Analytical and Numerical Modeling of an Acousto-Electromagnetic System for Buried Object Detection

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1 ABSTRACT

A novel numerical approach to calculate the electromagnetic scattering from vibrating metallic objects is presented using time-varying generalized impedance boundary conditions (GIBCs) in conjunction with the method of moments (MoM). The model has wide applicability, but is currently being used to model an acousto-electromagnetic buried object detection system. The numerical technique provides an accurate calculation of the scattered Doppler spectrum no matter how small the object vibration. Numerical results are shown to be in excellent agreement with existing analytical perturbation solutions.

2 INTRODUCTION

Electromagnetic (EM) scattering from moving objects is a classic topic that arises in many practical applications. Recently, a significant amount of interest in the scattering from vibrating objects has emerged in connection with acousto-electromagnetic methods applied to buried object detection. Results from an analytical study of the EM Doppler spectrum scattered from metallic and dielectric cylinders excited (acoustically) at resonance indicate that the method could be used to enhance detection and identification of buried objects [1]. What is currently lacking, however, is an efficient numerical method to calculate the EM scattering from a vibrating object of arbitrary shape once the acoustic or elastic-wave displacements have been determined. In this paper, we are interested in using IBCs to model the scattering from a vibrating object which can be considered a perturbation problem where the perturbation (and the corresponding IBC) varies with time. One of the most basic objects for which a scattering solution can be obtained is an infinite PEC cylinder. Using the exact solution for a circular cylinder, the IBCs needed to model the scattering from a perturbed PEC cylinder are derived in Section 3 for both the TM and TE polarizations. It is shown that the standard IBC (SIBC) is accurate to first order for the TM case, but a second order GIBC is needed to provide first order accuracy for the TE case. Results from a Method of Moments (MoM) implementation of the IBCs are presented in Section 4.

3 IMPEDANCE BOUNDARY CONDITIONS

3.1 TM CASE

In this section, the exact solution for a circular cylinder is used to derive IBCs for a stationary, perturbed cylinder. The goal is to accurately reproduce the field scattered from the perturbed cylinder using an IBC applied at the unperturbed boundary. Consider a

TM-polarized incident plane wave incident upon a PEC circular cylinder with unperturbed radius a. The eigenfunction solution for the scattered wave has the form [4]

$$E_z^s = \sum_{n=-\infty}^{\infty} j^{-n} a_n H_n^{(2)}(k\rho) e^{jn\phi} \quad \text{where} \quad a_n = -\frac{J_n(ka)}{H_n^{(2)}(ka)} \tag{1}$$

If the radius of the cylinder is perturbed by a small number δ,

$$\tilde{a}_n = -\frac{J_n(k(a+\delta))}{H_n^{(2)}(k(a+\delta))} \approx -\frac{J_n(ka) + k\delta J_n'(ka)}{H_n^{(2)}(ka) + k\delta H_n^{(2)\prime}(ka)} \tag{2}$$

where a first order Taylor series expansion of the numerator and denominator is employed. The approximation in (2) is valid provided that $\delta \ll a, \lambda$. Since our aim is to approximate the scattered field from the perturbed cylinder with a surface impedance applied at the unperturbed boundary, we need to employ the impedance boundary condition

$$\hat{n} \times \mathbf{E} = Z_s\, \hat{n} \times (\hat{n} \times \mathbf{H}) \tag{3}$$

where Z_s is the surface impedance on the boundary. Applying this condition to the eigenfunction scattering solution for a circular cylinder of radius a, it can easily be shown that the mode coefficients become

$$a_n = -\frac{J_n(ka) + jZ_s^{TM}/Z_0\, J_n'(ka)}{H_n^{(2)}(ka) + jZ_s^{TM}/Z_0\, H_n^{(2)\prime}(ka)} \tag{4}$$

where Z_s^{TM} is the surface impedance for the TM case, and Z_0 is the intrinsic impedance of the background medium. Comparing (4) to the mode coefficients for the perturbed cylinder in (2) shows that an equivalent solution (to first order in δ) is obtained when

$$Z_s^{TM} = -jk\delta Z_0 \tag{5}$$

3.2 TE CASE

Consider a TE-polarized incident plane wave incident upon a PEC circular cylinder with unperturbed radius a. The scattered field takes the form [4]

$$H_z^s = \sum_{n=-\infty}^{\infty} j^{-n} b_n H_n^{(2)}(k\rho) e^{jn\phi} \quad \text{where} \quad b_n = -\frac{J_n'(ka)}{H_n^{(2)\prime}(ka)} \tag{6}$$

