Using Simulated Annealing for Topological Optimisation of All-Optical Microwave Filters

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ABSTRACT

The application of simulated annealing to the combinatorial optimisation of all-optical microwave filter topologies is presented. The synthesis of topologies to meet predetermined filter specification is met through the random generation of IIR (infinite impulse response) structures which are then subjected to simulated annealing. This allows for all possible solutions to be explored including non-intuitive topologies, which until now has not been addressed. This approach shows that solutions can be found that are optimal in terms of the number of components.

INTRODUCTION

The processing of microwave and mm-wave signals using all-optical techniques has attracted considerable interest in recent years. This includes a considerable effort in the use of optical fibre components for microwave filtering functions [1-4]. These have traditionally been implemented using topologies based on tapped delay line structures involving several loops of optical fibre with moderately few couplers. However, in order to realise more demanding specifications, more components are often required, leading to relatively large structures. The analysis of these filters is fairly straight forward and utilises the *z*-transform commonly used in DSP analysis. However, there is very little in the way of synthesis techniques, with designs being variations of well known basic structures such as the Mach Zehnder section and windowing functions. The fact that these filters operate in the incoherent regime results in positive time domain solutions, which leaves the designer unable to use traditional design methodologies commonly used in discrete-time filter design.

Combinatorial optimisation problems require every possible combination, of all the component values, to be evaluated to ensure finding the optimal solution. All combinations are defined as the solution space, $\mathcal{S}$, and an individual solution is defined as, η. The total number of η, $N\mathcal{S}$, for a given topology depends upon the type and number of components, the precision of the values, and the topology itself. The minimum $N\mathcal{S}$ for a structure containing only couplers and delays in an IIR format is given by

$$N\mathcal{S} = Sk^{Nk} \times Sl^{Nl} \qquad (1)$$

where Sk and Sl are the sizes for the precision in the coupling coefficients, k, and delays, l, and Nk and Nl are the number of couplers and delays and $Nk = Nl$. However, it quickly becomes clear that small increases in the number of components leads to very large increases (approx. 3 orders of magnitude per coupler increase) in the solution space. The task of finding values for these to meet filter specifications soon becomes impossible and more modern computer aided approaches are necessary.

Combinatorial optimisation (in the guise of genetic algorithms) has been used to a limited extent by Cusick *et al* [1] on FIR filters which are optimum in a cascade of unbalanced Mach Zehnder sections. Later work considered the design of IIR structures using genetic algorithms [5]. Here the IIR synthesis consisted of small (maximum of three couplers) filters connected together in series to produce a larger, more complex filter. However, genetic algorithms when used for optimisation suffer from slow convergence towards the solution.

The aim of this work is to present a new approach to synthesising IIR filters that meet stringent specifications such as those in radar receivers. A design methodology is implemented that uses combinatorial programming, which is based on simulated annealing [6]. All-optical filter specifications, such as pass-band and stop-band characteristics as well as waveguide characteristics can be fed to the algorithm. From this any number of components can be connected together in a random fashion to produce novel filter topologies. Simulated annealing can operate on all features of the structure including the layout to produce unique solutions to the filter specification. This random generation and annealing is continued until a solution is found. Local minima traps which restrict the speed of global minimum search in GAs are overcome with an ability to “climb out”.

SIMULATED ANNEALING

Simulated annealing stems from an analogy between the thermodynamics of particles in a substance and the search for solutions in complex combinatorial optimisation problems. The arrangement of particles within a substance is reflected in a level of energy of the substance. The combination of variables values of components within a topology is reflected in a energy of the problem, and different combinations yield different energy levels. The probability of a topology arranged in combination η with energy $E(\eta)$ is controlled by a temperature, *T*, and is based on Boltzmann's probability distribution. Here a high energy rating results in a bad solution to the specification and an energy of zero when the specification is met.

$$P(E(\eta)) = \exp\left(\frac{-E(\eta)}{T}\right) \qquad (2)$$

Equation (2) expresses the idea that a system in thermal equilibrium at *T* has its energy probabilistically distributed among energy states $F(\eta)$. Fig. 1 shows the probability distribution for several temperatures. At very high temperatures, $T = \infty$, the topology has an equal chance of being in any combination. At very low temperatures, $T \approx 0$, the topology has no chance of being in a combination with a high $E(\eta)$. The temperatures in between show a greater probability of being in a low $E(\eta)$, whilst still allowing for high energy states.

