

CONFORMAL TRANSFORMATIONS APPLIED TO SEVERAL EMC PROBLEMS

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ABSTRACT

Conformal transformations are often applied to solve 2D Laplacian problems. We present results for a few examples in the field of electromagnetic compatibility. First we derive the mutual inductances M for a printed circuit board and the straight tracks, above a metal cabinet plane. Secondly, metal conduits protect cables inside them. A good measure is the transfer impedance Z_t , which is dominated by a position dependent mutual inductance M above a few kHz. Finally a cover on a conduit is calculated. All results compare well with the measurements, which have been reported elsewhere.

INTRODUCTION

The technique of conformal transformations is often applied to solve many two-dimensional (2D) Laplacian fields, which occur for instance in a quasi-static analysis of tracks on long printed circuit boards or other elongated sets of conductors. For boundaries of limited complexity, the transformations involve elementary analytical functions [1,2]. The inversion of the transformation often requires numerical procedures. The general Schwarz-Christoffel integral formula also allows more complex boundaries. In the last few years powerful tools [3] have been developed to numerically solve for the corresponding pre-vertices for singly connected regions. Simple approximations such as rapidly converging sums of elementary functions, can be derived from the accurate results. Often these approximations suffice for many practical engineering problems. Several applications are discussed hereunder.

PCB EXAMPLE

A printed circuit board has tracks on either side of a continuous ground plane (GP). The straight tracks are at various positions with respect to that GP, at the same side and on opposite sides. The tracks form closed circuits with the GP and the terminations at the track ends; in EMC terminology these circuits are called the differential mode (DM) circuits.

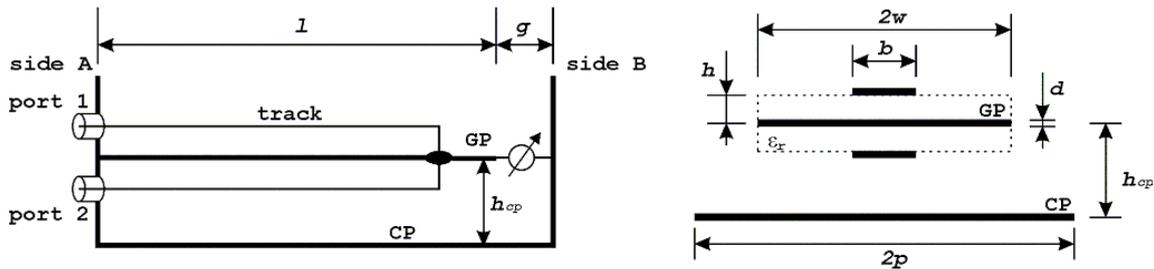


Fig. 1. Side-view and cross-section of a PCB in precompliance setup

The GP and the environment form a second circuit, the common mode (CM) circuit. Another plane, for instance a cabinet plane under the GP, can be a part of the environment. The induction coefficients M are calculated for a wide range of track positions and other parameters of the boards, and approximate expressions are formulated for a) the DM-DM or track-to-track coupling and b) the track to CM coupling. Figure 1 shows the so-called precompliance setup and also defines the parameters of geometry. The DM to CM transfer impedance Z_t is the ratio $V_{CM} / I_{1,2}$ where V_{CM} is measured over the gap g between the GP and the vertical side b , as indicated by the voltmeter in Fig. 1. The current $I_{1,2}$ is injected into port 1 or 2, where both tracks are shorted to the GP at the far end.

First the CP is regarded very far away. Then the Joukowski transform provides the M -coupling between two track circuits. Assume the GP to be located at the real axis in the complex z -plane between $-w < x < w$, and two tracks at position z_1 and z_2 . The transform $J: t = z / w + \sqrt{z/w+1} \cdot \sqrt{z/w-1}$ maps the z -plane outside the plate onto the t -space

outside the unit circle. The rotation $s = \exp j(\pi - \alpha_1)$ brings t_1 to the negative real axis; α_1 is the argument of t_1 . The current I_1 through track 1 returns through the GP. The complex potential Ω due to I_1 at s_1 and its image in the unit circle result in the M_d between the tracks:

$$\Omega(s_2) = (jI/2\pi)[\log(s + |t_1|) - \log(s - 1/|t_1|)] \quad \text{and} \quad M_d = -(\mu_0/I)[\text{Im}\Omega(s_2) - 1/(2\pi)\log|t_1|]. \quad (1)$$

Real explicit expressions for M_d cannot be obtained from (1), but the transformations only require elementary complex arithmetic. In case of the the CM to DM coupling, one assumes a current I through the GP with the return far away. In this case, Ω and the resulting M_c are simpler

$$\Omega(t) = (-jI/2\pi)\log t \quad \text{and} \quad M_c = (\mu_0/I)\text{Im}\Omega(t_2). \quad (2)$$