If the radius is perturbed by δ, the mode coefficients can be written as

$$\tilde{b}_n = -\frac{J_n'(k(a+\delta))}{H_n^{(2)\prime}(k(a+\delta))} \approx -\frac{J_n'(ka) + k\delta J_n''(ka)}{H_n^{(2)\prime}(ka) + k\delta H_n^{(2)\prime\prime}(ka)} \tag{7}$$

where a first order Taylor series expansion of the numerator and denominator is employed and is valid provided $\delta \ll a, \lambda$. As with the TM case, we wish to approximate the scattered field from the perturbed cylinder using a surface impedance applied at the unperturbed boundary. Applying the boundary condition in (3) to the eigenfunction solution for a circular cylinder of radius a, the TE mode coefficients become

$$b_n = -\frac{J_n'(ka) - jZ_s^{TE}/Z_0\, J_n(ka)}{H_n^{(2)\prime}(ka) - jZ_s^{TE}/Z_0\, H_n^{(2)}(ka)} \tag{8}$$

where Z_s^{TE} is the surface impedance for the TE case. Equating (8) to the mode coefficients for the perturbed cylinder in (7) and keeping only first order terms in δ, we can solve for Z_s^{TE}. Unlike the TM case, we cannot simply equate the numerators and denominators in these expressions. Instead, we set the entire expressions of (8) and (7) equal to each other and solve for Z_s^{TE}. It can be shown that

$$Z_s^{TE} = -jk\delta Z_0 \left[1 - \left(\frac{n}{ka}\right)^2\right] \tag{9}$$

Note that the surface impedance in (9) depends on the mode number n and, as such, is not applicable to a cylinder of arbitrary cross-section. The mode dependence suggests that a higher order IBC should be used instead. A general, second order GIBC for TE-polarization can be written [5]

$$E_s = -\eta_{zz} H_z + \frac{\partial}{\partial s}\left(A_0 \frac{\partial}{\partial s} H_z\right) \tag{10}$$

where E_s is the electric field in the x-y plane tangent to the surface. When applied to a circular cylinder whose surface impedance is independent of ϕ, this GIBC becomes

$$E_\phi = \left[-\eta_{zz} + A_0 \frac{1}{a^2}\frac{\partial^2}{\partial \phi^2}\right] H_z \tag{11}$$

When the GIBC in (11) is compared with the expression in (9) , recalling that $Z_s^{TE} = -E_\phi/H_z$ and that H_z has a ϕ-dependence of $e^{jn\phi}$, it becomes evident that an equivalent solution (to first order in δ) for the perturbed mode coefficients is achieved if

$$\eta_{zz} = -jk\delta Z_0 \quad \text{and} \quad A_0 = \frac{j\delta Z_0}{k} \tag{12}$$

This solution is valid for a circular cylinder with perturbed radius but can be easily extended for arbitrary perturbation of a cylinder having an arbitrary cross-section.

4 NUMERICAL RESULTS

For a PEC circular cylinder, an exact scattering solution is known for both the unperturbed cylinder with radius a and the perturbed cylinder with radius $a + \delta$. The impedance boundary conditions are applied at the unperturbed boundary but should accurately predict the scattering from the perturbed cylinder. As an example, consider a plane wave incident at $\phi^i = 0°$ upon a cylinder with radius $a = 0.5\lambda$ perturbed by $\delta = 0.05\lambda$. Figures 2 and 3 show the bistatic RCS for the TM and TE cases, respectively.

To model the scattering from a vibrating cylinder, the IBCs considered here must become time-varying. Using time-varying IBCs, it can be shown that the small time-varying component of the scattered field can be isolated from the large unperturbed term, allowing us to accurately compute the Doppler spectrum. To illustrate the scattering from a vibrating object, consider a PEC cylinder excited at a mechanical resonance. Since the time-varying nature of the object is manifest in the Doppler components of the scattered spectrum, we are interested in the scattering behavior of the 1st harmonic of the Doppler spectrum. For TE incidence upon a vibrating cylinder of radius $a = 1.5\lambda$, the bistatic scattering of the 1st harmonic is calculated with the MoM using both SIBC and GIBC and is shown in Fig. 4. The $n = 2$ mechanical mode of the cylinder is excited. Good agreement with the analytical solution exists only in the backscatter direction for the SIBC, but excellent agreement is maintained for all scattered angles using the higher order GIBC.

