SPECTRAL DOMAIN RAY TRACKING: AN ALTERNATIVE RAY METHOD FOR OPEN-ENDED WAVEGUIDES AND CAVITIES ANALYSIS

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ABSTRACT

The Spectral Ray Tracking (SRT) method is an original ray-based method, where rays represent samples of the source Plane Wave Spectrum. Arbitrary planar source distributions are admissible, even of limited extension. Backward ray launching and tracking in the spectral domain, allows to represent multiply reflected fields with a computational effort limited to one 2D discrete summation per observation point. The formulation of the method as well as its similarities and differences with existing methods (Shooting and Bouncing Rays, Generalized Ray Expansion) are presented. Its concept is validated in cases where exact solutions are known analytically.

INTRODUCTION

Ray-based methods have demonstrated their usefulness in many application fields, from optics to seismology and acoustics. Combined with powerful computing resources, they allow computation of fields in complex media, limited by nonuniform boundaries. Most of these methods rely on a far field assumption, and the source is represented as a radiating point source, from which rays are traced to each observation point. The major issue is then the computation of ray paths from point to point in a complex environment.

For reflector antennas or Radar Cross section computations, another method not relying on the same far field assumption has become very popular in the last ten years, the Shooting and Bouncing Rays (SBR) method [1]. In this approach, an incident plane wave is sampled into ray tubes which are tracked in the environment according to Geometrical Optics rules. The back scattered or radiated field is obtained via summation of the fields radiated by each spatial subaperture delimited by each exiting ray tube. This method, which is very simple in concept, has proven very efficient for it finally requires only one 2D summation, over the different tubes. However, it is limited to a source field in the form of a plane wave and does not take into account the field diffracted by the edges of the source aperture.

The Generalized Ray Expansion (GRE) method [2] is able to remedy these limitations, but at the expense of a larger number of rays to track: in this method, the source aperture is divided into a number of subapertures, and the far field assumption is then used for each of these apertures to represent its radiated fields in the form of rays. A cone of rays is launched from each subaperture and tracked within the environment. The summation of all the ray fields is thus a 2D times 2D summation (2D for the rays direction from a given subaperture times 2D for the number of subapertures).

In this paper, we propose still another ray tracking method, which allows to start from arbitrary source distributions, with a computational effort limited to one 2D discrete summation per observation point. This method is based on the discretization of radiating fields not in the spatial domain, but in the spectral domain. In the case of a plane aperture source, rays are tracked backwards from the observation point to the source plane, each ray representing a sample of the source Plane Wave Spectrum (PWS). This method can be viewed either as a generalization of the SBR method, taking advantage of the dual spatial-spectral significance of rays, or as an analog of the GRE method in the spectral domain. In the following, we shall call it the Spectral Ray Tracking (SRT) method.

We have already applied this method in the context of dielectric lens antenna analysis [3]. We propose here to demonstrate its potentialities in the context of open cavities or waveguides analysis. In this paper, we present the formulation of the method, and then validate its concept in cases where exact solutions are known. We address the specific problem of propagation in a waveguide in order to validate the technique in the context of multipath propagation in a confined volume, from a source of limited extension. Numerical results are shown for the case of parallel-plate waveguides. Calibration results regarding the number of rays to be tracked, and the accuracy of the solution as a function of the number of reflexions taken into account are presented.

FORMULATION OF THE METHOD

The general formulation of the method has been introduced in [3]. For the sake of simplicity, we shall briefly review it in the context of classical aperture theory. For definiteness and with no loss of generality, the source field distribution is supposed to be given in the plane $z = 0$, and y-polarized. $\tilde{E}_y(k_x, k_y)$ and $\tilde{E}_z(k_x, k_y)$ denote the y- and z-component of $\tilde{\boldsymbol{E}}(k_x, k_y)$, the source PWS, respectively, for the direction of propagation defined by the wave vector $\boldsymbol{k} = k_x\widehat{\boldsymbol{x}}+k_y\widehat{\boldsymbol{y}}+k_z\widehat{\boldsymbol{z}}$ with $k = (k_x^2 + k_y^2 + k_z^2)^{1/2} = 2\pi/\lambda$. $\tilde{E}_y(k_x, k_y)$ is obtained by a Fourier transform of the source field, and $\tilde{E}_z(k_x, k_y) = -k_x\tilde{E}_y(k_x, k_y)/k_z$. The electric field at any point in the half-space $z > 0$ is then given by:

$$\boldsymbol{E}(x, y, z) = \int_{-\infty}^{+\infty} \tilde{\boldsymbol{E}}(k_x, k_y) exp(-j(k_x x + k_y y + k_z z))dk_x dk_y \tag{1}$$

