ARRAY OPTIMAL FOCUSSING IN PRESENCE OF ARBITRARY UPPER BOUNDS

Ovidio M. Bucci(1,2), **Tommaso Isernia**(1)

(1)Dipartimento di Ingegneria Elettronica e delle Telecomunicazioni - Università di Napoli Federico II
via Claudio, 21 - 80125 Napoli (ITALY)
Phone: +39 081 7683140 – Fax: +39 081 5934448 – e-mail: {bucci, isernia}@unina.it

(2)Istituto per il Rilevamento Elettromagnetico dell'Ambiente (IREA) - CNR
via Diocleziano, 328 - 80124 Napoli (ITALY)

ABSTRACT

The problem of maximizing a field in a given direction (or point) subject to arbitrary upper bounds elsewhere is a classical problem in electromagnetics. Besides its obvious interest in radar applications, it has a potential impact on mobile communications (as properly designed pencil beams may allow frequency re-use in base stations) as well as in microwave hyperthermia applications.
In this communication we briefly review our recent theoretical and numerical results on the optimal focusing of fields radiated by an arbitrary (fixed geometry) array, and suggest possible extensions.

1. INTRODUCTION

The problem of synthesizing a field which is maximum in a given direction and subject to constant (or properly shaped) upper bounds in non target directions is a classical problem of electromagnetics. In canonical situations, i.e., uniformly spaced linear or planar arrays located in free space and subject to a constant upper bounds on sidelobes, classical solutions include the Dolph-Chebichev and Taylor methods for linear arrays, and Taylor or Tseng-Cheng approaches for planar arrays [1]. The latter have also been generalized in such a way to consider non uniform (albeit ring-shaped, and therefore not completely arbitrary) constraints on sidelobes.

On the other side, recent developments in array antennas technology push toward consideration of more complicated structures (such as conformal antennas or arrays located in a complex environment) and more selective constraints (such as, for instance, interference rejection by a single direction, or near field constraints for human safety or reduction of interactions with the platform). Finally, in some synthesis problems the geometry (for instance, the boundaries) of the array is not determined a priori, and the problem arises of determining the minimal dimensions and boundaries of an array such to fulfil the given design constraints
All these circumstances claim for new general and effective synthesis methods.

2. SOME RECENT RESULTS

A first step in such a direction has been done in [1], wherein it has been shown that the problem of the optimal focusing of one of the field components of a fixed geometry array subject to arbitrary upper bounds both in the far and near fields can be formulated as a Convex Programming problem. As a matter of fact, if E is the field component to be maximized in a given direction or location r_{max}, the problem can be formulated as [2,3]

$$\text{Minimize } \{\text{Real}[E(r_{max})]\} \qquad (1.a)$$

subject to

$$\text{Imag}\,[E(r_{max})] = 0 \qquad (1.b)$$

and

$$\left|\underline{E}(r_i)\right|^2 < SLL(r_i) \qquad i=1,.....M \qquad (1.c)$$

wherein (1.b) fixes the phase reference, $\underline{E}$ is the field, function SLL(r) define the constraints on the sidelobes, and the number M of sampling points r_i has to be taken on a sufficiently dense grid. As the real and imaginary parts of E are linear functions of the excitations coefficients, and each constraint (1.c) define an hypercylinder in the space of the unknowns, problem (1) amounts to minimize a linear function in a convex set, and it is therefore a Convex Programming (CP) problem. As such, it can be solved in a globally optimal fashion by using just local minimization techniques [4].
This simple result, and the 'ad hoc' developed numerical codes, allow to solve in an optimal fashion a number of synthesis problems. First, non uniform and conformal arrays can be dealt with in a straightforward fashion. In such a case, results of the proposed technique largely outperform previous synthesis techniques [3]. Second, arbitrary bounds on both the far and near fields can be dealt with [2]. In addition to that, superdirectivity constraints can be added to (1.c) without impairing global optimality of the approach. Finally, the generality of the approach allows to include mutual coupling in the synthesis stage without any approximation [5]. The presence of possible diffractions at the edges of the array and the presence of complex platforms could be taken into account in a straightforward fashion as well.

The general theory proposed in [2] has been then particularized to the relevant case of uniformly spaced linear or planar arrays. In such a case, and whenever only far field constraints are present, our recent results include three main results (see [6] for a thorough discussion).

First, it has been shown that the optimal far field focusing problem always admits a solution corresponding to a real far field pattern. As a consequence, if such a real solution is looked for, constraints (1.c) become linear, and the overall problem can be reduced, without any loss of performances, to a Linear Programming (LP) one. As a LP problem is much simpler to solve than a CP one, this considerably simplifies the actual solution of the problem.

Second, conditions for the existence of a unique solution to the Convex Programming problem (1) have been derived. In particular, it has been shown that if a unique solution does exist, it is real. Moreover, if there is a unique real solution to the problem, it is also the unique solution at all. As a consequence of the above, in the case at hand, conditions for the existence of a unique solution can be discussed for the simpler case of Linear Programming. By exploiting this circumstance one is able to develop uniqueness criteria. These latter shows that the solution is unique in case of linear arrays, while it could be not unique (as it is in fact the case, see next section) in case of 2-D arrays.

