JOSEPHSON ARBITRARY WAVEFORM SYNTHESIZER: ELECTRONICS FOR A QUANTUM STANDARD FOR AC VOLTAGE

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ABSTRACT

A new primary standard for AC voltage based on pulse driven Josephson junction arrays is under development. In this paper, preliminary spectral measurements and simulations are presented on a fast switching multiplexer unit built for the high precision synthesis of arbitrary waves. First order delta-sigma modulation is used to represent the wave to be synthesized. The first results are promising with respect to the conditions for high precision filtering of the quantization noise, i.e. a high signal-to-noise ratio is expected for the demodulated signal.

INTRODUCTION

Research in the field of electrical metrology mainly focuses on the development of so-called quantum standards. The main goal is to realize electrical quantities by means of laws of nature instead of "artifacts". A nice example is the primary standard of resistance: the Ohm can be related to the quantum Hall effect instead of a physical high quality resistor. The latter can be drifting in time, which is difficult to verify, or it might change its value when treated not too carefully. Relating standards to quantum mechanical phenomena in principle drastically decreases the uncertainty and makes them much more reliable.

The most familiar quantum standard is the DC Josephson array voltage standard. For two superconductors separated by a very thin insolating layer (a so-called Josephson junction), a DC voltage difference will occur when an RF bias current is supplied. The voltage V will only depend on the frequency f of the supplied signal and on constants of nature:

$$V = n\frac{2e}{h} f \ , \qquad (1)$$

where n is a tunable integer number that can be tuned using for example a DC bias current, e is the electron charge and h is Planck's constant. This voltage is known within a relative uncertainty of 10^{-9}, since frequency can be determined very accurately (relative uncertainty better than 10^{-14}).

In the last years research has been started to develop a quantum standard for AC voltage based on the Josephson effect [1-5]. The first problem was the hysteresis in the current-voltage (*I-V*) characteristics. Without special precautions, the response of an ordinary superconductor-insulator-superconductor (SIS) Josephson array to a DC bias is unpredictable, i.e. it is not possible to control the output of the array by simply varying the bias current. However, intrinsically shunted Josephson arrays, such as a superconductor-insulator-normal-insulator-superconductor (SINIS) array [6], show non-hysteretic *I-V* characteristics. This makes them suitable for the development of a programmable voltage standard.

One possibility to program the array is to divide it into binary sections [1,2,5] that can be biased with independent DC current sources. By activating a specific number of binary sections, an arbitrary DC voltage can be generated. By rapidly switching between the binary segments, the voltage can be changed in time, such that an AC voltage occurs. The main concern in using this technique is to overcome problems rising from fast switching events.

The second possibility is to modulate the drive frequency using a digital code [3,5]. A controlled train of pulses replaces

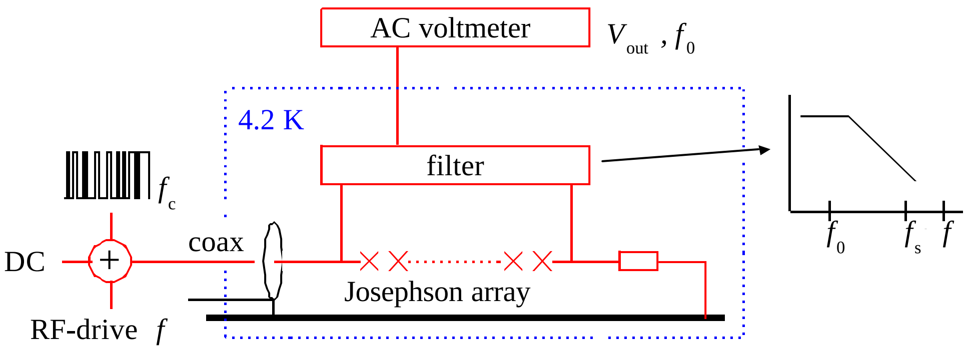

Fig.1: Schematic overview of the future pulse drive set-up. Digital code, DC offset and RF signal will be combined and fed to a Josephson array. The synthesized voltage has to be filter in order to recover the original signal.

the RF sine wave, consisting of a continuous series of pulses. Each individual pulse "excites" the Josephson array. When this train of pulses contains a delta-sigma modulated code for an arbitrary wave, after filtering the output of the Josephson array a signal of calculable arbitrary waveform will arise, see Fig.1. This Josephson arbitrary waveform synthesizer will be the basis of a new AC Josephson array voltage standard [8], which is the topic of this paper.

