WAVE PROPAGATION IN TVFD/TDFV COLD PLASMA

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ABSTRACT

The different concepts for frequency dependence and time varying characteristics in cold plasma are introduced. The field equations, which are suitable to use in applications of the wave phenomena in dispersive and time varying media, for the anisotropic cases are obtained. The convenient transformations for waves in dispersive and anisotropic materials are used in such cases. The solution is studied for gradient media. The cold plasma is modeled according to the introduced approach. The application of the method is given for a time dependent and frequency varying (TDFV) and/or time varying and frequency dependent (TVFD) cold plasma.

INTRODUCTION

There is a growing interest in frequency dependent materials in applications dealing with electromagnetic theory. Some of them are waveguides filled with dispersive materials, antenna structures, and microstrip systems based on dispersive substrates. In addition, propagation in human body is becoming an important aspect of electromagnetic studies in dispersive media in recent days. The well-known approximations are Debye's and Drude's dispersions. Although these approximations are defined up to nth order dispersion algorithms, the applications in literature and traditional softwares evaluate up to the contributions of 4^{th} order dispersions. There are some softwares up to 8^{th} order; however, they frequently produce difficulties when operating them greater than 4^{th} order.

Our perspective contributes new ideas including convenient transformations for waves in dispersive materials and covers most types of structures. The wave phenomena are understood thoroughly by exact calculations in regions filled by isotropic linear materials; because, the solution of Maxwell's equations are simple in this case. So, the simplest media is the isotropic linear material. There are linear relations between polarization vector $\vec{P}$ and electric field vector $\vec{E}$ in simple materials. We can apply the FDTD method to wide frequency band. We can obtain the results for such a wide band by applying the Fourier transform; however, some conditions related with the constitutive parameters put some limits on calculations. For example, we can apply the Fourier transform method if the constitutive parameters are constants only. The Fourier transform of electric susceptibility $\chi_e(\omega)$ will not be causal for first order Drude's dispersion. Luebbers studied to overcome this difficulty by defining the time domain susceptibility function by using unit step function. If we define susceptibilities with unit step function $u(t)$ then we can find Fourier transforms of susceptibility for first order Debye's materials and Drude's materials. The function $\chi_e(t)$ of first order Drude's material has a Fourier transform which is equal to $\chi_e(\omega)$ except at the frequency of ω=0 if and only if there is an unusual difference as $\pi\omega_q 2\delta(\omega)/v_c$ where $\delta(\omega)$ is the Dirac's distribution.

The above said traditional formalisms use some extra extensions to overcome the analytical difficulties. All of these extensions accepts certain discontinuities at t=0. These assumptions postulate a time domain discontinuity at t=0. Although, frequency domain characteristic has pole singularities at certain frequencies such a sharp discontinuity at t=0 cannot be explained by this point of view. If we want to explain the discontinuity of time domain susceptibility at t=0 then we have to discuss the generation of this matter in atomic level since the susceptibility is not an effect of external sources.

For a dispersive and anisotropic medium constitutive relations are written as locally [1]-[3] for covariant components of electromagnetic field by using the Einstein's summation rule.. The dielectric permittivity, permeability, and conductivity are written as contravariant tensors in third rank. We decompose field quantities in the spatial part and time part of and accept the separation of variables for the presentation of field functions and constitutive parameters. The dielectric permittivity tensor ε^{sjq} gives the electric susceptibility tensor $\chi_E{}^{sjq}$ as

$$\varepsilon^{sjq}(\vec{r}\,;t:\omega)=\varepsilon_0[\,I^{sjq}+\chi_E{}^{sjq}(\vec{r};t:\omega)\,] \tag{1}$$

where I^{sjq} is unit contravariant tensor in third rank.

The Fourier transform pair

$$F_t\{U(x;t)\}=\int_{-\infty}^{\infty}U(x;t)e^{i\omega t}dt\overset{\Delta}{=}\underline{U}(x:\omega) \tag{2a}$$

$$F_t^{-1}\{\underline{U}\,(x:\omega)\}=\frac{1}{2\pi}\int_{-\infty}^{\infty}\underline{U}(x:\omega)e^{-i\omega t}d\omega \tag{2b}$$

can give a suitable way to determine the effect of frequency variations of source on the characteristics of dispersive and anisotropic media [1, p.9]. Here $\underline{U}(x{:}\omega)$ is defined at a constant frequency ω. The function $\underline{U}(x{:}\omega)$ depends on variable x and changes according to parameter, which ω is fixed at a specific value.

