

# FREQUENCY PARAMETERIZATION TECHNIQUE APPLIED ON ELECTROMAGNETIC ANALYSIS OF RADIATING STRUCTURES

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## ABSTRACT

This paper presents the extension of a modal decomposition technique for the frequency parameterization of open structures. The modal decomposition based on the computation of the poles and modes of 3D geometries allows to get a broadband characterisation of a microwave structure. So far, this technique is restricted to closed structures. With the help of spherical modes for the outer region of open structures, full-wave 3D models parameterized in frequency can be obtained in case of the open structures.

## INTRODUCTION

The analysis of the microwave circuits requires rigorous computing methods. Furthermore, we usually need to determine the frequency performances of these circuits. For this purpose when applied on complex structures, full-wave 3D electromagnetic modelling is required. Time domain methods as the Finite Difference Time Domain can provide broadband frequency responses. Alternatively, frequency domain methods as the Finite Element Method (FEM) enhanced by techniques so-called fast frequency sweep can also derive efficiently broadband frequency responses computing the abrupt variation that can be mixed by using low resolution point by point simpling. A new method based on cavity modes decomposition has been successfully developed [1, 2]. This technique gave very good results for the closed microwave circuits analysis. Comparing to the point by point resolution method the modal decomposition allows obtaining frequency response of the studied circuits without solving linear system for each frequency point. Indeed, this technique enables building frequency parameterized transfer function of the studied structures. This technique is particularly competitive for analysing structures like filters which require an important number of time steps, when using time domain methods, to determine long transients or an important number of frequency steps, when using classical frequency domain methods to detect sharp resonances.

In this work, a FEM has been associated to modal decomposition method and spherical modes for building frequency parameterized model characterising the open structures. A general procedure can be followed to get the desired modal expansion with respect to the frequency provided solving the problem at two frequencies. The result is a 3D full-wave model of a radiating structure parameterized in frequency.

## THEORY

From system modeling point of view, an antenna in an open medium can be considered as a multiport with an excitation port and several spherical modes equivalent ports (Fig. 1).

Let's consider a transmitting open structure placed in a spherical coordinate system, we denote  $r_0$  as the radius of the smallest possible sphere circumscribing the radiating structure. The space outside the minimum sphere can be viewed as a space in which the propagation is done in the radial direction. And the "waveguide" modes are TE and TM spherical modes.

For the junction region, i.e. the inner region, we use the FEM. This technique leads to put linear equation mentioned above as [4]:

$$(R - k^2 M)e = jk\eta \sum_n J_{epn} \quad (1)$$

Where R and M are the rigidity and the mass matrices respectively and  $J_{epn}$  the excitation array containing the excitation input courant and the spherical modes of the output ports.

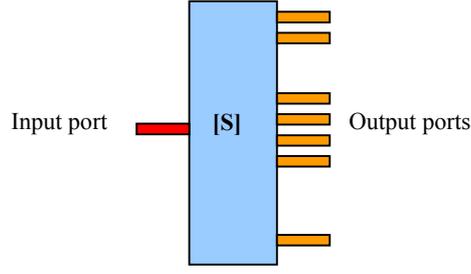


Fig. 1. Equivalent representation of an open structure in the open space

Recently, a method based on modal decomposition [2] has been proposed to solve equation (1). The method outputs a frequency and geometry-parameterized model which allows sensitivity analysis and optimization [2]. In this method, the E field propagating inside the cavity can be expressed as a decomposition on the cavity modes. Hence, for each excited port noted by  $n$  we obtain the solution as :

$$e_n = j\eta k \sum_i \frac{C_{ni}}{k_i^2 - k^2} V_i \quad (2)$$

Where  $k_i$  are the eigenvalues and the  $V_i$  eigenvectors of the system (1).  $C_{ni}$  is the scalar product between  $J_{epn}$  and  $V_i$  which corresponds to the coupling between the waveguide modes and the cavity modes.