DRIVING THE JOSEPHSON ARRAY

The conventional way of driving a Josephson array is by applying an RF signal, which results in a quantized voltage over the array, as in (1). However, the drive mechanism of Josephson junctions is based on the fact that instead of the voltage itself it is the superconducting phase that is quantized. The AC Josephson equation directly relates the time derivative of the difference in superconducting phase, ϕ, to the voltage over the junction,

$$\frac{1}{2\pi}\frac{d\phi}{dt} = \frac{2e}{h}V \; . \qquad (2)$$

A non-linear second order differential equation relating the phase to the applied current *I(t)* is obtained by combining (2) with the DC Josephson relation,

$$C\frac{dV}{dt} + \frac{V}{R} + I_c \sin\phi = I(t) \; , \qquad (3)$$

where C denotes the capacitance between the two superconductors, the resistance R reflects dissipation, and I_c is the maximum supercurrent the junction can support. Simulations show that the junction responds to current pulses above a certain threshold by a phase shift of 2π (or $n \cdot 2\pi$ when a current above the nth threshold is applied), just as a damped pendulum responding to an applied torque. Integrating (2) one can see that this corresponds to a quantized time integral of the voltage.

In order to change the voltage over the array, instead of a sine wave, an arbitrary sequence of pulses can be offered. However, managing a sequence of individual pulses with a bit rate comparable to a typical RF drive frequency of 10 GHz is very difficult. Instead, one can combine an RF signal just below the threshold with a two-level signal that lifts the pulses above the threshold. This way, from a simplified perspective the array in fact switches between the n=1 and the n=-1 plateau of the *I-V* curve. When the two-level signal contains the pulse code modulation of an arbitrary wave, after demodulation of the output of the Josephson array the desired arbitrary wave is recovered. Note that in this case the Josephson array only serves to regulate the amplitude of the synthesized arbitrary wave with high accuracy.

PULSE DRIVE ELECTRONICS

The pulse drive electronics forms the heart of the setup. It must provide proper switching speed, amplitude, bit pattern and synchronization. Synchronization between digital code and RF signal can be obtained only if the sinusoidal frequency, f, equals half-integral multiples of the sampling frequency, f_s. In order to avoid steeply rising edges this ratio should be 3/2, while the digital code is separated into only three pairs of consecutive bits, i.e. 00, 01 and 11, avoiding 10 (or using 10 and avoiding 01) [3]. The rise time between two successive bits should be smaller than half the period of the sine wave. The SINIS arrays are fabricated for frequencies around 10 GHz [7], which means that the rise time should not exceed 50 ps. The specification of the multiplexer fulfills this requirement; using a 4GHz real time oscilloscope we were only able to verify the rise time to be less than 100 ps. Previous measurements and simulations were limited to the time domain [9]; in this paper we focus on the spectrum of the digital code.

The dedicated pulse drive electronics receives the data from the computer through 16 parallel lines with a bit rate of 13.33 MHz. The electronics multiplexes the input such that its output contains segments of 32 repetitions of pairs of two

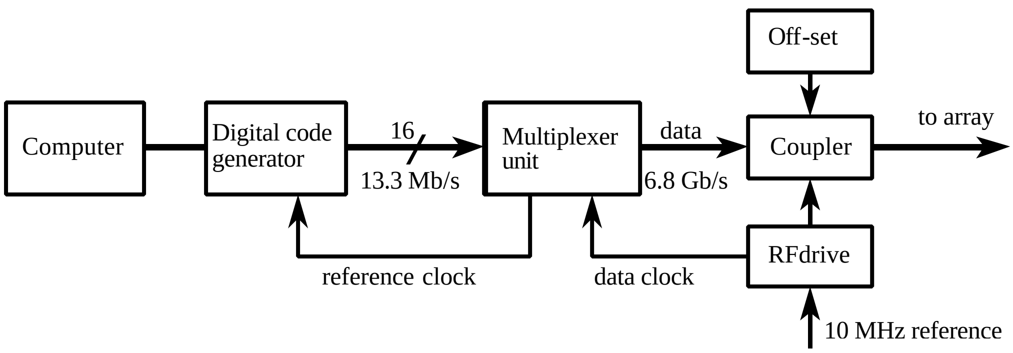

Fig.2: Schematic overview of the pulse drive electronics built for generation of the digital code.

consecutive bits, i.e. the input series A0,A1,A2,…,A7,B0,B1,…,B7 is converted into an output series 32*(A0,B0),32*(A1,B1),…32*(A7,B7). This way the output has a bit rate of 6.83 GHz, which is two third of the RF signal frequency of 10.24 GHz. However, the actual information rate is much lower, since this kind of multiplexing does not add information.

