ANALYSIS OF THE IONOSPHERIC INFLUENCE ON SIGNAL PROPAGATION AND TRACKING OF BINARY OFFSET CARRIER (BOC) SIGNALS FOR GALILEO AND GPS

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INTRODUCTION

The next generation European satellite based navigation system Galileo will make use of binary offset carrier (BOC) signals for precise pseudo range determination in addition to binary phase shift keying signals currently used by the GPS [3]. The main characteristic of this innovative BOC signal structure for satellite navigation systems is that the signal power is not concentrated around the carrier frequency. The main power is located in two main lobes to the left and right of the carrier frequency, whose separation is given by twice the subcarrier frequency. While this feature has many advantages, the dispersive character of the ionosphere may distort the signal significantly if the main lobes separation becomes large. We investigate the influence of a homogenous ionosphere, characterized only by the total electron content (TEC) value, on the deformation of the navigation signal autocorrelation function and on its delay (resp. advance). We will show that the influence is almost insignificant (despite the delay) if realistic values for the TEC are chosen. Only the standard formula for the ionospheric carrier phase advance shall be modified if the carrier phase of a BOC signal is tracked.

THE BOC SIGNAL

BOC modulations are characterised by two parameters: subcarrier frequency and spreading code rate, denoted by BOC(f_s, f_c), where f_s is the subcarrier frequency and the code rate is f_c. Both values are in MHz. According to the Galileo Signal Task Force, the European system might use a BOC(14,2) signal at E2-L1-E1 (1575.42 MHz) which has its main lobes of the power spectral density at E2 and E1 to have a spectral separation from the GPS signals at L1 (1575.42 MHz) [3]. We investigate only the BOC(14,2) signal, because this signal has the highest subcarrier frequency and consequently the deformation of the autocorrelation function and the correction to the standard ionospheric delay formula are highest. Our results can be easily generalized to all other BOC signals.

The E2-L1-E1 BOC(14,2) signal *s(t)* which might be emitted from the Galileo satellites is part of the Public Regulated Service and is described as

$$s(t)=\sum_{k=-\infty}^{\infty}c_k\,p(tf_c-k)\quad\text{with}\quad p(\phi)=\mathrm{rect}(\phi)\,\mathrm{sign}(\sin(2\pi\phi f_s/f_c))\cos(2\pi\phi f_{RF}/f_c)\,. \tag{1}$$

For the carrier frequency f_{RF} we use 1575.42 MHz, for the code rate f_c we use 2.046 MHz and for the subcarrier frequency f_s 14.322 MHz [3]. The time is denoted as t. The code sequence is c_k and it should be noted that its actual values are of no importance for this investigation as well as a possible Doppler shift due to the motion of the satellite and/or the receiver. Also the amplitude of the signal, resp. its signal to noise ratio has no influence on the results. The function “rect” represents a rectangular pulse of unit amplitude and duration.

Bandpass filtering of the signal within the satellite to keep the power spectral density within the allocated frequency band is not included in (1). However, it will be included in the navigation receiver frontend (cf. Fig. 1).

The signal *s(t)* is a convolution of the code sequence c_k with the waveform of a single chip $p(\phi)$. Therefore it is sufficient to consider the ionospheric influence on $p(\phi)$ only. This simplifies the calculations since $p(\phi)$ is different from 0 only if $0<\phi<1$.

IONOSPHERIC ADVANCE/DELAY AND DISPERSION

The influence of the ionosphere on the signal will be analyzed via a discrete Fourier transform. Therefore we write

$$p(t_j f_c) = \sum_{k=-N/2+1}^{N/2} \exp(2\pi i t_j f_k)\tilde{p}(f_k) \quad \text{with} \quad f_k = \frac{k-1}{N\Delta t} \quad \text{and} \quad t_j = (j-1)\Delta t\,. \qquad (2)$$

We choose N = 61460 and Δt = 0.079 ns to achieve a proper resolution of the Fourier transform of the single chip function $\tilde{p}(f_k)$.

