

# A Fiber Optic Phase Modulator with an Optical Frequency Shift of up to 20 GHz

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## ABSTRACT

A fiber optic phase modulator is described in which the phase of a light wave traveling in an optical fiber is modulated by a composite electromechanical resonator with  $Q \cong 1000$ . The instantaneous frequency of the wave at the modulator output varies according to a harmonic law. The interval of tuning of the instantaneous light wave frequency achieved in the experiment is 20 GHz, which corresponds to a modulation index of  $\cong 10^6$ . It is demonstrated that a maximum value of the frequency tuning range is determined by the elastic limit of the resonator material.

## INTRODUCTION

Numerous applications of the tunable lasers include the optical frequency domain reflectometry (OFDR) [1] (which has inspired this study), integral optical device testing [2], photodetector calibration [3], etc. The OFDR procedure involves a linear sweep of the light wave frequency, after which the beats between reference and scattered waves contain various spectral components uniquely corresponding to various parts of a tested line or a passive multiplex network of fiber optic sensors. However, a minimum linewidth of the existing tunable semiconductor lasers amounts to several hundred kilohertz, thus restricting the range of tested wavelengths to several hundred meters [1].

An alternative to the direct modulation was represented by an electrooptical modulator [4] in which the source emitted a fixed frequency and the modulation was performed behind the source. This approach requires using a high-rate processing system with a frequency band of a few gigahertz, since the optical wave frequency shift equals the frequency of the electric modulation signal. Another disadvantage of this scheme was the presence of modulation harmonics leading to nonlinear second-harmonic distortions of the photocurrent beat signal.

Previously [5], we proposed a scheme of the fiber optic modulator in which the phase of a light wave propagating in the fiber was changed by extending a part of this fiber coiled around a piezoceramic cylinder. This solution is intrinsically compatible with single-mode fiber optic devices and is wavelength independent and can therefore be applied to any type of light source. In addition, this scheme is free from the problem of harmonics and requires no wideband electronic processing system because of a low resonance frequency of the piezoceramic cylinder ( $\cong 25$  kHz). A disadvantage of this modulator scheme is a low resonator quality ( $Q \cong 25$ ) for which the frequency tuning in a sufficiently wide range (4 GHz) is achieved at the expense of a considerable power (8 W) dissipated in the modulator.

Below we describe a fiber optic phase modulator based on a composite resonator (Fig.1a) around which about 300 m of a single-mode optical fiber was wound.

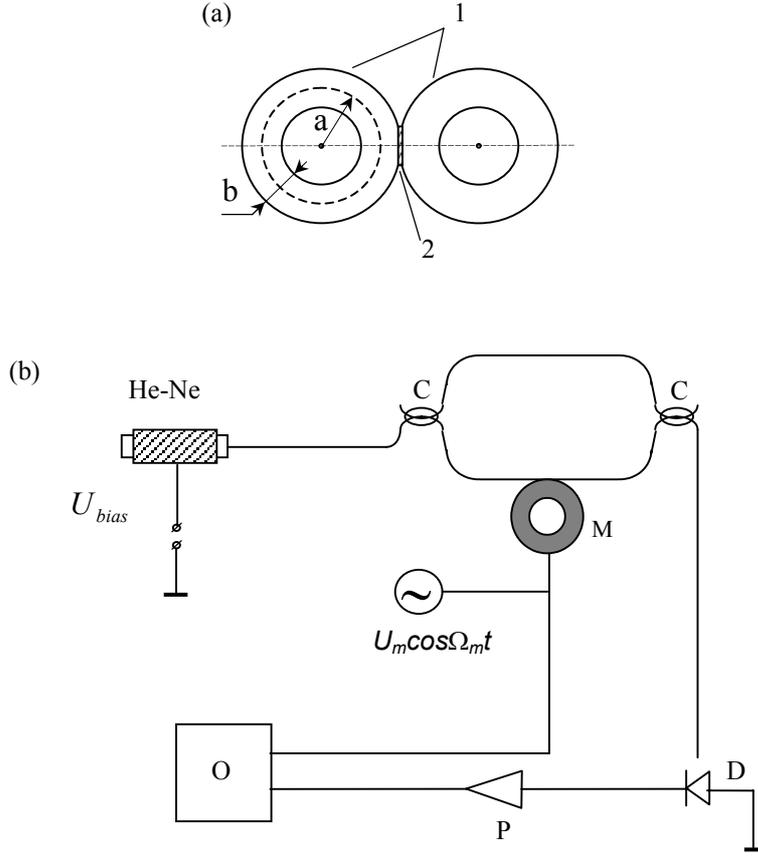
## THEORY

In order to study the light wave frequency modulation, we used a two-beam interferometer scheme (Fig.1b). In the case of a source with two longitudinal modes, the photocurrent  $i$  is determined by the interference of four waves:

$$i \approx \left\langle ra_1 \cos \omega t + ra_2 \cos(\omega t + \Delta \omega t) + a_1 t^2 \cos(\omega t + \varphi) + a_2 t^2 \cos(\omega t + \Delta \omega t + \varphi) \right\rangle^2, \quad (1)$$

where  $r$  and  $t$  are the amplitude reflection and transmission coefficients of the interferometer;  $a_1$  and  $a_2$  are the longitudinal mode amplitudes;  $\varphi$  is the phase increment;  $\omega$  is the cyclic frequency of the light wave; and  $\Delta \omega = 2\pi \Delta \nu$  is the mode spacing (angle brackets denote time average).

Let the interferometer to include a phase modulator such that  $\varphi = \varphi_m \cos(\Omega_m t)$ . Under the condition  $\varphi_m \gg 1$ , expression (1) reduces to



**Fig. 1.** Schematic diagrams showing : (a) the top view of a composite electromechanical resonator, (1) bronze cylinder; (2) piezoelectric transducer and (b) the experimental setup: (D) photodiode, (C) fiber couplers, (M) fiber optic phase modulator, (O) oscilloscope, (P) amplifier.

$$i(t) = G[\Omega(t)] \times \left\{ A \cos \int_0^t \Omega(t) dt + B \cos \int_0^t [\Omega(t) \pm \Delta\omega] dt \right\}, \quad (2)$$

where  $\Omega(t) = \varphi_m \Omega_m \sin \Omega_m t$  is the instantaneous frequency;  $A = r t^2 (a_1^2 + a_2^2)$  and  $B = r t^2 a_1 a_2$  are the beat amplitudes. The coefficient of proportionality in (2) represents a spectral response  $G[\Omega(t)]$  of the photodetector at the instantaneous frequency. For simplicity, the constant component of the photocurrent is omitted, which is equivalent to assuming  $G(0) = 0$ .

In the case of a single-mode laser operation ( $a_2 = 0$  and  $B = 0$ ), an amplitude-modulated wave is incident onto the photodetector. The modulation frequency of this wave exhibits periodic variations from 0 to  $\Omega_{\max} = \varphi_m \Omega_m$ . At the time instants such that  $\Omega_m t \ll 1$ , the frequency is modulated according to a linear law and the photocurrent envelope is

$$i(t) \cong \frac{A}{2} G(\Omega_{\max} \Omega_m t), \quad (3)$$

This expression is of practical interest for determining the amplitude–frequency characteristics of photodetector structures: the photocurrent envelope displayed on the oscilloscope visualizes the frequency response of the photodetector. A convenient means of calibrating the phase modulator is offered by a two-mode laser operation ( $a_1 = a_2, B \neq 0$ ). In this case, the photodetector measures the beats not only near the points of zero frequency, but at

the time instants  $\tau$  for which  $\Omega(\tau) = \Delta\omega$  as well. Thus, we can readily determine the light wave frequency deviation as

$$\Omega_{\max} = \frac{\Delta\omega}{\sin \Omega_m \tau}, \quad (4)$$

Let us analyze the factors determining the range of frequency deviation. The phase modulation index  $\varphi_m$  and, hence, the range of instantaneous frequency deviation  $\Omega_{\max}$  are proportional to the fiber strain amplitude. Denoting by  $\Delta l$  be the strain amplitude of the fiber halfturn, we obtain

$$\Omega_{\max} = \frac{4\pi n}{\lambda} N \Omega_m \Delta l, \quad (5)$$

where  $N$  is the number of turns,  $\lambda$  is the radiation wavelength, and  $n$  is the refractive index of the fiber core.

Proceeding from the general considerations, we expect that the resonator oscillation amplitude  $a_n$ , hence, the fiber strain  $\tilde{l}$  can be controlled by three parameters: (i) elastic limit of the fiber; (ii) power transmitted to the composite resonator; and (iii) elastic limit of the resonator material. The elastic limit of the fiber is reached only for a relative fiber strain on the order of 1% [1]. This level of dynamic straining can be attained only in metallic glasses or in a specially treated beryllium bronze. An analysis of the latter two parameters shows that, similar to the case of a piezoceramic modulator [5], the main factor limiting the strain amplitude in a low- $Q$  resonator is the transmitted power. However, the situation will principally change for a resonator possessing very high  $Q$ , in which the strains corresponding to the elastic limit can readily be achieved. Let us consider this case in more detail.