The discrete summation resulting from the discretization of this spectral domain integral can be viewed as a summation of ray fields with amplitudes $\boldsymbol{E}(k_x, k_y)dk_x dk_y$, propagating along the directions obtained by sampling the k_x and k_y variables.

From this point of view, the field at an observation point P can be found by sweeping the directions of arrival to that point with ray tubes launched backwards from P. In a multi-reflecting and/or refracting environment, each ray launched from the observation point is tracked through successive local refractions and reflections. For each given “tube” of directions of arrival, a ray path is saved after this backward launching step. When the four rays defining a tube reach the source plane, their directions are projected on the transverse plane (k_x, k_y) of the wave vectors space, and define a transverse differential surface in the spectral domain. With the knowledge of both this spectral surface and the source PWS, the field amplitude associated to the ray tube is calculated. This field is then transformed along the ray path previously saved, following the usual Geometrical Optics rules: in multi-reflecting and/or refracting environments, propagation of the field along a ray tube not only changes the phase of the field, but also its amplitude and direction, through reflection and transmission operators, and through phase front transformations at curved interfaces.

This backward ray launching and tracking technique in the spectral domain, allows to represent multiply reflected fields generated by a source of limited extension. We shall illustrate this feature of the method, in the case of wave propagation in a metallic perfectly conducting waveguide. We take a parallel-plate waveguide, with z-axis and with its walls at $y = \pm a/2$. The source excitation is y-polarized, and uniform along the y direction. A ray directed along the wave vector $(k_x, 0, k_z)$ and arriving at point $P(x, y, z)$ after n reflections inside the guide, can be viewed as a ray coming from a translated source, equal or symmetric to the original source according to the number of reflections. In such a simple configuration as the guide studied here, it is easy to establish that the y-component of the field associated to that ray is given by the following expression:

$$E_y(x, y, z) = (-1)^n \tilde{E}_0((-1)^n k_x) \exp(-j(|k_x|na + k_x x + k_z z))dk_x dk_y \tag{2}$$

where $\tilde{E}_0(k_x)$ is the y-component of the source PWS. The source PWS is defined as the PWS of the field incident on the guide aperture, bounded by the aperture walls, so that it accounts for the effect of the finite dimensions of the guide aperture on the incident field. In the case under study, the source PWS $\tilde{E}_0(k_x)$ is obtained by convolving the PWS of the incident field in the plane of the guide aperture, xOy, with $(2\pi/a)\text{sinc}(k_x a/2)$.

Summing up all the ray contributions arriving at point P after any number of reflections, for a given value of k_x (with $|k_x| \leq k$), yields:

$$E_y(x, y, z) = \sum_{n=0}^{+\infty}(-1)^n \tilde{E}_0((-1)^n k_x) \exp(-j(|k_x|na + k_x x + k_z z))dk_x dk_y \tag{3}$$

The PWS of this field is:

$$\tilde{E}_{eq} = \sum_{n=0}^{+\infty}(-1)^n \tilde{E}_0((-1)^n k_x) \exp(-j|k_x|na) \tag{4}$$

This PWS appears to be the PWS of an “equivalent” source field, composed of the excitation field in the guide aperture, and of its “images” representing reflected fields. For instance, let us take a source field equal to:

$$E_0(x) = \cos(\pi x/a)\Pi_a(x) \tag{5}$$

where $\Pi_a(x)$ denotes a unit square pulse of width a, centered at the origin. This source field excites the TE1 mode of the parallel-plate waveguide, whose equivalent source is $E_0(x) = \cos(\pi x/a)$. This equivalent source can be constructed by summing up the source field bounded by the guide aperture and its images. Bouncing the spectral rays in the guide is thus exactly equivalent to constructing the "equivalent" source representing both the effect of the excitation field and of the boundary conditions on the guide walls. Of course, in cases which are not amenable to analytic or modal solutions, SRT benefits from the localization property which is used to bounce rays, in the same way as SBR or GRE methods. This allows to analyze bent or non uniform configurations.