When the CP approaches the GP, two cases can be considered, first a very wide CP or $2p \rightarrow \infty$, and secondly a CP with $2p$ of the same magnitude as $2w$. The CM to DM M_c -coupling can be described by transforming the GP and the CP in the z -plane into two opposite sides of a K - K' rectangle in the ζ -plane; this rectangle is then regarded as a part of an infinitely wide transmission line. For the first case, the very wide CP, one has $z = Z(\zeta, m)$. Here K and K' are the complete elliptic integrals with modulus m and its complement; Z is the Jacobi ζ -function. The value of m depends on the ratio h_{CP}/w (see Fig. 1). The determination of m and the inversion of the track position z still require numerical procedures. Simple approximations hold if $h_{CP}/w < 1$, e.g. for the mid-position $(0, h_1)$ of the track above the GP: $M_c(0, h_1) \approx \mu_0(\pi - 2K/K')^{-1} \times h/h_{CP}$ in which $K'/K \approx w/h_{CP} + \{1 + \ln(2\pi w/h_{CP})\}/\pi$, and where $L_{CM} = \mu_0 K/2K'$ is the selfinductance of the CM circuit. The field between the GP and CP is nearly homogeneous for $2w > h_{CP}$. This results in $M_c(x, h_1) \approx \mu_0 h_1/2w$ for most x -positions except near the edges. The mapping for the second case $2p \approx 2w$ is discussed in the literature, but the parameters are difficult to evaluate; see [4] and references therein. The MATLAB Schwarz-Christoffel toolbox [3] solves the required prevertices numerically up to the desired accuracy; this software also allows easy inversion of the position z . In Fig. 2 we present M_c as a function of h_{CP} for a $2w = 50$ mm wide GP above a $2p = 200$ mm wide CP, for various track position at 1.5 mm distance from the GP. The solid line shows the conformal mapping results; the circles are obtained by the Method of Moments (MoM). The DM to DM coupling requires the Schwarz-Christoffel analysis for multiply connected regions. Numerical techniques exist, but their discussion is outside the scope of this contribution. However, MoM and experimental results are presented in [4].

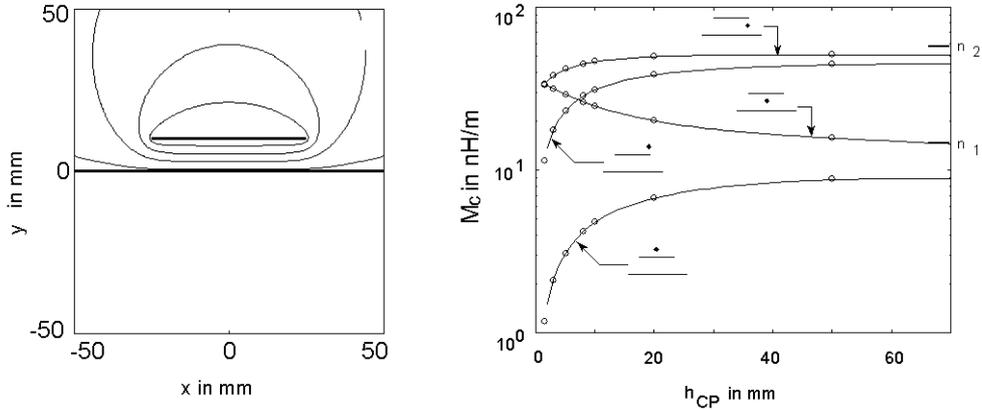


Fig. 2 Left: Magnetic field lines at high frequency, for a current I through the CM loop GP-CP. Right: Mutual inductance M_c between the CM loop and the DM circuit of a track at the indicated positions with respect to the GP. The 50 mm wide GP is at the distance h_{CP} above a 200 mm wide CP. The solid lines are the conformal mapping results, the circles result from the Method of Moments.