DELTA-SIGMA MODULATION

For high-resolution representation of low frequency signals delta-sigma modulation is an efficient technique. The quantization noise of this type of modulation is sinusoidal instead of constant in frequency: lower for low frequencies but increasing for higher frequencies [10]. Obviously, for such a spectrum it is easier to suppress the unwanted frequency components.
The working principle of a first order delta-sigma modulator is based on an integrator and a feedback loop. The integrator accumulates the difference between the average input and the actual output; eventually the feedback loop will correct for this difference.
In order to modulate an arbitrary wave as discussed in the previous section, avoiding steeply rising edges, first we apply a three level delta-sigma modulation. The three levels -1, 0, 1 are then converted into the two-bit level pairs 00, 01, 11, respectively. Hence, the actual sampling rate f_s of the modulator equals eight times the update frequency of the generator, i.e. f_s = 106.7 MHz.
We performed measurements and simulations on a first order delta-sigma modulated 10.4 kHz sine wave, such that (apart from the 10.4 kHz peak) the spectrum only contains modulation noise. A 10240 digit representation of one cycle of the sine wave is constructed. The number of digits N is one-to-one related to the frequency f_0 of the signal, due to the fixed output rate of the generator: $f_0 = f_s/N$. The digital representation of the signal is three-level delta-sigma modulated and converted into a two-level digital code. This two-level code is then generated and multiplexed.
As can be seen from Fig.3 and Fig.4 respectively, experiments and simulations are in good qualitative agreement. Just above the signal band (i.e. from DC to 10.4 kHz) the quantization noise is given by the noise floor of the spectrum analyzer. The noise increases in a sinusoidal way with frequency, up to half the sampling frequency, as predicted by theory [10]. The multiplexing unit extends the usual delta-sigma spectrum from a quarter period of a single sine towards a diminishing repetitive structure. The repetition rate of this structure is the sampling frequency f_s = 106.7 MHz.
Demodulation of the digital code can be performed by high precision low pass filtering. Since close to the signal band

