

# Signal Distribution in the Angle-of-Arrival and Delay Domains and Doppler Spread for Various Outdoor Communication Links

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A unified three-dimension probabilistic model is presented to describe multipath phenomena in various built-up areas with randomly distributed buildings placed on rough terrain. Using a combination of the statistical description of the built-up terrain and the 3D deterministic approach based on Kirchhoff approximation, the effects of scattering and diffraction phenomena on signal power spatial and temporal distribution are examined. The joint information regarding angle-of-arrival, delay and Doppler spread is obtained through study of the main propagation mechanisms in urban wireless communication channels. Comparison with other models and with measurements carried out in different urban environments is presented. It is shown that in dense urban areas the distribution of the obstructions surrounding the base station and the moving vehicle, as well as the elevations of both antennas contribute to the signal power profiles in the space, time and frequency domains.

## 1. Introduction

For UHF/X-band radio waves most large city buildings are essentially opaque and their sizes larger than the wavelength,  $\lambda$ . In such a situation, a wide spectrum of shadow zones is observed at the street level and very sharp boundaries between light and shadow zones are created [1-4]. Moreover, in the cases when buildings are randomly distributed on the ground surface, which is also considered to be rough, all the specific peculiarities of city topography form the particular conditions of wave propagation at the street level [5, 6]. In such an urban environment we cannot use well known deterministic models (see bibliography in [5]) and must use a statistical description of the real building pattern inside the city in order to derive the field strength. The multipath effects is described below analytically using the unified stochastic approach based on the so-called multiparametric model first introduced in [5] and then improved by taking into account in the 3D case the diffraction phenomenon and the actual buildings' overlay profile [6]. This approach is based on a combination of statistical description of the built-up terrain and a deterministic description of average signal power after scattering and diffraction from the obstructions around the base station and any subscribers, stationary or movement.

Below we present the spatial and time delay joint correlation of signal fading and the signal power spectrum distribution in the angle-of-arrival, time delay and Doppler spread domains using the proposed multiparametric model of wave propagation above built-up terrain with randomly distributed obstructions and taking into account various terminal antennas' height with respect to the rooftops' level. For mobile communication channels the more important case is when the single-scattering and diffraction are predominant phenomena [5, 6]. Therefore, we examine the characteristics of multipath field components and of total power spectrum, using formulas obtained for the process of independent scattering and diffraction of each uncorrelated multipath field component from randomly distributed obstructions placed on rough terrain.

## 2. Statistical description of city relief

Below we consider an array of randomly distributed buildings placed on the rough ground surface, which together form the buildings' overlay profile.

*Spatial distribution of city buildings.* The statistical functions constructed in [6] allow us to calculate the probability of direct visibility between two observed points  $\mathbf{r}_1$  and  $\mathbf{r}_2$  within the layer of city buildings:

$$P_{12} = \exp\{-2\langle L \rangle v r_{12}/\pi\}, \quad (1)$$

from which we can easily define the parameter  $\gamma_0$

$$\gamma_0 = 2\langle L \rangle v/\pi \quad (2)$$

which determines the density of buildings in the horizontal plane  $z=0$ . Here  $\nu$  is the density of buildings (per 1 km<sup>2</sup>) in the investigated area,  $\langle L \rangle$  is the average length of buildings (screens) in the investigated area and  $r_{12}=|\mathbf{r}_2-\mathbf{r}_1|$ .

*Distribution of reflected sections at the building surfaces.* We also introduce, according to [5, 6], the probability function that any building's segment with length  $l$  and width  $l$ :

$$P_{cd} = \exp\{-\gamma_0 \gamma_{12} r_{12} - l / \langle l \rangle\}, \quad (3)$$

where

$$\gamma_{12} = (z_2 - z_1)^{-1} \int_{z_1}^{z_2} P_h(z) [1 - X P_h(z_1)]^{-1} dz, \quad z_2 > z_1 \quad (4a)$$

$$\varepsilon_{12} = (z_2 - z_1)^{-1} \int_{z_1}^{z_2} (z - z_1)(z_2 - z_1)^{-1} P_h(z) [1 - X P_h(z_1)]^{-1} dz \quad (4b)$$

and  $\langle l \rangle = [\nu r_{12} \varepsilon_{12} |\sin \Psi_s|]^{-1}$  is an average scale of the segment; all other parameters and functions are defined above.