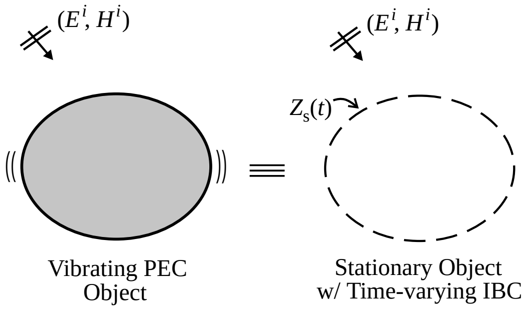

Figure 1: Modeling of vibrating PEC object with a stationary object having time-varying surface impedance $Z_s(t)$.

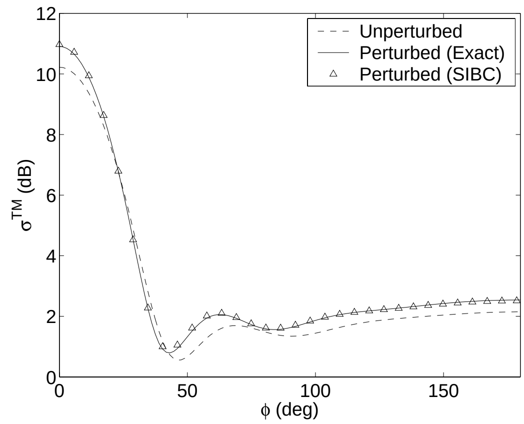

Figure 2: SIBC approximation of TM scattering from a circular cylinder with a perturbed radius. ($a = 0.5\lambda$, $\delta = 0.05\lambda$)

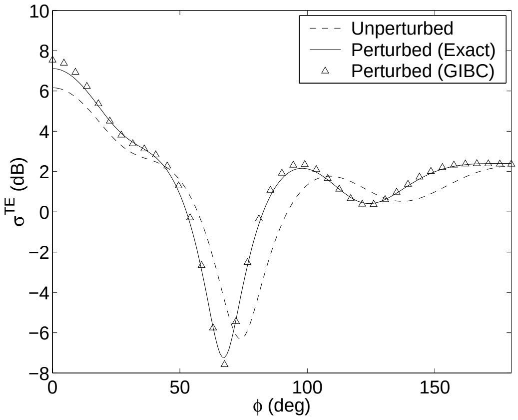

Figure 3: GIBC approximation of TE scattering from a circular cylinder with a perturbed radius. ($a = 0.5\lambda$, $\delta = 0.05\lambda$)

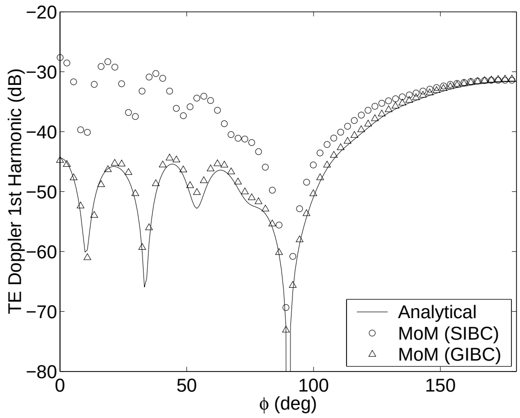

Figure 4: Comparison of MoM and the analytical solution for TE Doppler scattering from a vibrating circular cylinder. ($a = 1.5\lambda$)

References

[1] D. E. Lawrence and K. Sarabandi, "Acoustic and electromagnetic wave interaction: analytical formulation for acousto-electromagnetic scattering behavior of a dielectric cylinder," *IEEE Trans. Antennas Propagat.*, vol. 49, no. 10, Oct. 2001.

[2] M. A. Leontovich, *Investigations on Radiowave Propagation - Part II.* Moscow, USSR: Academy of Sciences, 1948.

[3] S. N. Karp and F. C. Karal Jr., "Generalized impedance boundary conditions with applications to surface wave structures," in *Electromagnetic Wave Theory*, Pt. 1 (J. Brown, Ed.), Pergamon Press, New York, pp.479-483, 1965.

[4] R. F. Harrington, *Time-Harmonic Electromagnetic Fields.* McGraw-Hill, Inc., 1961.

[5] T. B. A. Senior and J. L. Volakis, *Approximate Boundary Conditions in Electromagnetics.* London, U.K.:IEE Press, 1995.