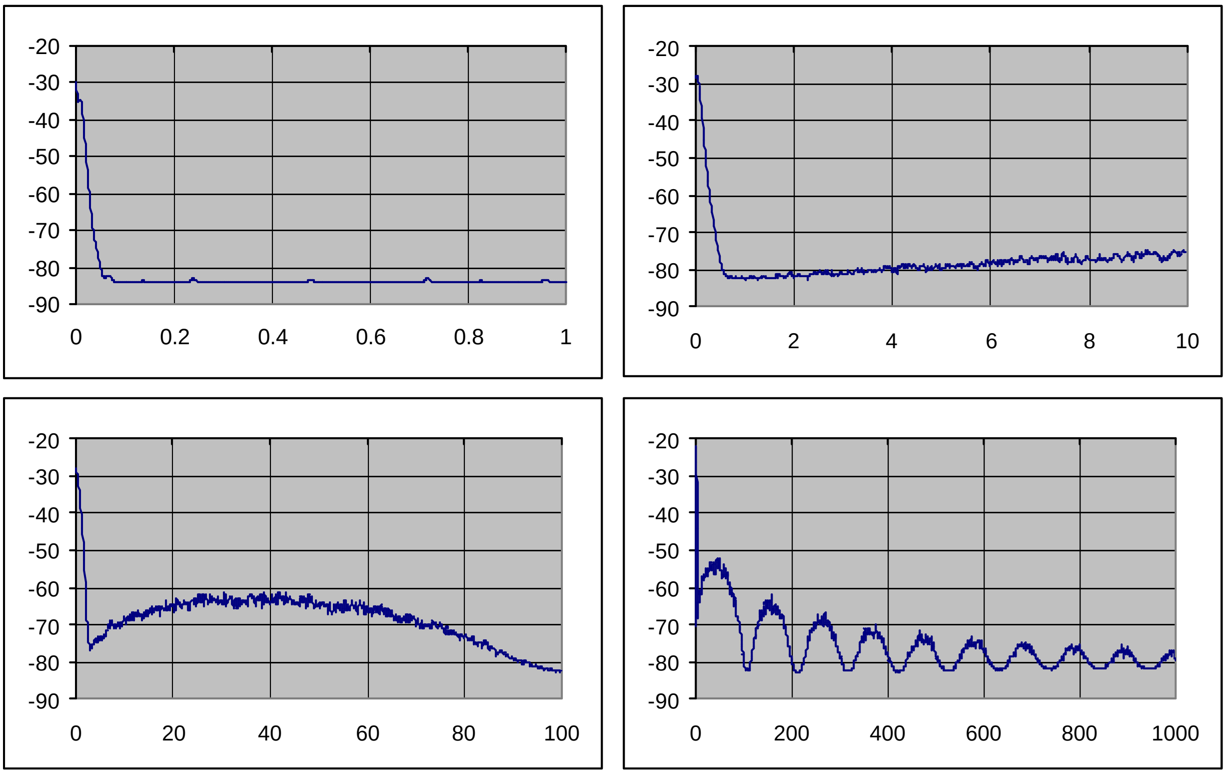

Fig.3: Measured spectrum of a multiplexed delta-sigma modulated 10.4 kHz sine wave from DC to 1 MHz (top left), 10 MHz (top right), 100 MHz (bottom left) and 1 GHz (bottom right). The power is indicated in dB.

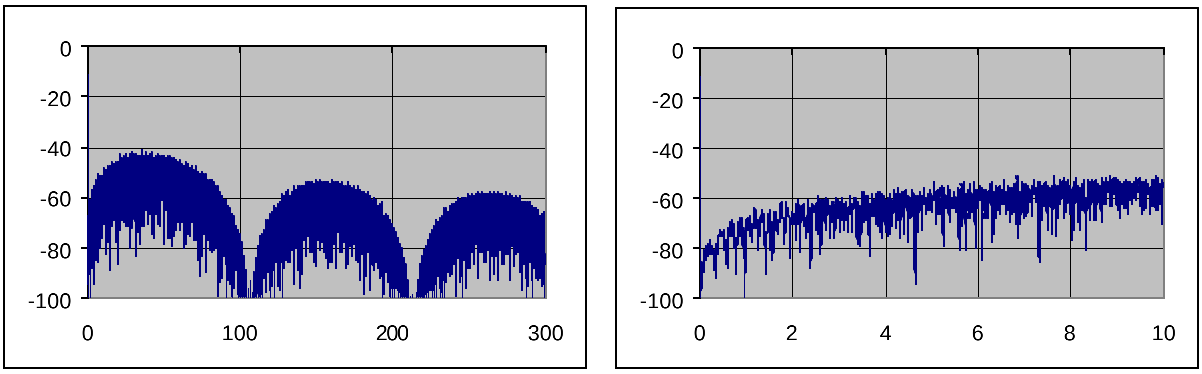

Fig.4: Simulated spectrum of a delta-sigma modulated 10.4 kHz sine wave up to 300 MHz (left) and 10 MHz (right). On the vertical axis the power is indicated in dB. The spectrum is based on a direct Fourier transform (DFT), which means that it contains no absolute amplitude information.

the quantization noise is at minimum, a high signal-to-noise ratio is expected.
These preliminary results show that the use of our electronics in combination with a delta-sigma modulated signal is promising. The insertion of a Josephson array will regulate the amplitude of the arbitrary wave to be synthesized. Losses in the cables and the filters for the suppression of the quantization noise will be the main sources of uncertainty.

CONCLUSION

A new standard for AC voltage based on pulse driven Josephson junction arrays is under development. As a first step, preliminary spectral measurements and simulations have been presented on a fast switching multiplexer unit built for the high precision synthesis of arbitrary waves. First order delta-sigma modulation is used to represent the wave to be synthesized. The first results are promising with respect to the conditions for high precision filtering of the quantization noise. Future work will concentrate on the development of suitable low-loss filters. Finally, all parts of the set-up, including Josephson array, have to be assembled in order to perform precision measurements. The intended Josephson arbitrary waveform synthesizer (JAWS) has the potential to become the primary standard for AC voltage.

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