### 3. Signal power fading in space, time and frequency domain

*Angle-of-arrival power spectrum distribution.* We derive a function  $W(\varphi)$  which determines the power spectrum of single-scattered waves for the observer placed at the mobile transmitting point:

$$W(\varphi) = \frac{\Gamma \lambda \ell \nu h}{16\pi^2 [\lambda^2 + (2\pi \ell \nu \gamma_0 h)^2] d^3} \{f_1(\varphi) + f_2(\varphi)\} = W_1(\varphi) + W_2(\varphi) \quad (5)$$

where

$$f_1(\varphi) = \frac{2z_1^2 (\gamma_0 d)^2 \zeta' (1 - \cos \varphi)}{(z_1 + h) h} \frac{\exp\left[-\gamma_0 d \left(\frac{h}{z_1} + \frac{\zeta'}{2} \left[1 + \frac{\gamma_0 d}{2} \left(1 + \frac{h}{z_1}\right) (1 - \cos \varphi)\right]\right)\right]}{\left[1 + \frac{\gamma_0 d}{2} \left(1 + \frac{h}{z_1}\right) (1 - \cos \varphi)\right]} \quad (6a)$$

$$f_2(\varphi) = \frac{2h(\gamma_0 d)^2}{(z_1 + h)} \left[1 + \left(\frac{h}{z_1}\right)^2 \frac{1 + (k \ell \nu \gamma_0 d)^2}{1 + (\gamma_0 \zeta' d)^2}\right] \frac{\exp\left[-\gamma_0 d \left(\frac{h}{z_1} + \frac{\zeta'}{2} \left[1 + \frac{\gamma_0 d}{2} \left(1 + \frac{h}{z_1}\right) (1 - \cos \varphi)\right]\right)\right]}{\left[1 + \frac{\gamma_0 d}{2} \left(1 + \frac{h}{z_1}\right) (1 - \cos \varphi)\right]} \quad (6b)$$

Here  $\zeta' = \frac{[(\lambda d / 4\pi^3) + (z_1 - h)^2]^{1/2}}{z_1}$ . Formulas (6a) and (6b) consist of a term  $\sim (\lambda d / 4\pi^3)$ , taking into account the process of diffraction, which earlier [5] was not investigated.

*The power spectrum of time delay.* To obtain the energetic spectrum in time domain we must use the distribution of incoming moments of single scattered waves obtained in Section 2 and described by formulas (2.28) and (2.29). Using the same procedure of derivations as for the angle-of-arrival power spectrum, it is easy to obtain the complete expression for the power spectrum of time delay of arriving waves:

$$W(\tau) = \frac{\Gamma}{8\pi^2 d^2} \frac{k \ell \nu \gamma_0 h}{1 + (k \ell \nu \gamma_0 h)^2} \left\{ (1 - \zeta') \left[ 1 + (1 - \zeta')^2 \frac{1 + (k \ell \nu \gamma_0 h)^2}{1 + (\zeta' \gamma_0 h)^2} \right] f_1(\tau) + \frac{\zeta'}{(1 - \zeta')} f_2(\tau) \right\} \\ = W_1(\tau) + W_2(\tau) \quad (7)$$

where the function  $f_1(\tau)$  is given by:

$$f_1(\tau) = \frac{(\gamma_0 d)^2 \sqrt{\tau^2 - 1}}{4\tau^2} \exp\left\{-\gamma_0 \tau \frac{(2 - \zeta')}{2} d\right\} I_0\left(\frac{\gamma_0 \zeta' d}{2}\right) \quad (8a)$$

Here, as above,  $\zeta' = [(\lambda d / 4\pi^3) + (z_1 - h)^2]^{1/2} / z_1$ , and the function  $f_2(\tau)$  for  $\gamma_0 d \gg 1$  is given by:

$$f_2(\tau) = \frac{\gamma_0 d}{2} \exp\left\{-\frac{\gamma_0 \tau d}{2}\right\} \left[ \exp\left\{-\frac{\gamma_0 \tau d}{2}\right\} + \frac{\sqrt{\tau-1}}{\sqrt{\tau+1}} I_0\left(\frac{\gamma_0 d}{2}\right) \right] \quad (8b)$$

Once more, formulas (7)-(8) take into account the process of diffraction, which earlier [5] has not investigated.