If we apply the alternate Fourier transform of [1, p.12], [2, p.3-5], [3, p.1]

$$F_{\underset{\sim}{\omega}}\{\,U(x;\underset{\sim}{\omega}:\omega)\,\}=\int_{-\infty}^{\infty}U(x;\underset{\sim}{\omega}:\omega)e^{i\underset{\sim}{\omega}t}d\underset{\sim}{\omega}\overset{\Delta}{=}U(x;t:\omega) \tag{3a}$$

$$F_{\underset{\sim}{\omega}}^{-1}\{U(x;t:\omega)\}=\frac{1}{2\pi}\int_{-\infty}^{\infty}U(x;t:\omega)e^{-i\underset{\sim}{\omega}t}d\omega\overset{\Delta}{=}U(x;\underset{\sim}{\omega}:\omega) \tag{3b}$$

with respect to parameter $\underset{\sim}{\omega}$ then the wave equation can be separated in spatial and temporal parts.

The $\tilde{\varepsilon}_{pq}(\vec{r}\,;\underset{\sim}{\omega}:\omega)$ is defined as the frequency varying dielectric permittivity tensor of anisotropic and dispersive media around a constant frequency ω. The $\tilde{\mu}_{pq}(\vec{r}\,;\underset{\sim}{\omega}:\omega)$ will define the frequency varying magnetic permeability tensor of anisotropic and dispersive media around a constant frequency ω. These constitutive parameters are defined by causal functions in anisotropic media having frequency varying and time dependent parameters. We have not to postulate irregularities, which adds a distribution to a classical function and/or a discontinuity at physical properties of material in time domain, in the case of anisotropy.

The permittivity ε_0 and permeability μ_0 of free space are transformed to suitable parameters in dispersive and anisotropic media. So, time domain electric and magnetic susceptibilities are transformed to corresponding parameters at variable frequency $\underset{\sim}{\omega}$ around a constant frequency ω in frequency domain for anisotropic and dispersive media.

THE TDFV/TVFD COLD PLASMA MODEL

The simplest closed system of transport equations is given as cold plasma. We can write the basic equations in cold plasma as

$$\frac{\partial\rho_{m\alpha}}{\partial t}+div(\rho_{m\alpha}\vec{v}_\alpha)=S_\alpha \tag{4a}$$

$$\rho_{m\alpha}\frac{\partial\vec{v}_\alpha}{\partial t}=n_\alpha q_\alpha(\vec{E}+\vec{v}_\alpha\wedge\vec{B}_0)+\rho_{m\alpha}\vec{g}+\vec{A}_\alpha-\vec{v}_\alpha S_\alpha \tag{4b}$$

Here $\rho_{m\alpha}$ is the mass density of particle type with α, q_α is the heat flow, $\vec{v}_\alpha$ is the average fluid velocity of particle type with α, S_α illustrates the collisions leading to production or loss of particles, $\vec{A}_\alpha$ is the rate of change of mean momentum per unit volume due to collision, n_α is number density, and $\vec{g}$ is gravity acceleration. The $\vec{E}$ is external electric field and the $\vec{B}_0$ is the plasma magnetic field. If the plasma is non-magnetized then we have $\vec{B}_0$ =0. If we ignore the ion motion then it will be S_α=0, $\rho_{m\alpha}\vec{g}\cong 0$, and $\vec{A}_\alpha$ =0.

The $\vec{E}$ and $\vec{B}$ satisfy (2.29a)-(2.29c) and (2.30a)-(2.30c) of [2], respectively, for an isotropic and non-conducting TDFV/TVFD plasma.

Equations (4a) and (4b) is written as

$$i\omega\underline{\rho}_{m\alpha}(\vec{r}:\omega)+\mathrm{div}[\underset{\sim}{\rho}_{m\alpha}(\vec{r};\underset{\sim}{\omega}:\omega)*\underset{\sim}{\vec{v}}_{\alpha}(\vec{r};\underset{\sim}{\omega}:\omega)]-\underline{S}_\alpha(\vec{r}:\omega)=0 \qquad \text{when } \underset{\sim}{\omega}=-\omega \tag{5a}$$