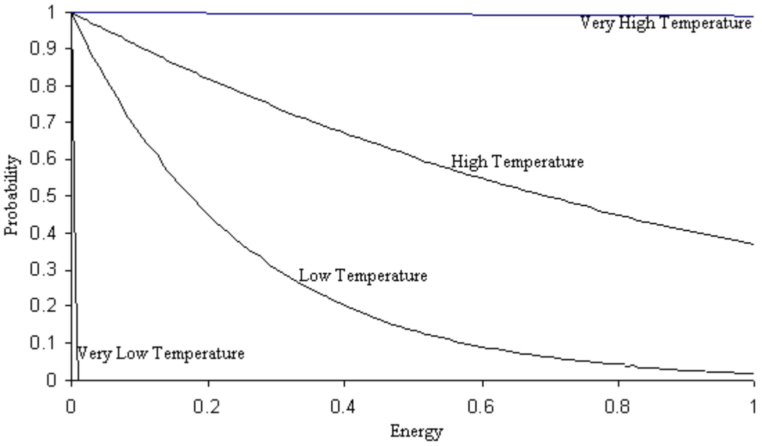

Fig 1 Probability distribution of energy states for different temperatures, *T*.

The transition of the combination η_i with energy $E(\eta_i)$ to combination η_j with energy $E(\eta_j)$ is governed by the probability

$$P = \exp\left(\frac{-(E(\eta_j) - E(\eta_i))}{T}\right) \qquad (3)$$

Equation (3) states that the probability of moving to a lower $E(\eta)$is greater than 1, therefore a probability of 1 is assigned. The probability of moving to a higher $E(\eta)$, which can be seen in Fig. 1 where the x-axis is now delta energy, is greater at low T. The higher the temperature the more likely an "up-hill" transition is. It also shows that simulated annealing behaves like the greedy search algorithm [7], while allowing for the ability to "climb-out" of local optima with a certain probability, which decreases with temperature. It can be shown that for a long cooling period a global minimum will be found [6].

DESIGN METHODOLOGY

The simulated annealing operation was used to optimise all-optical microwave filters containing the following components.

1. Filter specification (Fig 2)
2. Topology to be optimised
3. Solution space S (or a set of boundaries that contain S)
4. String to describe η (component values) bounded by S
5. Function to generated a neighbourhood of individuals $\mathcal{N}\eta$, for each η, bounded by S
6. Function to generate random individuals η_{rand} bounded by S
7. Energy function to evaluate the individuals η
8. Control parameter temperature T