The emitted signal propagates through the ionosphere which is considered to be homogeneous. Thus the total effect of the ionosphere on the signal is determined by the TEC along the signal propagation path. Scintillation effects, diffraction or ray bending are not considered here. It is well known that the ionosphere advances the sinusoidal waves by 40.3 TEC/f^2 m [4]. We neglect higher order effects f^{-n}, n>2 which cause an advance below a few cm [4]. Thus the Fourier transform of the single chip function after propagation through the ionosphere *p'* has the form

$$\tilde{p}'(f_k) = \tilde{p}(f_k)\exp(iI(f_k)) \quad \text{with} \quad I(f_k) = \frac{2\pi 40.3\text{TEC}}{cf_k} \qquad (3)$$

and the signal as a function of code phase (resp. time) can be obtained via the inverse discrete Fourier transform. The constant *c* is the speed of light. The ionospheric advance function *I(f)* has to fulfil certain symmetry properties which, however, shall not be discussed here. To analyze the ionospheric dispersion we rewrite the advance function as

$$I(f_k) = \frac{2\pi 40.3\text{TEC}}{c}\left(\frac{1}{f_{RF}} - \frac{f_k - f_{RF}}{f_{RF}^2} + \left(\frac{1}{f_k} - \frac{1}{f_{RF}} + \frac{f_k - f_{RF}}{f_{RF}^2}\right)\right). \qquad (4)$$

The three terms in the main parenthesis of (4) will be discussed in the following. The first term is constant and is a simple multiplication of the whole signal. The second term is linear in the frequency f_k and causes a constant group delay of the signal, independent of the frequency. This is the delay which is usually referred to as the ionospheric delay. For a high TEC value of 10^{18} m^{-2} the group delay is about 16 m. The third term (the parenthesis within the main parenthesis) is nearly quadratic in (f_k -f_{RF}) and distorts the shape of the single chip function. Its influence becomes larger the wider the signal spectrum is.

SIMULATION OF THE NAVIGATION RECEIVER FRONTEND

The single chip function *p'* is fed into a Matlab/Simulink simulation of a receiver frontend. The frontend converts the real-valued signal from f_{RF} to the intermediate frequency (IF) and is shown in Fig. 1. Note that we use an IF of 0 MHz. The incoming signal is bandpass filtered by a finite impulse response filter of order 1024 and multiplied by the complex nominal carrier with the frequency of f_{RF}. The output of the multiplier is lowpass filtered and the resulting complex signal p_{IF} is stored for further processing. For two exemplary TEC values the output is shown in Fig. 2.

It should be noted that from the single chip function output of the frontend the entire received navigation signal at the IF level can be reconstructed by convoluting it with the code sequence.

CODE TRACKING, THE S-FUNCTION

We determine the ionospheric delay (plus the receiver hardware delay) by evaluating the correlation function

$$R(\phi) = \int_{\phi'=-\infty}^{\infty} p_{IF}(\phi' + \phi)\text{rect}(\phi')\text{sign}(\sin(2\pi\phi' f_s / f_c))d\phi' \qquad (5)$$

between the single chip function p_{IF} at IF level and the internally generated reference chip function.

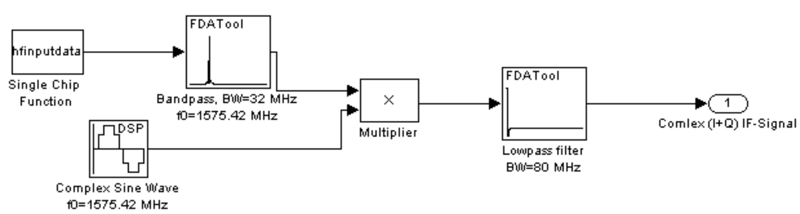

Fig. 1 Matlab/Simulink simulation of the navigation receiver frontend.

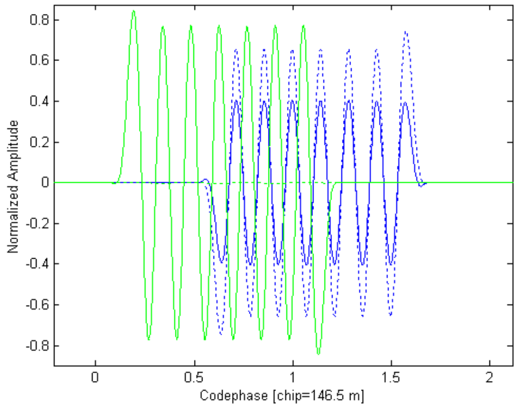

Fig. 2 Real part (solid line) and imaginary part (dotted line) of the single chip function at IF level p_{IF}. The green line refers to a vanishing TEC value, the blue line to a TEC value of $4{\cdot}10^{18}$ m^{-2}. The receiver hardware code delay of this simulation is 24.36 m or 0.1662 chips.

