k_y y)} dk_x dk_y \tag{2}$$

The integration is efficiently evaluated by the fast fourier transform.

In our simulation, it is assumed that the distribution of the amplitude of the incident wave forms the Gaussian shape in $x$-$y$ plane, and the phase is equally random in order to simulate the stationary random wave. When the center of the wave distribution is taken at the origin, the complex amplitude is written by

$$g(x, y) \sim \frac{1}{2\pi\sigma_x\sigma_y} e^{-\left(\frac{x^2}{2\sigma_x^2} + \frac{y^2}{2\sigma_y^2}\right)} e^{i\psi(x,y)} \tag{3}$$

where $\sigma_x$ and $\sigma_y$ indicate the longitudinal and latitudinal scales of the distribution, and $\psi(x, y)$ is the uniform random deviate on $(0, 2\pi)$.

The wave distribution on $k_x$-$k_y$ plane corresponding to $g(x, y)$ is determined by the Fourier transform of $g(x, y)$ as

$$G(k_x, k_y) = \frac{1}{2\pi} \iint g(x, y) \exp\{i(k_x x + k_y y)\} dx dy \tag{4}$$

Additionally, it is taken into account that the magnetospheric wave has a beam form in the wave number space. Assuming the beam form as a Gaussian function centered at $(k_{x_c}, k_{y_c})$ with width $\sigma_{k_x}$ and $\sigma_{k_y}$, $G(k_x, k_y)$ weighted by the Gaussian function is used as the complex amplitude of the downgoing whistler wave.

We use the altitude profile of the ionosphere shown in Fig.1 throughout the simulations. The nighttime electron density is refered to the international reference ionosphere (IRI-95) as the representative models. Ion effects such as lower hybrid resonance (LHR) do not concern in our calculation. The geomagnetic field model is obtained from the international geomagnetic reference field (IGRF).

## SIMULATION RESULTS AND DISCUSSION

Fig.2 is an example of the distribution of the field energy density on the ground surface where the parameters of the incident wave distribution are shown in Table 1. The field energy density is defined as

$$w = \left\langle \frac{\varepsilon_0}{4} \left( |\boldsymbol{E}|^2 + |Z_0 \boldsymbol{H}|^2 \right) \right\rangle \tag{5}$$

$\langle \cdot \rangle$ indicates the expectation, and we estimate the value by averaging 100 samples of random wave field. The center of the distribution resides polarward on the ground as the wave propagates downward along the geomagnetic field line.

Table. 1. Parameters of incident wave distribution.

| | |
|---|---|
| Frequency | 23.9kHz |
| Incident altitude | 500km |
| Scale of the incident area | $\sigma_x = \sigma_y = 30$km |
| Beam width | $\sigma_{k_x} = \sigma_{k_y} = 0.1k_0$ |
| Center of the beam | $k_{x_c} = 0, \; k_{y_c} = 0.3k_0$ |

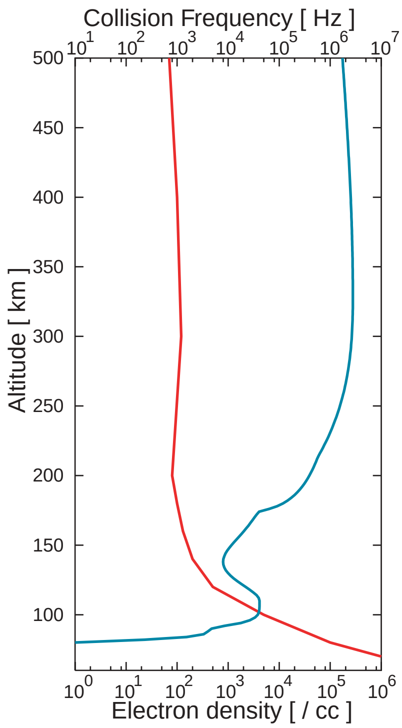


Fig. 1. Profiles of electron density and mean collision frequency.

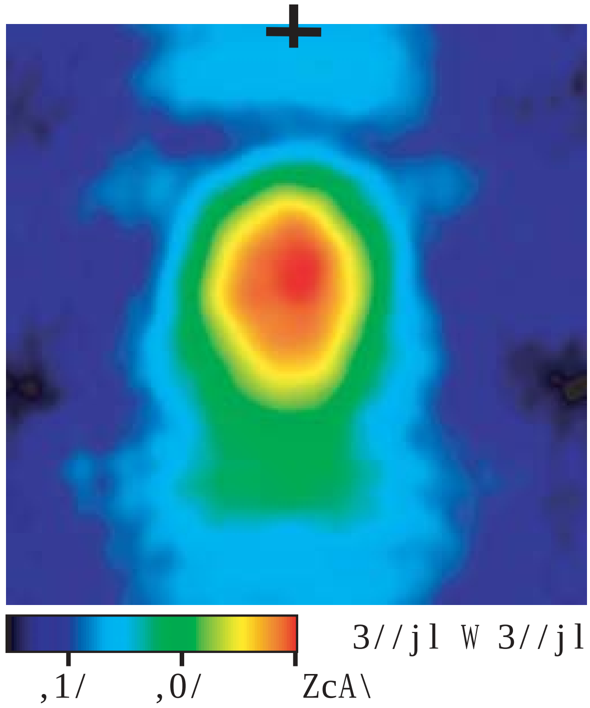


Fig. 2. Wave energy distributions on the ground. The cross "+" indicates the center of the Gaussian wave distribution at the incident altitude.