CONDUITS

Metallic conduits provide protection for cables placed inside against electromagnetic interference caused by a CM current through the conduit. A good measure for this protection is the conduit transfer impedance Z_t , which depends on the shape of the conduit and on the position of the cable in it. Figure 3 shows how this $Z_t = V/I$ can be defined for a U-shaped conduit. Already at not too high frequencies, usually a few kHz, a constant mutual inductance M starts to

dominate the Z_t ; ref. [5] discusses the transition from the resistively dominated Z_t and the associated changes in current distribution in the conduit wall. The conformal transformation for the flat plate is again the Joukowski transform mentioned before. The transformation for the U and H-shape are an extension of the Jacobi ζ -function [2]: D maps the rectangle between $\zeta = \pm(K+jK')$ onto the conduit and C maps the ζ -rectangle onto the unit-circle in the t -plane (Fig. 3):

$$D: z = Z(\zeta) + \frac{\text{cn}\zeta \text{dn}\zeta}{\text{sn}\zeta - f} \quad \text{and} \quad C: t = \frac{1 - f \text{sn}\zeta + \sqrt{1 - f^2} \text{cn}\zeta}{\text{sn}\zeta - f} \quad (3)$$

in which $f = (G - 1 + m)/\sqrt{m}G$ for the U shape and $f = 0$ for the symmetrical H-shape. The functions sn, cn and dn are the Jacobi elliptic functions; $G = E/K$ with E the complete elliptic integral of the second kind.

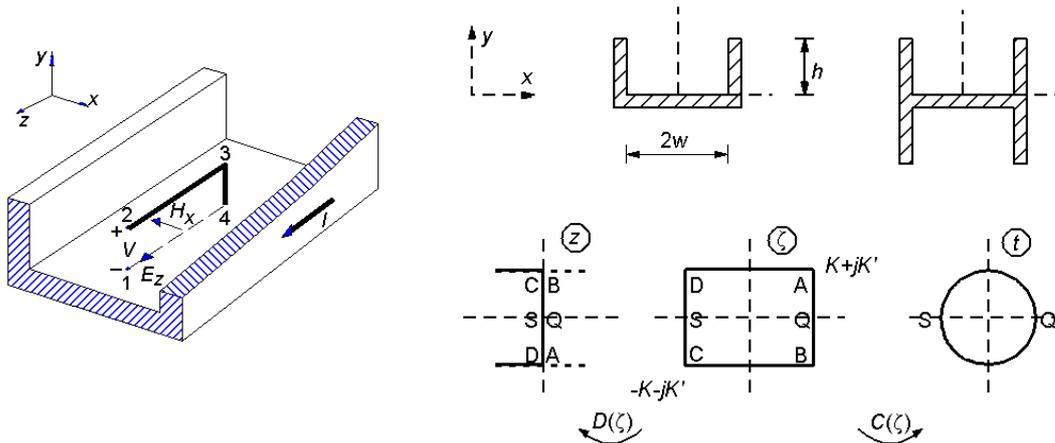


Fig. 3 Left: A U-shaped conduit with a lead to define the transfer impedance Z_t . Upper right: The U and H-shape conduit with parameters. Lower right: The three spaces z , ζ and t for the conformal mapping.

Three magnetic field lines around conduits are depicted in Fig. 4. Inside the conduit a simple series expansion holds:

$$M(x, y) = \sum_{m=0,1,2,\dots}^{\infty} (-1)^m p_m (h/w) \cos(m_a x) \sinh(m_a y) / \sinh(m_a h) \quad m_a = (2m + 1)\pi / 2w, \quad (4)$$

which has been derived from a projection of each term in the sum onto the $M(x, y)$ at the straight line between the upper conduit edges. Table I gives numerical values of p_m for $h/w = 0.5, 1.0$, and 2.0 . The p_m values only depend on the conduit shape; interpolation between the values given results in about 10 percent accuracy for M , except near the edges where the field diverges. For higher accuracy, expressions (3), or the numerical procedures of [3] should be used.

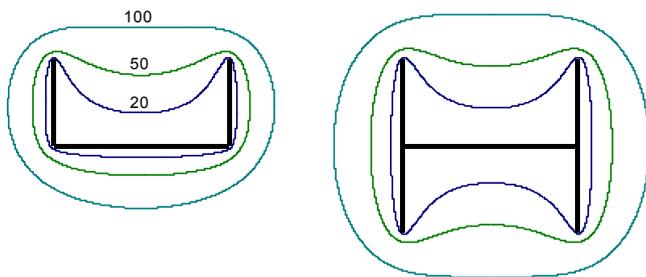


Fig. 4 Magnetic field lines as equi- M curves for 100, 50 and 20 nH/m. In case of a U and H-shaped conduit with $h/w = 1$.

Table I
First p_m values in nH/m for a U and H-shaped conduit, at three values for h/w .