The position of the maximum of the (complex) correlation function $R(\phi)$ determines the estimated ionospheric plus receiver hardware delay. Since direct determination of the maximum is impractical within a real navigation receiver, the so-called S-function is formed by correlating the incoming signal with a slightly delayed and a slightly advanced replica of the code. The distance (in units of chips) of the early and late code replica is called correlator spacing d. If the output of both correlation processes is equal the maximum of the correlation function is found.

For this analysis we consider an early-power minus late-power code discriminator [1] and the corresponding S-function is given by

$$S(\phi) = \gamma\left(\left|R(\phi - d/2)\right|^2 - \left|R(\phi + d/2)\right|^2\right). \tag{6}$$

The value of the constant γ must be chosen such that $S(\phi)=\phi$ for small values of ϕ. The position of the zero-crossing of the S-function determines the estimated delay. In Fig. 3 we show the S-function for various TEC values and a correlator spacing d=0.071, chosen to track the main peak of the BOC(14,2) correlation function [5]. For a better comparison we shift all S-functions to the same origin. Thus the delay due to the different TEC values is not visible in Fig. 3.

From Fig. 3 one clearly sees that the S-function does not change its shape for realistic TEC values. Only for the extreme value of $50{\cdot}10^{18}$ m^{-2} it deforms significantly. In Fig. 4 we show the deformation of the correlation function if the TEC assumes unrealistic high values. For realistic values no changes in the correlation function are visible.

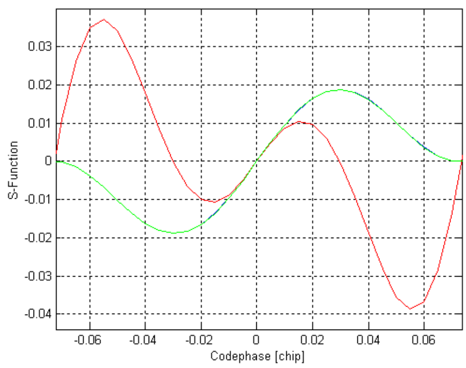

Fig. 3 Early-power minus late-power S-function. All functions are shifted to the same origin. The green line refers to a vanishing TEC value, the blue line to a TEC value of $4{\cdot}10^{18}$ m^{-2} but nearly no difference is visible to the green line. The red line refers to an unrealistic high TEC value of $50{\cdot}10^{18}$ m^{-2}.

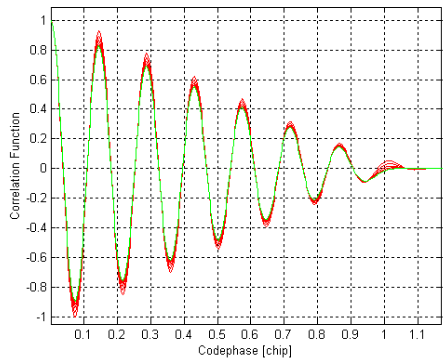

Fig. 4 Correlation function. All functions are shifted to the same origin. The green line refers to a vanishing TEC value, the red lines to unrealistic TEC values of $10{\cdot}10^{18}$ m^{-2}, $20{\cdot}10^{18}$ m^{-2}, $30{\cdot}10^{18}$ m^{-2}, $40{\cdot}10^{18}$ m^{-2} and $50{\cdot}10^{18}$ m^{-2}.

The ionospheric code delay is obtained by calculating the (central) zero-crossing of the S-function. Its value for a vanishing TEC determines the delay caused by the bandpass and lowpass filter (i.e. the receiver hardware delay). This value has to be subtracted from all other delays to obtain only the ionospheric delay.

RECEIVER TRACKING CHANNEL SIMULATION

To determine the ionospheric code delay and the carrier phase advance in another way we simulate a BOC(14,2) tracking channel via Matlab/Simulink. The simulation runs at a rate of 32 MHz. It uses the signal p_{IF} to reconstruct the full navigation signal at IF level. The GPS C/A code sequence PRN 5 is used. The carrier phase is tracked using an "arctan" phase discriminator [1] and the phase-lock-loop bandwidth is 18 Hz. The code is tracked using an early-power minus late-power discriminator and the delay-lock-loop bandwidth is also 18 Hz [1]. No thermal noise is included in the simulation since it has no influence on the ionospheric delay (resp. advance). For more details see [5].