*Power distribution in the Doppler spread domain.* Using this relation between the spectral functions in the frequency and time domains

$$\tilde{W}_\tau(\tau, \varphi_0) = \frac{1}{v} \tilde{W}\left(\frac{\omega}{v}, \varphi_0\right) \quad (9)$$

we can examine the signal's distribution in the Doppler spread domain for various built-up areas with randomly distributed obstacles. Let us, as above, consider two typical situations in the built-up scene.

For the *first case*, when the stationary transmitter/receiver antenna is *at the roofs level or below it*, that is,  $z_1 \leq h$ , we can obtain the following expression for the spectral function of signal temporal fluctuations according to the procedure described in [6]:

$$\tilde{W}_\tau(\omega, \varphi_0) = \frac{2 \cdot sh\chi \cdot \omega_{d \max}}{\sqrt{\omega_{d \max}^2 - \omega^2}} \left[ \frac{(\omega_{d \max} \cdot ch\chi - \omega \cdot \cos \varphi_0)}{[(\omega_{d \max} \cdot ch\chi - \omega \cdot \cos \varphi_0)^2 - (\omega_{d \max}^2 - \omega^2) \sin^2 \varphi_0]} \right] \quad (10)$$

where the parameter  $\chi$  depends on the building density and the range between the terminal antennas:

$$\chi = \ln \left[ \left( 1 + \frac{1}{\gamma_0 d} \right) - \left( \left( 1 + \frac{1}{\gamma_0 d} \right)^2 - 1 \right)^{1/2} \right]$$

For the *second case*, when the antenna height is *higher than the rooftops* of buildings, i.e.,  $z_1 > h$ , and  $\gamma_0 d \geq 10$  we get following the analytical procedure described in [6]:

$$\tilde{W}_\tau(\omega, \varphi_0) \approx \frac{2(\omega_{d \max} - \omega \cdot \cos \varphi_0)}{\omega_{d \max} \sqrt{\omega_{d \max}^2 - \omega^2}} \quad (11)$$

From this expression is clear seen that the power spectrum in the frequency domain in this case can be presented with an accuracy of constant value by function:

$$\sim \left[ \left( \omega_0 \frac{v}{c} \right)^2 - \omega^2 \right]^{-1/2} \quad (12)$$

which is close to the classical 2D Clarke's "U-shaped" spectrum and "quasi-3D" Aulin's spectrum described in [7]. Investigations of power spectrum distribution described in [7] have shown that the spectrum is not symmetrical and differs from the classical symmetrical "U-shaped" Clarke's distribution. The effect of asymmetry depends on angle-of-arrival distribution and influence of multipath components in total field spectrum. However, the approach proposed in [7] cannot explain the influence of terrain features and of antenna position with respect to buildings surrounding it, as well as diffraction phenomena, the effects, which follow from the stochastic approach that is described above. All these effects can be explained from the latter approach taking into account the shadowing effect by buildings and multipath phenomena, as well as the position of the radio port antenna with respect to buildings surrounding ( $z_1 > h$  or  $z_1 > h$ ).

