THREE-DIMENSIONAL PROFILE INVERSION OF PENETRABLE OBJECTS FROM TIME-DOMAIN DATA

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ABSTRACT

An optimization approach to imaging of electrical property distributions of inhomogeneous anisotropic objects is developed using time-domain total field data. The approach is the extension of the forward-backward time-stepping method previously described for a two-dimensional case. It is shown that the gradient of the functional given by a difference between the measured field data and the calculated field data for estimated objects is explicitly derived using adjoint fields. Direct and adjoint problems are solved numerically using the finite-difference time-domain method. As a numerical example, slice images of reconstructed results for an inhomogeneous isotropic dielectric cube are given.

INTRODUCTION

In several areas of applied sciences such as geophysical exploration, medical imaging, and nondestructive testing, we often encounter nonlinear inverse scattering problems. Therefore, substantial interest has developed in the effort to reconstruct the shape, location, internal structure, and material parameters of an unknown object from measurements of the scattered field exterior to the object. In the last two decades, a variety of inversion algorithms for the problems in both the frequency and time domains have been proposed with the aim of widening the range of the contrast of electrical parameters and the size of a scatterer to be reconstructed. Some of the inversion algorithms in the frequency domain have been applied to three-dimensional (3D) objects[1]-[6], while few have considered 3D inverse scattering problems in the time domain. In this paper, we consider reconstruction of electrical parameter profile of 3D penetrable objects embedded in a homogeneous medium from the knowledge of measured transient total electromagnetic field data. We describe a forward-backward time-stepping (FBTS) method to solve the 3D time-domain inverse scattering problem. The FBTS method was previously applied to 2D isotropic and anisotropic objects and its effectiveness was shown for noise-free and noise-contaminated data [7], [8]. This work extends the FBTS method to a 3D case.

PROBLEM STATEMENT

Maxwell's equations in a nondispersive medium are represented in a matrix form as

$$\mathcal{L}\mathbf{v} = \mathbf{j}, \tag{1}$$

where

$$\mathbf{v} = (\, E_x \quad E_y \quad E_z \quad \eta H_x \quad \eta H_y \quad \eta H_z \,)^t, \tag{2}$$

$$\mathbf{j} = (\, \eta J_x \quad \eta J_y \quad \eta J_z \quad 0 \quad 0 \quad 0 \,)^t. \tag{3}$$

The differential operator $\mathcal{L}$ is defined by

$$\mathcal{L} \equiv \bar{A}\frac{\partial}{\partial x} + \bar{B}\frac{\partial}{\partial y} + \bar{C}\frac{\partial}{\partial z} - \bar{F}\frac{\partial}{\partial(ct)} - \bar{G}, \tag{4}$$

where

$$\bar{A}=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix},\ \bar{B}=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},\ \bar{C}=\begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\bar{F}=\begin{pmatrix} \bar{\bar{\varepsilon}}_r & \mathbf{0} \\ \mathbf{0} & \bar{\bar{\mu}}_r \end{pmatrix},\quad \bar{G}=\begin{pmatrix} \eta\bar{\bar{\sigma}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}. \tag{5}$$

The tensors $\bar{\bar{\varepsilon}}_r$, $\bar{\bar{\mu}}_r$, $\bar{\bar{\sigma}}$ are the tensor relative permittivity, permeability, and conductivity, respectively. The quatity η denotes the intrinsic impedance in free space.