For a cylindrical modulator operating on the principal bending mode, one can readily check that [6]

$$\Delta l = \frac{4}{3} \frac{\sigma}{E} \frac{r_0^2}{d}, \quad (6)$$

where  $\sigma$  is the elastic strain amplitude,  $E$  is the elastic modulus of the modulator material,  $r_0$  is the cylinder radius, and  $d$  is the cylinder wall thickness. Formula (6) is derived under the assumption that the cylinder wall thickness  $d$  is small as compared to the radius  $r_0$ . Therefore, all subsequent expressions will also refer to the case of a thin-wall cylinder.

The principal bending mode frequency of a thinwall cylinder is given by the formula

$$\Omega_m \cong 0.48 \frac{d}{r_0^2} \sqrt{\frac{E}{\rho}}, \quad (7)$$

where  $\rho$  is the modulator material density. Using formulas (5)–(7), we readily obtain an expression relating the instantaneous frequency deviation to the physicochemical parameters of the modulator material:

$$\frac{\Omega_{\max}}{2\pi} = 1.28 \frac{n}{\lambda} N \sqrt{\frac{\sigma^2}{E\rho}}, \quad (8)$$

The term under square root sign in (8) has a physical meaning of the specific elastic energy stored per unit mass of the strained element. Therefore, this relationship implies that the range of the instantaneous frequency deviation is determined by the elastic modulus  $E$ , modulator material density  $\rho$ , and elastic stress  $\sigma$ . In practice, the maximum possible elastic stress  $\sigma$  is determined by the elastic limit  $\sigma_{el}$  of the modulator material. Exceeding this limit leads to violation of the Hooke law, whereby the oscillator becomes nonlinear and the system escapes from resonance. As is known [7], the  $\sqrt{\sigma_{el}^2 / \rho E}$  value is maximum for copper-based alloys. In addition, there is a specially developed commercial technology of thermal treatment additionally increasing the  $\sigma \geq \sigma_{el}$  value by more than one order of magnitude.

In order to determine the transmitter power, we have calculated mechanical energy  $U$  stored in the strained resonator using the explicit form of the principal bending mode:

$$U = \frac{13}{16} M \Omega_m^2 S^2, \quad (9)$$

Here,  $M$  is the cylinder mass,  $\Omega_m$  is the resonance frequency, and  $S$  is the maximum amplitude of the cylinder bending oscillations. At the same time, the oscillation amplitude  $S$  is related to the frequency tuning range

$v_{\max} = \Omega_{\max} / 2\pi$  by the following expression:

$$S = \frac{v_{\max} \lambda}{4N\Omega_m n}, \quad (10)$$

Using (9) and (10) together with a formula for the transmitted power,  $P = \Omega_m U / Q$ , we obtain an expression for the frequency deviation range:

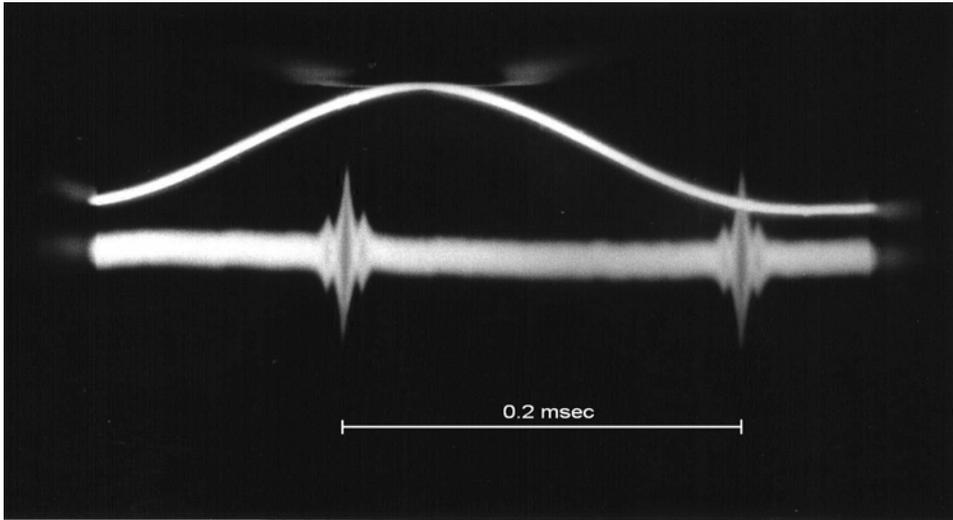
$$v_{\max} = \frac{16Nn}{\lambda} \sqrt{\frac{PQ}{M\Omega_m}}, \quad (11)$$

As can be seen from this expression, the frequency tuning range can be increased by reducing the resonance frequency  $m$  and increasing the  $Q$  value and the power transmitted to the resonator.