The simulation runs for 3 s, sufficient to determine the estimated carrier phase and code values. The code chip function at IF level p_{IF} corresponding to a vanishing TEC value is used to determine the receiver hardware delays for code and phase separately. After that calibration run we use p_{IF} functions corresponding to various TEC values and subtract the hardware delays from the estimated carrier phase and code values to obtain the ionospheric delay (resp. advance).

RESULTS

The standard formula predicts for the ionospheric code delay and for the phase advance in m the values (cf. [4])

$$I_{\text{CODE}} = \frac{40.3\text{TEC}}{f_{RF}^2} \quad \text{and} \quad I_{\text{PHASE}} = -\frac{40.3\text{TEC}}{f_{RF}^2}. \tag{7}$$

A more accurate formula can be obtained if the BOC signal is approximated by the addition of two sinusoidal waves with a frequency of $f_{RF}\pm f_s$. From this assumption we derive (without explaining it here)

$$I_{\text{CODE}}^{\text{BOC}} = \frac{40.3\text{TEC}}{2 f_s}\left(\frac{1}{f_{RF}-f_s} - \frac{1}{f_{RF}+f_s}\right) \quad \text{and} \quad I_{\text{PHASE}}^{\text{BOC}} = -\frac{40.3\text{TEC}}{2 f_{RF}}\left(\frac{1}{f_{RF}-f_s} + \frac{1}{f_{RF}+f_s}\right). \tag{8}$$

In the limit $f_s \rightarrow 0$ (8) equals (7).

For Tab. 1 the ionospheric code delay is calculated by all four different methods discussed above. One sees that all values agree within a few mm.

The ionospheric phase advance is shown in Tab. 2. No S-function analysis has been performed. The simulation agrees at the sub mm-level with (8) and it should be noted that the standard formula (7) for the ionospheric advance differs from the other two columns by a few mm. Since the phase measurement noise is usually less than 1 mm, this difference is significant.

Tab. 1 Ionospheric code delay in mm for a BOC(14,2) signal at the Galileo E2-L1-E1 band. The values of the last three columns are differences with respect to (8). Add the value of the second column to obtain the full delay.

TEC [10^{16} m^{-2}]	Equation (8)	Equation (7)	S-Function Analysis	Simulation
80	12990.87	-1.07	0.00	5.52 +/- 0.04
160	25981.74	-2.15	-0.00	1.81 +/- 0.03
240	38972.61	-3.22	-0.02	2.05 +/- 0.02
320	51963.48	-4.29	-0.03	4.94 +/- 0.03
400	64954.35	-5.37	-0.03	-1.35 +/- 0.04

Tab. 2 Ionospheric phase advance in mm for a BOC(14,2) signal at the Galileo E2-L1-E1 band. All values are modulo $c/(2 f_{RF}) = 95.1$ mm because the Costas phase-lock-loop used in the simulation gives the advance modulo half the carrier wavelength. The values of the last two columns are differences with respect to (8). Add the value of the second column to obtain the full advance.

TEC [10^{16} m^{-2}]	Equation (8)	Equation (7)	Simulation
80	44.25	1.07	0.07 +/- 0.00
160	88.49	2.15	0.15 +/- 0.00
240	37.59	3.22	0.22 +/- 0.00
320	81.84	4.29	0.30 +/- 0.00
400	30.94	5.37	0.38 +/- 0.00

CONCLUSIONS

We investigated the ionospheric code delay, the phase advance and the deformation of the code tracking S-function for a BOC(14,2) signal at the Galileo E2-L1-E1 band. For code tracking we find that the S-function does not change its shape significantly even for high TEC values such that code tracking is possible. Furthermore, the standard formula (7) for the ionospheric code delay gives the delay with mm-accuracy. Since code measurement errors due to thermal noise or multipath are much higher no modification to (7) is necessary.

We also investigated the case of an unrealistic high TEC. Although such high a TEC value does not occur in nature it became clear that the frontend of the receiver must have a reasonable constant group delay over the whole frequency band, otherwise code tracking might become difficult [2].

The conclusions are different for the ionospheric phase advance since the phase measurement error due to thermal noise is below 1 mm. Therefore we suggest to replace the standard ionospheric advance formula (7) by (8) to account for the separation of the two main peaks of the BOC(14,2) signal spectrum. The process of carrier phase tracking itself (cycle-slips, signal-to-noise ratio) is not negatively influenced by the ionospheric advance.

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