Consider an object of inhomogeneous anisotropic material surrounded by a homogeneous isotropic medium. The object is successively illuminated by a short pulsed wave generated by an electric dipole source located at a transmitter points $\mathbf{r}_m^t$ $(m = 1,\ 2,\ \cdots,\ M)$:

$$\mathbf{j}_m = \mathbf{J}(t)\delta(\mathbf{r} - \mathbf{r}_m^t), \tag{6}$$

where the time factor $\mathbf{J}(t)$ is assumed to be null vector before time $t = 0$. We want to determine the unknown constitutive parameter distributions of the object from the knowledge of the incident pulsed wave and the time domain data of electromagnetic waves measured at receiver points $\mathbf{r}_n^r$ $(n = 1,\ 2,\ \cdots,\ N)$. The inverse scattering problem considered here can be formulated as an optimization problem of finding the parameter distributions which minimize the following cost functional :

$$F(\mathbf{p}) = \int_0^T \sum_{m=1}^{M}\sum_{n=1}^{N} K_{mn}(t)\left|\mathbf{v}_m(\mathbf{p};\ \mathbf{r}_n^r,\ t) - \tilde{\mathbf{v}}_m(\mathbf{r}_n^r,\ t)\right|^2 dt, \tag{7}$$

where $\mathbf{p}$ is the parameter vector consisting of the elements of $\bar{\bar{\varepsilon}}_r$, $\bar{\bar{\mu}}_r$, and $\eta\bar{\bar{\sigma}}$. The vector $\tilde{\mathbf{v}}_m(\mathbf{r}_n^r,\ t)$ is the measured electromagnetic fields at $\mathbf{r}_n^r$ due to the source $\mathbf{j}_m$ and the vector $\mathbf{v}_m(\mathbf{p};\ \mathbf{r}_n^r,\ t)$ is the calculated electromagnetic fields for an estimated parameter $\mathbf{p}$ at the same receiver position under the same source excitation. The function $K_{mn}(t)$ is a nonnegative weighting function which takes a value of zero at $t = T$, where T is the duration of the measurement.

GRADIENT OF THE FUNCTIONAL

Let us denote a parameter variation as $\delta\mathbf{p}$. Then, the Fréchet derivative of the functional $F(\mathbf{p})$ at $\mathbf{p}$ is given by

$$F'(\mathbf{p})\delta\mathbf{p} = 2\int_0^T \sum_{m=1}^{M}\sum_{n=1}^{N} \mathbf{u}_m(\mathbf{p};\ \mathbf{r}_n^r,\ t)^t\delta\mathbf{v}_m(\mathbf{p};\ \mathbf{r}_n^r,\ t)dt, \tag{8}$$

where

$$\mathbf{u}_m(\mathbf{p};\ \mathbf{r}_n^r,\ t) = K_{mn}(t)\left[\mathbf{v}_m(\mathbf{p};\ \mathbf{r}_n^r,\ t) - \tilde{\mathbf{v}}_m(\mathbf{r}_n^r,\ t)\right], \tag{9}$$

and $\delta\mathbf{v}_m(\mathbf{p};\ \mathbf{r}_n^r,\ t)$ is the Fréchet differential of the nonlinear operator which maps the parameter vector $\mathbf{p}$ to the field vector $\mathbf{v}_m$ at the receiver position $\mathbf{r} = \mathbf{r}_n^r$ and is the solution of the following equation:

$$\mathcal{L}\left(\delta\mathbf{v}_m\right) = \left[\delta\bar{F}\frac{\partial}{\partial(ct)} + \delta\bar{G}\right]\mathbf{v}_m, \tag{10}$$

under the condition $\delta\mathbf{v}_m(\mathbf{p};\ \mathbf{r},\ 0) = \mathbf{0}$, where $\delta\bar{F}$ and $\delta\bar{G}$ are the variations of $\bar{F}$ and $\bar{G}$.

In order to represent the Fréchet differential $F'(\mathbf{p})\delta\mathbf{p}$ as an inner product of $\delta\mathbf{p}$ and the gradient vector $\mathbf{g}$, we introduce the adjoint operator $\mathcal{L}^*$ defined by

$$(\mathcal{L}^*\mathbf{w},\ \delta\mathbf{v}) = (\mathbf{w},\ \mathcal{L}(\delta\mathbf{v}))\,, \tag{11}$$

where the symbol $(\mathbf{f},\ \mathbf{g})$ denotes an inner product:

$$(\mathbf{f},\ \mathbf{g}) \equiv \int_0^T\int_\infty \mathbf{f}(\mathbf{r},\ t)^t\mathbf{g}(\mathbf{r},\ t)dvdt, \tag{12}$$

and where the space integral is performed over the whole space. Using the initial condition $\delta\mathbf{v}_m(\mathbf{p};\ \mathbf{r},\ 0) = \mathbf{0}$ and the causality:

$$\lim_{|\mathbf{r}|\to\infty} \delta\mathbf{v}(\mathbf{p};\ \mathbf{r},\ t) = \mathbf{0}, \qquad \text{for } 0 \le t \le T, \tag{13}$$

the adjoint operator is found to be given by

$$\mathcal{L}^*\mathbf{w} = \left[-\frac{\partial}{\partial x}\bar{A}^t - \frac{\partial}{\partial y}\bar{B}^t - \frac{\partial}{\partial z}\bar{C}^t + \frac{\partial}{\partial(ct)}\bar{F}^t - \bar{G}^t\right]\mathbf{w} = \left[-\bar{A}^t\frac{\partial}{\partial x} - \bar{B}^t\frac{\partial}{\partial y} - \bar{C}^t\frac{\partial}{\partial z} + \bar{F}^t\frac{\partial}{\partial(ct)} - \bar{G}^t\right]\mathbf{w}, \tag{14}$$

subject to

$$\mathbf{w}(\mathbf{p};\ \mathbf{r},\ T) = \mathbf{0}. \tag{15}$$

Let $\mathbf{w}_{mn}(\mathbf{p};\ \mathbf{r},\ t)$ be the solution of the following equation:

$$L^*\mathbf{w}_{mn} = \mathbf{u}_m(\mathbf{p};\ \mathbf{r}_n^r,\ t)\delta(\mathbf{r} - \mathbf{r}_n^r). \tag{16}$$

Note that the right-hand side of (16) becomes zero at $t = T$ due to the choice $K_{mn}(T) = 0$, and thus we can find the solution $\mathbf{w}_{mn}(\mathbf{p};\ \mathbf{r},\ t)$ which satisfies the condition (15). Substituting (10) and (16) into (11), and comparing the resulting equation with (8), it is found that

$$F'(\mathbf{p})\delta\mathbf{p} = \sum_{i=1}^{3}\sum_{j=1}^{3}\left(\langle g_{\varepsilon\mathrm{ij}},\ \delta\varepsilon_{\mathrm{rij}}\rangle + \langle g_{\mu\mathrm{ij}},\ \delta\mu_{\mathrm{rij}}\rangle + \langle g_{\sigma\mathrm{ij}},\ \delta\eta\sigma_{\mathrm{ij}}\rangle\right), \tag{17}$$

where the inner product is defined by

$$\langle a(\mathbf{r}),\ b(\mathbf{r})\rangle \equiv \int_V a(\mathbf{r})b(\mathbf{r})dv. \tag{18}$$

The domain V of integration is an estimation region which contains the unknown object. Note that the domain of integration with respect to the space coordinates has been changed from an infinite extent to the finite region V since we can set $\delta\mathbf{p}(\mathbf{r})$ to be zero outside the region V. Because of the space limitation, the explicit expression of the gradients $g_{\varepsilon\mathrm{ij}}$, $g_{\mu\mathrm{ij}}$, and $g_{\sigma\mathrm{ij}}$ are not shown here. Those are expressed in terms of the adjoint field vector $\mathbf{w}_m(\mathbf{p};\ \mathbf{r},\ t)$. The expressions of gradients for a two-dimensional anisotropic case are found in [8].