Using (2), the impedance matrix  $[Z]$  can be calculated, since each impedance term associated to the ports noted  $n$  and  $m$ , is given by the relation ( $Z_{nm} = {}^t J_{epn} e_m$ ) [4]. The pole expansion of the impedance matrix is therefore written in the form [1]:

$$Z_{nm} = j\eta k \sum_i \frac{C_{ni} C_{mi}}{k_i^2 - k^2} \quad (3)$$

## MODAL EXPANTION FOR THE RADIATING STRUCTURES

In case of the open structures, the impedance matrix allows to calculate the scattering matrix, which contains all spherical modes coefficients. The spherical mode theory lets suppose that the field generated by an antenna can be written as the sum of spherical modes multiplied by the corresponding spherical modes coefficients.

The relation (3) can be further decomposed as [1] :

$$Z_{mn} = \frac{jA_{mn}}{k} + jkB_{mn} + jk^3\eta \sum_i^Q \frac{C_{ni} C_{mi}}{k_i^2 (k_i^2 - k^2)} \quad (4)$$

$$\text{Where: } \begin{cases} A_{nm} = \sum_{i=1}^Q C_{ni} C_{mi} \\ B_{nm} = \sum_{i=1}^Q \frac{C_{ni} C_{mi}}{k_i^2} \end{cases} \quad (5)$$

The relation (4) highlights the frequency independent  $A_{nm}$  and  $B_{nm}$  terms.  $A_{nm}$  represents the eventual contribution of the poles at the zero frequency and may be determined by using asymptotic approximation.  $B_{nm}$  converges slowly and thus requires a lot of eigenvalues to be determined. Hence, it's better to determine it by an asymptotic approach too. To illustrate this and as a test, we consider an homogeneous free space sphere. The impedance matrix is known analytically. The truncated series expansion of the spherical TE and TM modes impedance are :

$$Z_{TE_{mn}} = -j \left( \frac{kr}{n+1} + o(k^3 r^3) \right) \quad (5)$$

$$Z_{TM_{mn}} = j \left( \frac{n+1}{kr} - \frac{kr}{2n+3} + o(k^3 r^3) \right) \quad (6)$$

Where  $r$  is the radius of the sphere.

Figure 2 (resp. 3) shows in red the impedance of some TE (resp. TM) spherical modes calculated with the relation (5) (resp. 6) in the frequency band going from 0 to 20 GHz. These curves are compared to the theoretical curves (in blue). We notice that the curves are coincident.

These results shows that it is not possible to use this method to characterize an open structure surrounded by a sphere, since we do not have such available formulas [(5) and (6)] in the general case; To overcome this problem, the proposed method is as follows. First, we calculate the impedance matrix of the studied structure for two different frequencies. Second, we calculate the  $C_{nm}$  coefficients characterizing the same structure. Finally, with these known parameters we can deduce the terms  $A_{nm}$  and  $B_{nm}$ .

## RESULT

As a test case, we consider a Hertz dipole. After determining the terms  $A_{nm}$  and  $B_{nm}$ , then using the equation (4), we find the frequency-parameterized impedance matrix and calculate the scattering matrix of the dipole. Moreover, we have established the radiation patterns for many frequencies (fig 4 at left). These patterns are compared with the ones calculated by a direct frequency solving using FEM (fig 4 at right). We notice that figures 4 represent the well-known radiation pattern of a dipole calculated by of the two methods, and the results agree very well. Also table I compares the real part of the input impedance of the dipole given by the theoretical relation ( $R_e = \frac{2\pi}{3} \eta \frac{l^2}{\lambda^2}$ ) with the real part of the input impedance given by direct resolution on one hand and modal decomposition on the other. We observe the good concordance of results for each chosen frequency.

## CONCLUSION

The modal decomposition applied to the radiating structures analysis gives good results. This method allows to build efficiently frequency parameterized model of radiating structures.

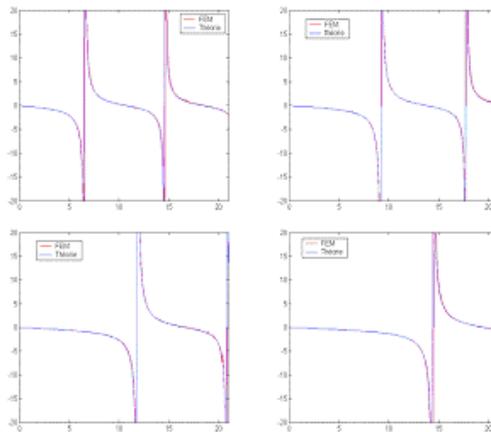


Fig. 2. Impedance of the  $TE_{m1}$ ,  $TE_{m2}$ ,  $TE_{m3}$ ,  $TE_{m4}$ , spherical modes