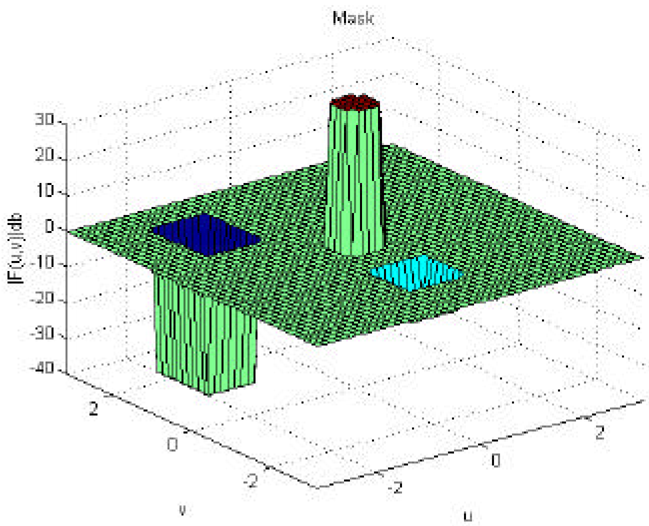
The fact that the solution of the basic problem is not unique in case of 2-D arrays suggests that it makes sense trying to optimise some further parameter in the (convex) set of equivalent solutions. As a third result about uniformly spaced linear or planar arrays, we have shown that if this further performance parameter fits some general and practically relevant condition, one of the optimal solutions must be (again) real. As a consequence, one can show that the the directivity (as well as other parameters of interest) can be optimized in a globally optimal fashion through the solution of a relatively simple additional optimization problem.
In fact, the basic focusing problem can be dealt with as a Linear Programming problem. Let M be the set of solutions to such a problem, which means that all the elements of M give rise to the same maximum for the field. Then, it suffices to minimise the radiated power in M in order to maximise the directivity in the set of equivalent solutions to the basic problem. As M is defined by linear constraints and the objective function is a positive definite quadratic function, the latter is a globally solvable Quadratic Programming problem. A similar reasoning can be exploited in order to optimize the smoothness of excitations [6], which is of interest for both slot and microstrip arrays.

3. NUMERICAL EXAMPLES

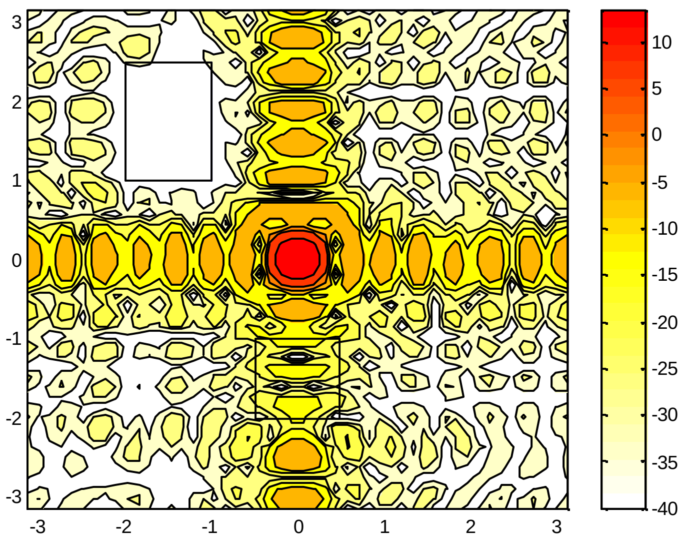
In order to show usefulness and effectiveness of the above results, we performed a large series of numerical experiments. With reference to uniformly spaced planar arrays, two examples are briefly reported in the following to show respectively non uniqueness of a solution to the basic focusing problem, and flexibility of the approach with respect to the kind of sidelobe behaviour one is enforcing.

As a first step, we compared results of our approach and codes with the analytical ones which are available in case of uniform sidelobes. In all the examples, we verified non-uniqueness of a solution to the problem (1), as the patterns furnished by the Tseng-Cheng procedure [1] and our approach are different, although they both satisfy the constraints and achieve the same field ma ximum.

For example, let us consider the case of an array with 15x15 elements, half a wavelength spaced, and a 16° circular beamwidth. The pattern synthesized with our approach and codes [6] exactly coincides with the Tseng-Cheng pattern along the main cuts (where a Chebichev, i.e. equiripple solution is expected), but it achieves a different and somehow better behaviour along the other cuts. As a matter of fact, the faster decrease along these other cuts (which can be attributed to the different electrical dimensions of the array along the different cuts) implies an higher directivity. In fact, it would be 1.93 times higher in case of isotropic sources, 1.56 higher in case of Huygens source, and 1.31 times higher in case of a cos(teta) element pattern. It is worth to note that non-uniqueness result in quite different patterns, so that the further optimization step proposed at the end of the previous section has the chance of considerably ameliorating the performances of the array being synthesized.