For the multiple signal classification (MUSIC)[2] and the wave distribution function (WDF)[3] estimation, we construct the spectral matrix $\mathbf{S} = \langle \boldsymbol{x}\boldsymbol{x}^\dagger \rangle$, where $\boldsymbol{x} = [B_x, B_y, E_z/c]$. $\mathbf{S}$ is approximated by the average from 100 samples. Noise is added at every observing point equally so that the SN ratio is 20dB at the maximum point of the energy density. Thus the characteristics of the direction finding techniques are assessed at any point on the ground by using the simulated random wave field.

The arriving direction and the WDF are examined at every 50km point in the area (400km $\times$ 400km) of Fig.2. Fig.3 is the estimated arriving directions and the WDFs. It is found that the MUSIC method fails to estimate the arriving direction and polarisation in case that the observing point is more than $2\sigma_x$ or $2\sigma_y$ [km] apart from the peak of the energy density where the noise exceeds the VLF signal. However, the WDF method is effective at any observable point on the ground to estimate the location of the exit region. We have also confirmed that the width of the WDF corresponds to the scale of the exit region. These simulations are useful to interpret the direction finding results which contain the systematic errors.

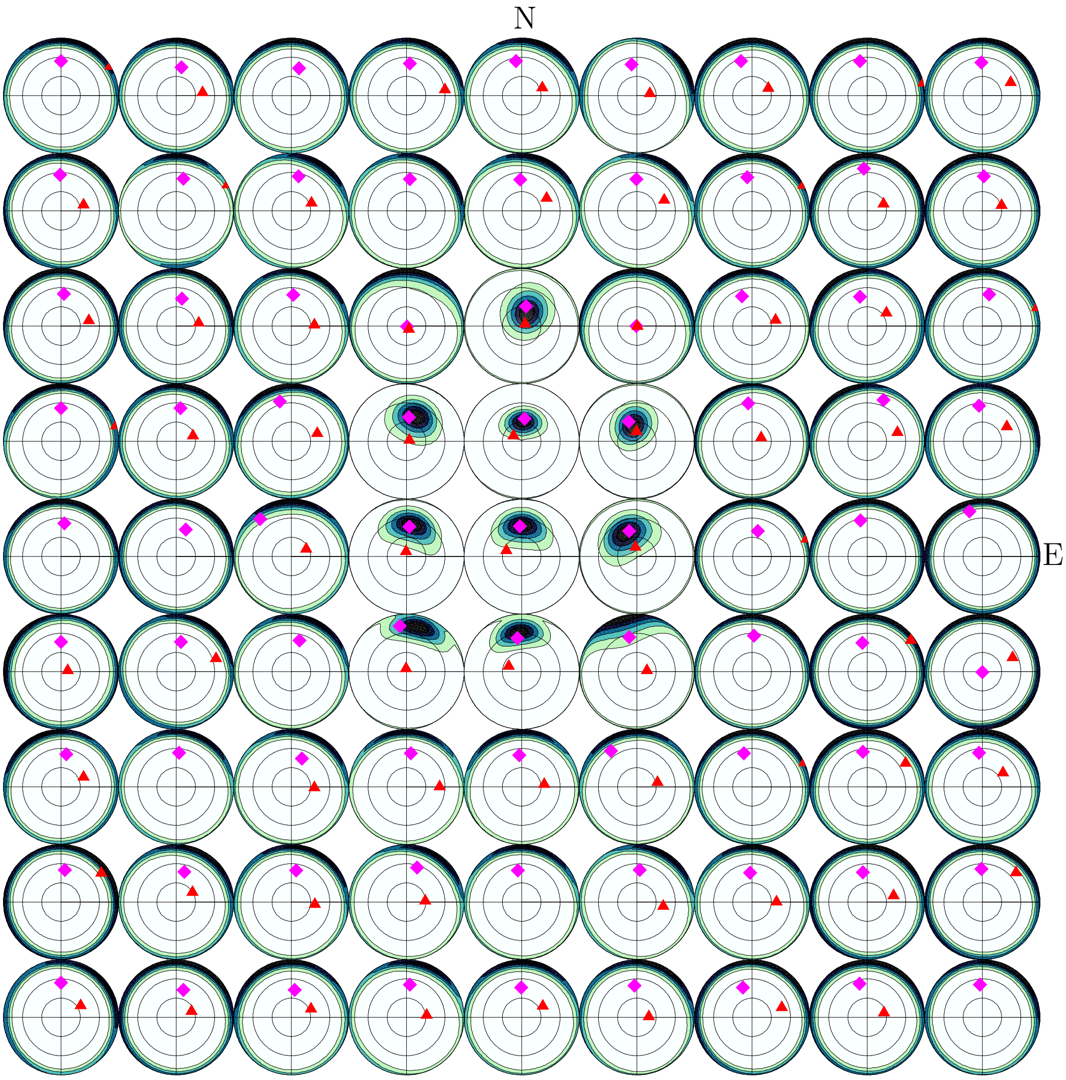


Fig. 3. Direction finding results for the simulated wave field by the full-wave analysis in 400km×400km area. The red triangle is the MUSIC results, and the contour maps are the estimated WDF.

## REFERENCES


[1] Nagano, I., Mambo, M. and Hutatsuishi, G., "Numerical calculation of electromagnetic waves in an anisotropic multilayered medium", Radio Sci., Vol. 10, No. 6, pp. 611–617, 1975.

[2] R. O. Schmidt, "Multiple emitter location and signal parameter estimation", IEEE Trans. Antennas Propagat., Vol. AP-34, pp. 276–280, Mar 1986.

[3] Shimakura, S., M. Hayakawa, F. Lefeuvre and D. Lagoutte, "On the estimation of wave energy distribution of magnetospheric VLF waves at the ionospheric base with with ground-based multiple electromagnetic field components", J. Geomag. Geoelectr., **44**, 573-590, 1992.