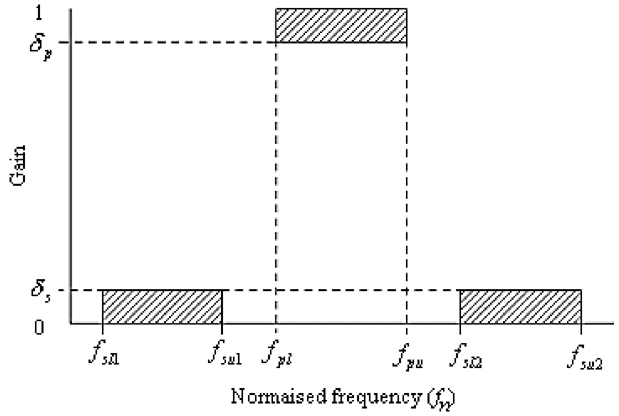

Fig.2 Filter specification Shaded area shows where the response must lie

Step 1: Generate a random topology of y components.
Step 2: Randomly select a population of x individuals $\eta_{pop.}$ This increases the chances of a search being close to the global minimum.
Step 3: Evaluate the energy of η_{pop} $E(\eta_{pop})$
Step 4: If a solution is found stop
Step 5: If stopping criteria has been met terminate topology and return to step 1
Step 6: Assign a probability of being chosen based on $P = \exp\left(\frac{-(E(\eta_{pop}) - E(\eta))}{T}\right)$ (for initial population $E(\eta) = 0$).
Step 7: Choose η based on probability.
Step 8: Generate a neighbourhood of solutions $\mathcal{N}\eta$ for η and a random solution $\eta_{rand,}$ evaluate energies $E(\mathcal{N}\eta, \eta_{rand})$ and insert $\mathcal{N}\eta$, η_{rand}, *and* $E(\mathcal{N}\eta, \eta_{rand})$ into η_{pop} *and* $E(\eta_{pop})$.
The generation of η_{rand} ensures searches are close to the global minimum.
Step 9: Update T and return to step 4.

EXAMPLE

As an example of the simulated annealing applied to IIR filter design, the following specification was provided:
$f_{sl1} = 0.5$, $f_{su1} = 0.75$, $f_{sl2} = 1.25$, $f_{su2} = 1.5$ $\delta_s = -15$dBo, $f_{pl} = 0.95$, $f_{pu} = 1.05$, $\delta_p = -3$dBo.
The resulting topology produced the response shown in Fig. 4

The solution was given by (Fig 3):

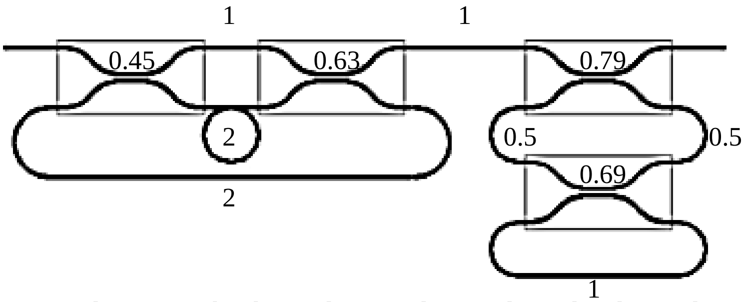

Fig. 3 Topology given by the random topology and simulated annealing

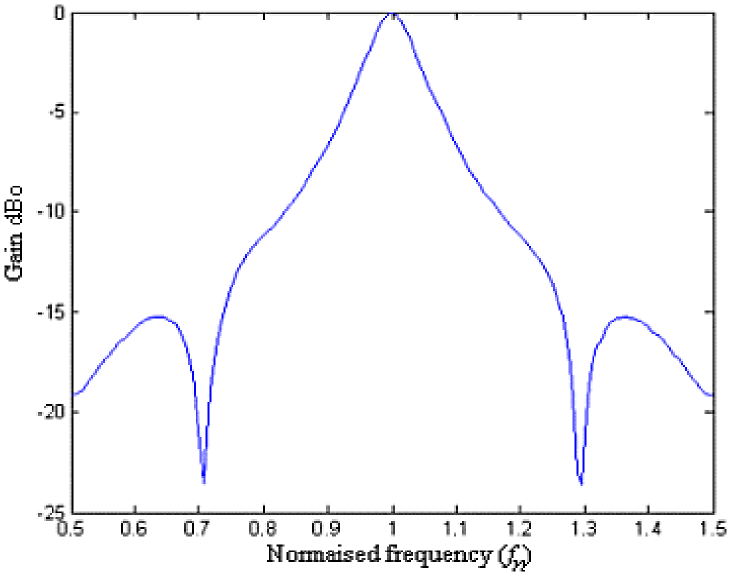

Fig 4 Response of topology given by the random topology and simulated annealing

CONCLUSION

A design methodology for all optical microwave filters has been developed using simulated annealing. This approach differs from previous design techniques[1,5] in that it allows for random topologies to be generated and optimised which can produce novel, non-intuitive structures.

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