and

$$\begin{aligned}&\underset{\sim}{\rho}_{m\alpha}(\vec{r};\underset{\sim}{\omega}:\omega)*[i\underset{\sim}{\omega}*\underset{\sim}{\vec{v}}_{\alpha}(\vec{r};\underset{\sim}{\omega}:\omega)]-[\underset{\sim}{n}_{\alpha}(\vec{r};\underset{\sim}{\omega}:\omega)*\underset{\sim}{q}_{\alpha}(\vec{r};\underset{\sim}{\omega}:\omega)]*\vec{E}(\vec{r};\underset{\sim}{\omega}:\omega)+\\ &+(-1)[\underset{\sim}{n}_{\alpha}(\vec{r};\underset{\sim}{\omega}:\omega)*\underset{\sim}{q}_{\alpha}(\vec{r};\underset{\sim}{\omega}:\omega)]*[\underset{\sim}{\vec{v}}_{\alpha}(\vec{r};\underset{\sim}{\omega}:\omega)\otimes\underset{\sim}{\vec{B}}_{0}(\vec{r};\underset{\sim}{\omega}:\omega)]+\\ &+(-1)\underset{\sim}{\rho}_{m\alpha}(\vec{r};\underset{\sim}{\omega}:\omega)*\vec{g}(\vec{r};\underset{\sim}{\omega}:\omega)-\underline{\vec{A}}_\alpha(\vec{r};\underset{\sim}{\omega}:\omega)+\\ &+\underset{\sim}{\vec{v}}_{\alpha}(\vec{r};\underset{\sim}{\omega}:\omega)*\underset{\sim}{S}_{\alpha}(\vec{r};\underset{\sim}{\omega}:\omega)]=0 \qquad \text{when } \underset{\sim}{\omega}=-\omega\end{aligned} \tag{5b}$$

in cold plasma by similar calculations with the results in [1]-[3]. The convolutions in (5a)-(5b) are calculated with respect to $\underset{\sim}{\omega}$.

In the case of non-gradient, non-conducting, and isotropic TDFV/TVFD plasma we obtain

$$\underline{T}_x^e(:\omega)\,\Delta\, U_x^e(\vec{r}:\omega) - \omega f_\mu(\vec{r}:\omega)\, f_\varepsilon(\vec{r}:\omega)\, U_x^e(\vec{r}:\omega)\underset{\sim}{T}_\mu(\underset{\sim}{\omega}:\omega)*\{\underset{\sim}{\omega}\,[\underset{\sim}{T}_\varepsilon(\underset{\sim}{\omega}:\omega)*\underset{\sim}{T}^e_x(\underset{\sim}{\omega}:\omega)]\} =$$

$$= \frac{\partial\,(f_\eta f_{\rho^e})}{\partial\,x}\,[\underset{\sim}{T}_\eta(\underset{\sim}{\omega}:\omega)*\underset{\sim}{T}_{\rho^e}(\underset{\sim}{\omega}:\omega)] - i\omega\, f_\mu U_x^J(\vec{r}:\omega)[\underset{\sim}{T}_\mu(\underset{\sim}{\omega}:\omega)*\underset{\sim}{T}^J_x(\underset{\sim}{\omega}:\omega)] \tag{6a}$$

$$\underline{T}_y^e(:\omega)\,\Delta\, U_y^e(\vec{r}:\omega) - \omega f_\mu(\vec{r}:\omega)\, f_\varepsilon(\vec{r}:\omega)\, U_y^e(\vec{r}:\omega)\underset{\sim}{T}_\mu(\underset{\sim}{\omega}:\omega)*\{\underset{\sim}{\omega}\,[\underset{\sim}{T}_\varepsilon(\underset{\sim}{\omega}:\omega)*\underset{\sim}{T}^e_y(\underset{\sim}{\omega}:\omega)]\} =$$

$$= \frac{\partial\,(f_\eta f_{\rho^e})}{\partial\,y}\,[\underset{\sim}{T}_\eta(\underset{\sim}{\omega}:\omega)*\underset{\sim}{T}_{\rho^e}(\underset{\sim}{\omega}:\omega)] - i\omega\, f_\mu\, U_y^J(\vec{r}:\omega)[\underset{\sim}{T}_\mu(\underset{\sim}{\omega}:\omega)*\underset{\sim}{T}^J_y(\underset{\sim}{\omega}:\omega)] \tag{6b}$$