NUMERICAL EXAMPLES

The conjugate gradient method is applied to the minimization of the functional F. At each step of iteration, direct and adjoint problems are solved numerically using the finite-difference time-domain (FDTD) method. To illustrate the effectiveness of the FBTS method, we consider the reconstruction of a 0.62λ inhomogeneous isotropic cube having sinelike relative permittivity profile with maximum value 5, where λ is the wavelength in free space corresponding to the highest frequency contained in the incident pulsed wave with the time excitation function:

$$J(t) = \frac{d^3}{dt^3}\left[e^{-\alpha^2(t-\tau)^2}\right], \tag{19}$$

where $\tau = \beta\Delta t$, $\alpha = 4/\tau$, $\beta = 132$. The reconstruction domain containing the object is a 0.89λ cubic region. Six transmitter points on the surface of a 1.43λ cube are used and two orthogonal electric dipoles located at each transmitter point illuminate the object separately. For each illumination, all components of electromagnetic fields are collected at 318 receiver points on the surface of the cube. The FDTD solution space consists of $43 \times 43 \times 43$ cells with cell size $\Delta x = \Delta y = \Delta z = 4.0$ mm . The time duration of the measurement is $500\Delta t$ with the time step size $\Delta t = 7.55$ps. The weighting function is chosen to be $K_{mn}(t) = \cos(\pi t/2T)$. The initial guess for the relative permittivity is chosen as that of the background medium, i.e., free space. Fig. 1(a) shows the sixth, eleventh, and sixteenth slice in the z axis of the reconstruction cube containing the true object, while Fig. 1(b) shows the corresponding slices of reconstructed results at the $200th$ iteration. The reletvie permittivity profiles are found to be reconstructed quite accurately. We have also tested the iterative reconstruction method for several examples of high-contrast objects and found that the method can reconstruct the relative permittivity successfully.

CONCLUSIONS

We have proposed an optimization approach to reconstructing the electrical parameter profiles of three-dimensional penetrable objects from time-domain field data. The gradient of the functional given by a difference between the measured field data and the calculated field data for estimated objects have been explicitly derived using the adjoint fields. Numerical simulations have demonstrated the validity of the reconstruction method.

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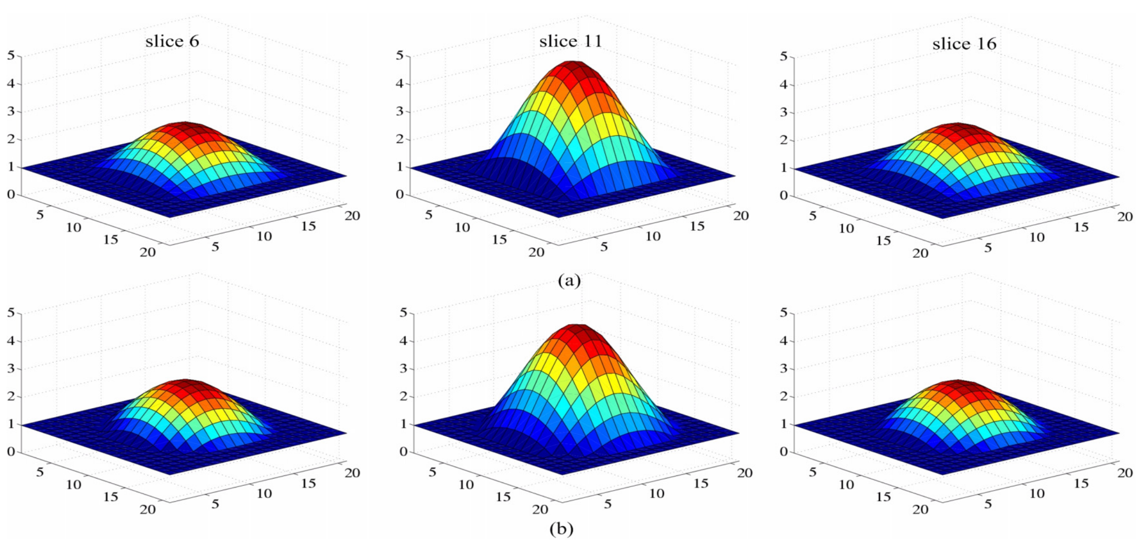

Fig. 1 Slice images in the z axis of a dielectric cube with a smooth sine-like relative permittivity profile: (a) true object, (b) reconstructed results after 200 iterations.