$\downarrow m$ $h/w \rightarrow$	U-shape			H-shape		
	0.5	1.0	2.0	0.5	1.0	2.0
0	70.3	76.5	67.2	63.4	65.7	54.9
1	17.3	15.7	13.1	15.3	13.3	10.6
2	7.7	6.8	5.6	6.8	5.7	4.5
3	5.1	4.6	3.8	4.5	3.9	3.1

CONDUIT WITH NON-CONTACTING COVER

Figure 5 shows the conduit and the parameters; the cover carries no net current. This situation can also be analyzed by conformal mapping [6]. First the conduit and cover are regarded as a two-conductor transmission line, I_c through the

conduit and $-I_c$ through the cover. The transformation D maps the inside of a rectangular box in the complex ζ -plane upon the half conduit OPQR and half cover ST. In the toolbox of [3] D has 13 vertices when the wall thickness d is taken into account, and 11 in the limit $d \rightarrow 0$. The flux is $\Phi = \mu_0 I_c h' / 2w'$, where h' / w' results from the determination of the vertices. The field distribution inside leads to $M(x,y)$ as in (4), with $4\Phi/I_c\pi(2m+1)$ substituted for p_m . Four magnetic field lines are shown in Fig. 5; the conduit profiles are exaggerated for the sake of clarity. The flux and the current are concentrated in the slit. For narrow slits ($s, t \ll l$) one can approximate I_s , the part of I_c in both slits, by $I_s = 2\Phi / L_s$ with $L_s \approx \mu_0 s / l_{PQ}$ the inductance localized in a single slit; $l_{PQ} = 1 + d + (s+t)/2$ is the total slit length measured midway between cover and conduit from P to Q. I_s and L_s are clearly very sensitive to variations in s . The remaining part of the distributed current, $I_c - I_s = \Phi / L_c$, has been calculated separately for conduits with a $2w$ wide cover without flanges, and with thickness $d = 0$. For a wide conduit, $2w \gg h$, one expects an inside contribution proportional to $2w/h$. The $1/(2m+1)$ term in p_m indicates a slow convergence of the sum in (4) near the corner P, and suggests an $\ln(t)$ slit contribution. The outside contributes less for larger w/h . A least square fit to $2w'/h' = \mu_0 / L_c$ for 81 shapes with $1 < 2w/h < 16$ and $0.01 < t/h < 0.2$ yields

$$\mu_0 / L_c = -1.69 \ln(t/h) + 2.03w/h + 1.16 - 0.31h/w \quad (5)$$

The second transformation C provides the ratio of the cover current I_c to the total current I . A circle with unit radius is mapped upon the full outer boundary of conduit and cover. Then no net magnetic flux enters the inside region through the slit; in fact the slit can be completely neglected, which reduces the number of vertices to eight. The left part of Fig. 5 shows nine lines corresponding to the current density; the orthogonal magnetic field lines are omitted. The return for the current I is assumed to flow at a large distance; the current distribution is then symmetrical with respect to the vertical axis. The ratio I_c/I is the arc NSN' in the ζ -plane between N at the mid-slit position and its mirror image N', divided by 2π . In practice I_c/I will range between 0.5 for $l = h/2$ overlap and 0.25 for a nonoverlapping cover on a square conduit. A fixed value of 0.38 will likely be within 3 dB of the actual I_c/I .

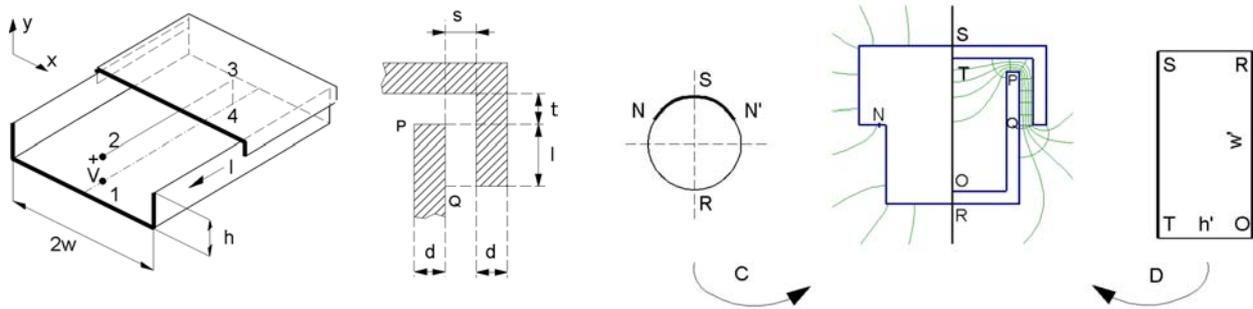


Fig. 5 Left: Cut-away view of the conduit with cover, and its parameters. Right: the three spaces and the transformations C and D. Several magnetic field lines are indicated inside.

Summarizing, conformal transformation provided mutual inductance values M as a function of position. When compared to measurements, agreement was observed to within the accuracy of the measurement or the calculations based on the actual parameters. Details have been published elsewhere [4,5,6].

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