## EXPERIMENT

The radiation source in our experiments was a He-Ne laser of the LGN-207 type ( $\lambda = 0.63\mu\text{m}$ ). In comparison with the commercial device, our setup allowed the resonator frequency to be changed relative to the amplification circuit, thus switching the source from double- to single-mode lasing. The interferometer scheme assumed a two-beam configuration (Fig.1b) with a phase modulator in the signal arm.

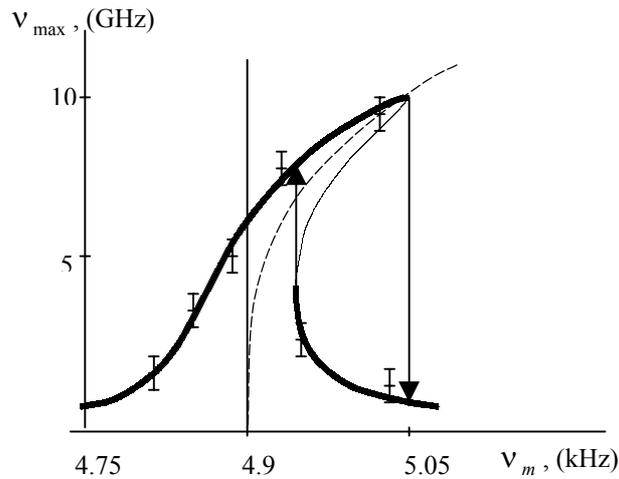
The fiber optic phase modulator represented a  $\cong 300\text{-m}$ -long single-mode optical fiber tightly coiled in seven layers ( $\cong 1000$  turns) on a composite electromechanical resonator. The resonator comprised a pair of elastic cylindrical elements made of bronze, touching one another along the cylinder generatrix. The principal bending mode of the cylindrical elements ( $\cong 4.9$  kHz) was excited with the aid of piezoceramic transducers. In an unloaded state ( $\sim 10$  turns), the modulator had  $Q \sim 1000$ . The piezoceramic transducers were driven by a unipolar ac voltage with an amplitude of about 150 V applied to the electrodes.



**Fig. 2.** Oscillograms of the photodetector signal observed in a double-mode lasing regime.

When the laser operated in a double-mode regime, the oscillogram contained symmetric spikes (Fig. 2) described by the second term in (2). The spikes correspond to an intermode beat frequency of  $\Delta\nu = 650$  MHz. The instantaneous frequency deviation produced by the phase modulator was calculated as described in [5]. Formula (4) yields 10 GHz, which corresponds to the total frequency tuning range of 20 GHz. As demonstrated above, the frequency deviation in our system was determined by the interval of a linear relationship between the elastic strain amplitude and the external force magnitude. Calculation of the  $\Omega_{\max}$  value for a modulator with  $\rho = 8000$  kg/m<sup>3</sup>,  $\sigma_{el} \cong 100$  MPa,  $E = 120$  GPa,  $\lambda = 0.63$   $\mu\text{m}$ ,  $n = 1.45$ , and  $N = 1000$  yields  $\Omega_{\max} / 2\pi \cong 9.4$  GHz, in agreement with the experimental value. If the oscillation amplitude increases so that  $\sigma \geq \sigma_{el}$ , the anharmonicity of the restoring force becomes an important factor

leading to a nonlinear relationship between the resonance frequency and the external force amplitude.



**Fig. 3.** The plot of instantaneous frequency deviation versus frequency of the applied voltage ( $U_m = 150$  V).

As a result, the resonance curve exhibits bending typical of the nonlinear oscillator. Fig.3 shows a resonance curve characteristic of the oscillations with a finite amplitude. The frequency detuning in our case was  $\cong 150$  HZ.

The transmitted power estimated by formula (11) for the instantaneous frequency deviation  $\Delta\nu = 10$  GHz amounts to  $P \cong 12W$ . However, no heating of the resonator was observed in experiment. A thorough analysis of the energy losses showed that the resonator quality is mostly determined by a response to the acoustic emission from the oscillating system to the environment.

In our opinion, the use of a specially heat-treated beryllium bronze element will allow the instantaneous frequency deviation range up to 100 GHz (with a total frequency tuning range of 200 GHz) for a modulator coil containing 1000 turns. This is comparable to the frequency tuning range of a single-frequency dynamic semiconductor laser.

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