$$\underline{T}_z^e(:\omega)\,\Delta\, U_z^e(\vec{r}:\omega) - \omega f_\mu(\vec{r}:\omega)\, f_\varepsilon(\vec{r}:\omega)\, U_z^e(\vec{r}:\omega)\underset{\sim}{T}_\mu(\underset{\sim}{\omega}:\omega)*\{\underset{\sim}{\omega}\,[\underset{\sim}{T}_\varepsilon(\underset{\sim}{\omega}:\omega)*\underset{\sim}{T}^e_z(\underset{\sim}{\omega}:\omega)]\} =$$

$$= \frac{\partial\,(f_\eta f_{\rho^e})}{\partial\,z}\,[\underset{\sim}{T}_\eta(\underset{\sim}{\omega}:\omega)*\underset{\sim}{T}_{\rho^e}(\underset{\sim}{\omega}:\omega)] - i\omega\, f_\mu\, U_z^J(\vec{r}:\omega)[\underset{\sim}{T}_\mu(\underset{\sim}{\omega}:\omega)*\underset{\sim}{T}^J_z(\underset{\sim}{\omega}:\omega)] \tag{6c}$$

$$\underline{T}_x^h(:\omega)\,\Delta\, U_x^h(\vec{r}:\omega) - \omega f_\mu(\vec{r}:\omega)\, f_\varepsilon(\vec{r}:\omega)\, U_x^h(\vec{r}:\omega)\, \underset{\sim\varepsilon}{T}(\underset{\sim}{\omega}:\omega)*\{\underset{\sim}{\omega}\,[\underset{\sim\mu}{T}(\underset{\sim}{\omega}:\omega)*\underset{\sim x}{T^h}(\underset{\sim}{\omega}:\omega)]\}=$$

$$= -\,[U^J_{z,y}(\vec{r}:\omega)\,\underline{T}_z^J(:\omega) - U^J_{y,z}\,\underline{T}_y^J(:\omega)] \qquad \text{(6d)}$$

$$\underline{T}_y^h(:\omega)\,\Delta\, U_y^h(\vec{r}:\omega) - \omega f_\mu(\vec{r}:\omega)\, f_\varepsilon(\vec{r}:\omega)\, U_y^h(\vec{r}:\omega)\, \underset{\sim\varepsilon}{T}(\underset{\sim}{\omega}:\omega)*\{\underset{\sim}{\omega}\,[\underset{\sim\mu}{T}(\underset{\sim}{\omega}:\omega)*\underset{\sim y}{T^h}(\underset{\sim}{\omega}:\omega)]\}=$$

$$= [U^J_{z,x}(\vec{r}:\omega)\,\underline{T}_z^J(:\omega) - U^J_{x,z}\,\underline{T}_x^J(:\omega)] \qquad \text{(6e)}$$

$$\underline{T}_z^h(:\omega)\,\Delta\, U_z^h(\vec{r}:\omega) - \omega f_\mu(\vec{r}:\omega)\, f_\varepsilon(\vec{r}:\omega)\, U_z^h(\vec{r}:\omega)\, \underset{\sim\varepsilon}{T}(\underset{\sim}{\omega}:\omega)*\{\underset{\sim}{\omega}\,[\underset{\sim\mu}{T}(\underset{\sim}{\omega}:\omega)*\underset{\sim z}{T^h}(\underset{\sim}{\omega}:\omega)]\}=$$

$$= [U^J_{x,y}(\vec{r}:\omega)\,\underline{T}_x^J(:\omega) - U^J_{y,x}\,\underline{T}_y^J(:\omega)] \qquad \text{(6f)}$$

by [2,(2.29a)-(2.30c)]. Equations (6a)-(6b) can be solved by the solution technique of modified separation of variables given in [2, §3.2.1].

The contravariant and covariant tensors are corresponded to vector and scalar quantities in cold plasma equations for the anisotropic case [4, §3.2]. These tensors are separated into their spatial and time parts. The equations are solved by the modified separation of variables [2, pp. 16-26].

CONCLUSIONS

The different concepts for frequency dependence and time varying characteristics in cold plasma are introduced. The field equations, which are suitable to use in applications of the wave phenomena in dispersive and time varying media, for the anisotropic cases are obtained. The convenient transformations for waves in dispersive and anisotropic materials are used in such cases. The solution is studied for gradient media. The cold plasma is modeled according to the introduced approach. The application of the method is given for a time dependent and frequency varying (TDFV) and/or time varying and frequency dependent (TVFD) cold plasma.

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