NUMERICAL RESULTS

The truncated cosine source (5) is applied to the parallel-plate conducting waveguide defined above, and the E-field is calculated along this waveguide using the SRT method. Fig. 1 illustrates convergence with the number of spectral samples n_0 at a given distance z, the number of reflections, n, being fixed. The curves are plotted for two cases corresponding to two different values of the distance z: (a) $z = 25\lambda$ and (b) $z = 250\lambda$.

Fig. 2 shows the evolution of the E-field magnitude when the number of reflexions is increased. As expected from the equivalent spectrum expression (4), the sampling rate must be higher when the number of reflexions is increased. Fig. 2 (c) and (d) show that very small absolute errors can be obtained with the SRT method, even in such critical cases as perfectly conducting guiding structures. If the waveguide walls were not perfectly conducting, or in the case of dielectric waveguides with dielectric losses, convergence would be reached with smaller numbers of reflexions. Part of the error is caused by evanescent waves, which are not taken into account in this method. However their contribution can be shown to be rather negligible.

Fig. 3 presents results in configurations which have been previously analyzed with the SBR method in [4]. An open-ended parallel-plate waveguide with separation $a = 3\lambda$ is illuminated by a plane wave with incidence angles $\theta = 0°, 30^o$ and 45^o. The results presented in (3)(a)(c)(e) are calculated by the SRT method and show a very high resemblance with those presented in (3)(b)(d)(f), which are calculated by a modal analysis, taking into account all propagating modes (5 for $a = 3\lambda$). In particular, the blurring effect which was observed in [1,4] for waveguides with small transverse dimensions is correctly represented. E-fields calculated with the SBR method did not agree so well with the actual fields obtained with the modal analysis, in the case of apertures smaller than 50λ [1,4]. The reason why the SRT method yields better results in such cases stems from the fact that the source PWS implicitly takes into account diffraction effects caused by the finite size of the source aperture.

CONCLUSION

In this paper, an original ray tracking method is introduced and compared with existing methods. The method is validated both conceptually and numerically, in cases where exact solutions are known.

This method could represent an interesting alternative to existing methods when fields have to be known at a limited number of predefined observation points, for only one 2D summation per point is needed. Being able to calculate fields at predefined points also allows for the use of Fast Fourier Transform to calculate either the far field of the exit aperture, or the plane wave spectrum in this aperture, in the course of a marching on procedure. The Spectral Ray Tracking method will certainly appear to be more efficient when the source aperture is large and the exit aperture is small, whereas the Generalized Ray Expansion method is certainly more efficient for smaller source apertures and larger exit apertures.

Further developments of the SRT method include its validation when boundary interfaces are curved, and when the source fields are not given on a planar surface.

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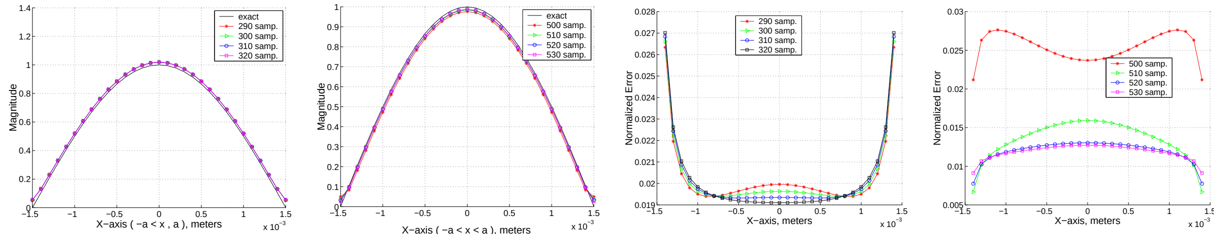
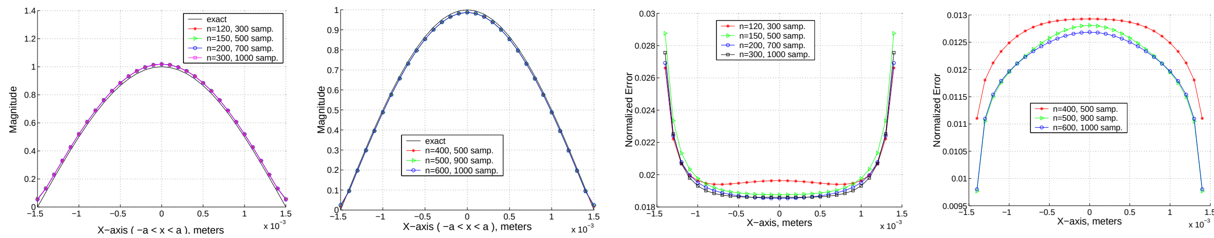