As a second example, in order to show the flexibility and generality of the proposed approach, we considered the optimal synthesis of a planar array of 15x15 isotropic elements with half a wavelength spacing subject to the asimmetric mask in the figure on the left. . The enforced constraints require a circular simmetric 16° degree wide main beam. Furthermore, in order to filter out unwanted interfering signals (in the receiving mode) or reducing the field in a given zone (for compatibility or human safety reasons), we enforced that the sidelobes in the rectangular regions $\Omega:\{-0.5 \le u \le 0.5, \quad -2 \le v \le -1\}$ and $\Gamma:\{-2 \le u \le -1, \quad 1 \le v \le 2.5\}$ are at a lower level, say –10 db and –40 db with respect to the maximum value of the sidelobes.



It is worth to note that such a situation cannot be managed in an optimal fashion by either the Tseng Cheng approach [1] or the extension provided by Kim and Elliott [7], as both of them would enforce unnecessary low ring sidelobes.

Results achieved by ou approach are reported in the figure on the left. It is worth to note that the approach proposed allows a maximum of 19.4 db to be achieved, which is just 1.4 db lower than the maximum one would achieve with an equilevel sidelobe mask of 0 db. Also note that the approach [7], by enforcing unnecessary low ring sidelobes, would result in a maximum about 6 db lower.

4. POSSIBLE IMPROVEMENTS AND EXTENSIONS

Previous results can be extended following two main directions, which are the object of current activity.

The first one is the achievement of computational efficiency (in the same way as [6]), while preserving as much generality as possible (as in [2,3]). To this end, the problem of the optimal focusing of a generic (for example a conformal) array can be splitted in two different steps. The first one takes advantages of minimum redundancy representations of fields radiated by non-superdirective sources enclosed in given hulls [8]. In fact, as these latter can be reinterpreted in terms of the field radiated by an auxiliary planar array, the optimal far field which can be actually radiated by sources in that hull can be found in the same effective way as in [6]. Then, this optimal field is used as reference in the second step, which is a relatively simpler field synthesis problem.

It is worth to note that minimum redundancy representations of fields radiated by sources enclosed in given hulls also represent a powerful tool to deal with array geometry synthesis problems in an optimal fashion. In fact, for a fixed hull, synthesis of the auxiliary array fix the maximum performances one can achieve with a source of that size, thus suggesting enlargement or reduction of the dimensions of the antenna.

A different way to achieve computational efficiency could be, in case of large antennas on complex platforms, the use of field representations such as [9].

A second possible extension is to the optimal synthesis of difference patterns in the presence of arbitrary upper bounds in non-target directions. This problem amounts to maximimize slope of a null in a given direction, rather than the field amplitude as in the previous case. Because of the similarity amongst the two problems, many of the results we developed in [3,6] for the optimal synthesis of sum patterns can be extended in a conceptually easy fashion to the optimal synthesis of difference patterns.

These results could then be exploited in order to get array antennas such to be easily reconfigurable and nearly optimal in both the sum and difference modes.

A further possible extension include possible hybridizations amongst the above approach and genetic algorithms in order to synthesize in an optimal fashion sum patterns in case of non uniformly spaced arrays with unknown spacings.

REFERENCES

[1] R.S. Elliott 'Antenna theory and design', Prentice-Hall, Englewood Cliffs, NJ, 1981

[2] T.Isernia, G.Panariello, "Optimal Focussing of scalar fields subject to arbitrary upper bounds", Electronic Letters, 22nd Jan. 1998 - Vol. 34, No.2, pp.162-164.

[3] T.Isernia, P. Di Iorio, F.Soldovieri; "An effective approach for the optimal focusing of array fields subject to arbitrary upper bounds"; IEEE Trans. on Antennas and Propagation, pp. 1837-1847; dec.; 2000

[4] R. Fletcher, "Practical methods of optimization", Wiley, New York, 1990.

[5] L.Caccavale, T. Isernia, F. Soldovieri; "Methods for the optimal focusing of microstrip array antennas including mutual coupling"; IEE Proc. on Microwaves Antennas and Propagation, Vol. 147, N. 2 pp. 199-202; june; 2000

[6] O.M. Bucci, L. Caccavale, T. Isernia "Optimal far field focusing of uniformly spaced arrays subject to arbitrary upper bounds in non-target directions", "; IEEE Trans. on Antennas and Propagation, september, 2002.

[7] Y.U.Kim, R.S.Elliott, "Extensions of the Tseng-Cheng pattern synthesis technique", *Journal of Electromagnetic Waves and Applications*, Vol.2, No.3/4, pp. 255-268, 1988

[8] O.M. Bucci, C. Gennarelli, C. Savarese "Representation of electromagnetic fields over arbitrary surfaces by a finite and non-redundant numbre of samples", IEEE Trans. on AP, vol. 46, pp.351-359, 1998

[9] F. Capolino, S. Maci, F. Mariottini, L.B.Felsen "High frequency Green's function for array of dipoles with weakly varying tapered excitations", Proc. AP symposium 2001, Boston 2001.