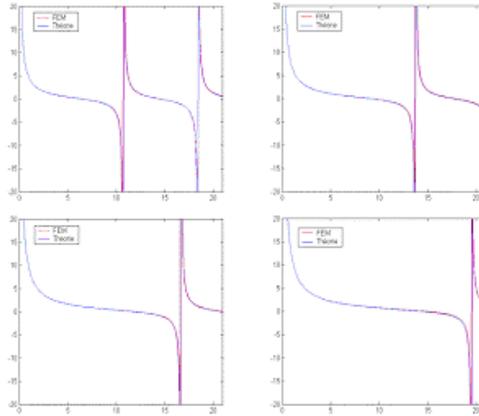


Fig. 3 . Impedance of the  $TM_{m1}$ ,  $TM_{m2}$ ,  $TM_{m3}$ ,  $TM_{m4}$ , spherical modes

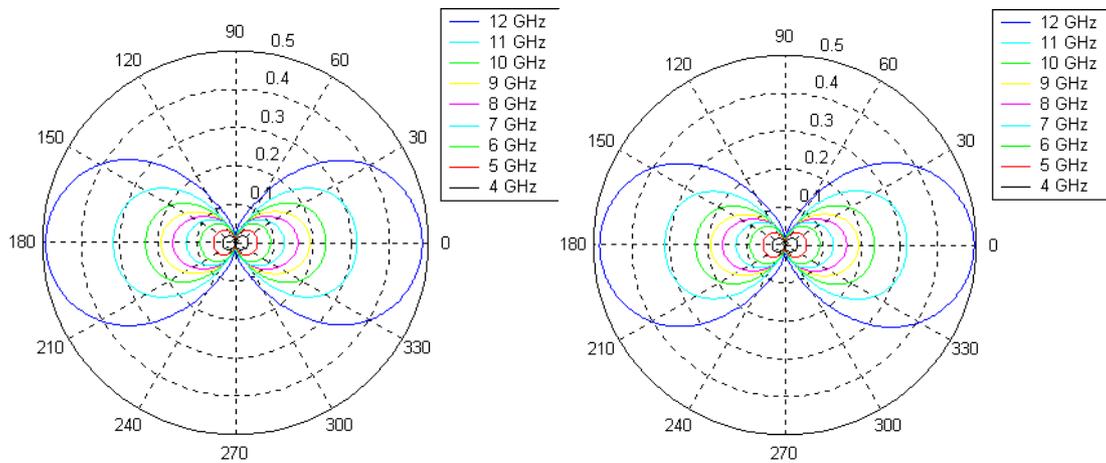


Fig. 4 . Representation of the electric field of the dipole obtained with a current of 1 A, calculated by modal decomposition at left and by direct resolution at right

Table 1. Real part of the input impedance of the dipole determined by theoretical relation, direct resolution and modal decomposition

	Theory	Direct resolution	Modal decomposition
1 GHz	0.1404	0.13091	0.1309
3 GHz	1.2633	1.2568	1.1603
6 GHz	5.0532	5.8651	5.8649
9 GHz	11.3698	11.048	11.046
12 GHz	20.2129	18.9913	18.9823

## REFERENCES

- [1] P. Arcioni, G. Conciauro. "Combination of generalized admittance matrices in the form of pole expansions". IEEE Trans. on MTT. Vol. 47, N° 10, 1999. PP 1990-1995.
- [2] A. Gati, M. F. Wong, and V. Fouad Hanna, «New technique using poles and modes derivatives for frequency and geometry parameterization of microwaves structures», IEEE MTT-S 2001.
- [3] E. Richalot, M. F. Wong, V. Fouad Hanna, and H. Baudrand, «Rigorous analysis of radiating structures using finite elements and spherical modes expansion», Annales des Télécommunications yearbooks, n. 3-4, vol.53, pp.130-137, 1998.
- [4] J. 1988. M. F. Wong, O. Picon, and V. Fouad Hanna, «Three dimensional finite element analysis of n-port waveguide junctions using edge-elements», in IEEE MTT-S June 1992, vol. 2, pp. 709-712.