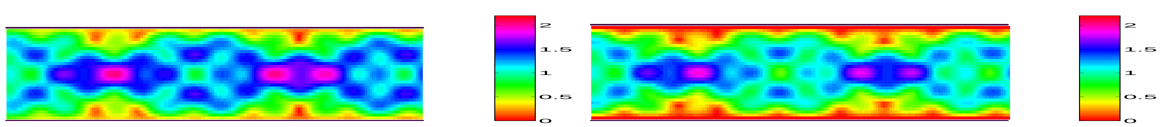
(a) $z = 25\lambda$ (b) $z = 250\lambda$ (c) $z = 25\lambda$ (d) $z = 250\lambda$

Fig. 1. E-field computed with SRT, and associated normalized absolute error, for a perfectly conducting parallel-plate waveguide (spacing $a = 0.5\lambda$) with truncated cosine source, $\lambda = 3mm$.
E-field magnitude for different numbers of samples (a) at $z = 25\lambda$ for 120 reflections (b) at $z = 250\lambda$ for 500 reflections.
(c)(d): normalized absolute error for SRT E-fields shown in (a)(b).

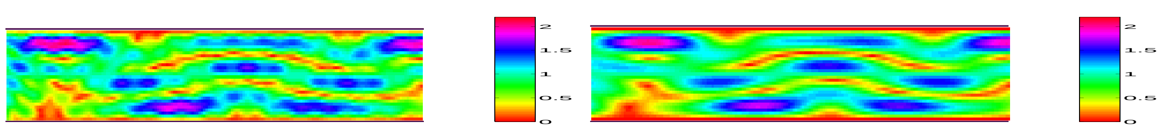


(a) $z = 25\lambda$ (b) $z = 250\lambda$ (c) $z = 25\lambda$ (d) $z = 250\lambda$

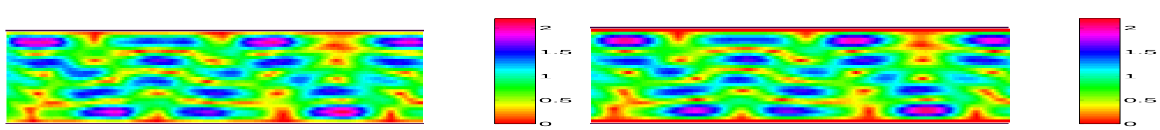
Fig. 2. E-field computed with SRT, and associated normalized absolute error, for a perfectly conducting parallel-plate waveguide (spacing $a = 0.5\lambda$) with truncated cosine source, $\lambda = 3mm$.
(a)(b): E-field magnitude for different numbers of reflections (SRT), compared to reference solution.
(c)(d): normalized absolute error for the E-fields computed via SRT, shown in (a)(b).



(a) SRT $\theta_0 = 0^o$ (b) Modal $\theta_0 = 0^o$



(c) SRT $\theta_0 = 30^o$ (d) Modal $\theta_0 = 30^o$



(e) SRT $\theta_0 = 45^o$ (f) Modal $\theta_0 = 45^o$

Fig. 3. Electric field magnitude distribution inside a parallel-plate waveguide (separation $= 3\lambda$, length $= 18\lambda$) with a θ_0-rotated plane wave excitation, computed with SRT and through modal analysis (5 propagating modes).