#### 4. Comparison with experimental data

*Power azimuth and time delay spectra distributions.* Both built-up environments around the experimental sites can be characterized as high dense and obstructive, i.e., the NLOS urban areas. The first experimental site of Aarhus is the built-up area with 4-7 floor buildings ( $h = 20 - 21$  m) placed on rough terrain and with an irregular street grid. The base station (BS) antenna was mounted at two different heights:  $z_1 = 20$  m and  $z_1 = 32$  m, that is, at the same level and higher with respect to the buildings' rooftops. The mobile vehicle (MV) had the antenna height of  $z_2 = 2$  m. The Stockholm site area was more heavily built up and obstructive than the first one consisting 4-6 floor buildings (with an average height  $h = 15 - 17$  m) surrounding BS antenna which was mounted at the height of  $z_1 = 21$  m above the terrain. According to topographic maps of the both experimental sites obtained

from [3, 4], the building contour density parameter  $\gamma_0$  was estimated from (2) for Arhus site area  $\gamma_0 = 6 \text{ km}^{-1} - 10 \text{ km}^{-1}$  with the mean value of  $8 \text{ km}^{-1}$ ; for the Stockholm site area  $\gamma_0 = 8 \text{ km}^{-1} - 12 \text{ km}^{-1}$  with the mean value of  $10 \text{ km}^{-1}$ . As were obtained from measurements, the arriving signal power is strongly concentrated around the direction to MV, but for Stockholm its distribution is wider than that obtained in Arhus. This result is clearly understood by use of the results of calculations presented by formulas (5)-(6), from which follows that with increase of BS antenna height  $z_1$  with respect to the average building height  $h$  (for Arhus it was 21 m and 35 m while for Stockholm it was 21 m) the distribution of power azimuth spectrum (PAS) function becomes closer to a Laplacian [8, 10], that is,  $W(\varphi) \sim \exp\{-p\varphi\}$ , where  $p$  is a constant. In other words, with increase of BS antenna height the obstructions which are closer to the MV are important and their influence on PAS is describe by the diffraction term of (5)-(6). Conversely, with decrease of BS antenna height with respect to the average building height  $h$  the PAS shape becomes closer to the Gaussian distribution  $W(\varphi) \sim \exp\{-q\varphi^2\}$  ( $q = \text{const}$ ), which describes the prevailing multiple scattering phenomena.

The same results for the power delay spectrum (PDS) function distribution were obtained both experimentally and theoretically by use formulas (7)-(8). In fact, for high BS antenna elevation function  $f_1(\tau)$  from (8a) prevails with respect to  $f_2(\tau)$  and gives the significant diffraction effect in formula (7) for the PDS. In this case the shape of the PDS limits to the Laplacian PDS  $W(\tau) \sim \exp\{-a\tau\}$ , where  $a$  is a constant. Conversely, with decrease of BS antenna height  $z_1$  the function  $f_2(\tau)$  from formula (8b) becomes larger than  $f_1(\tau)$ , and its contribution in the total PDS is described by formula (8b). In this case the shape of PDS limits to the Gaussian PDS  $W(\tau) \sim \exp\{-b\tau^2\}$ , where  $b$  is a constant, when the multipath component of the resulting signal is predominant with respect to that of diffraction. Hence, our predicting theoretically obtained formulas (5)-(8) satisfactorily describes not only a spread of the PAS and PDS in the angle-of-arrival and delay domain, but also changes of its shape and width with changes of the BS antenna elevation.

*Power Doppler spectrum distribution.* For experimental verification of the complicated signal power spectrum distribution in the Doppler spread domain, a special experiment was carried at  $f = 920 \text{ MHz}$  using the same experimental site described in [5]. This tested site consists mostly three-five-floor buildings, which is homogeneously distributed around both terminal antennas. The transmitter antenna was assembled at the top of mobile vehicle at the height of 2m; the receiver antenna was assembled on the roof of building at the height of 30m, as well as at the height of 3m. The vehicle speed was varied from 10 km/h to 40 km/h. The experimentally obtained signal power spectrum distribution gave an asymmetry of the spectrum and an addition local maximum due to effects of diffraction from buildings around the terminal antennas and due to their random distribution along the radio path for each position of the vehicle with respect to mutual radio path between terminals and existence of the LOS component of the total signal when one of the antenna was higher than building roofs, which were in full agreement with theoretical results follow from formulas (10)-(12).

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