ANALYSIS OF DIELECTRIC RESONATORS AND FINITE SIZE MICROSTRIP ANTENNAS USING THE INTEGRAL EQUATION TECHNIQUE

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ABSTRACT

This contribution presents the analysis of arbitrary 2D and 3D dielectric objects using the integral equation formulation. The integral equation presented uses the volume equivalence principle, and it is solved with the Method of Moments (MoM) technique using roof top basis functions. The novelty of the formulation proposed resides in that a Galerkin technique is used in which the testing procedure is applied rigorously without introducing any approximations. This fact adds an additional variational degree to the numerical algorithm. In consequence the numerical instabilities associated to the analysis of dielectric bodies using other integral equations techniques, and previously reported in the literature, are avoided. The use of a rigorous testing procedure introduces in the formulation new line and surface integrals which can be singular inside the dielectric bodies. In this contribution we present an efficient way to numerically extract and treat these new singularities.

INTRODUCTION

The electromagnetic analysis of arbitrary shaped dielectric bodies is very interesting from the practical point of view, since they are commonly used in numerous communications devices. They are for instance integral parts in dielectric loaded resonators, which highly increases the quality factor of the resonance due to a reduced losses in new dielectric materials as compared to traditional metallic resonators. Also dielectric bodies are encountered in many antenna systems including radomes or finite size printed microstip antennas.

For the analysis of dielectric bodies the integral equation formulation has been used in the past in its volume or surface equivalence versions [1, 2]. For the volume equivalence technique the integral equation has usually been solved using pulse basis functions combined with a point matching technique [3, 4]. Although this formulation has been shown to work, it exhibits numerical instabilities for high relative dielectric constant values. This has often been attributed to the artificial surface charges that are introduced by the pulse basis functions and to the rather simple point matching testing mechanism [4, 5]. Similar numerical instabilities are reported in [2], where linear piece wise basis functions are used, although the results seem to converge to the right values when enough unknowns are introduced in the algorithm. The use of roof-top basis functions has been presented in [1], although with very simplified one point testing mechanism, thus resulting in numerical inaccuracies.

To avoid the numerical problems associated to these methods, we proposed in this contribution a formulation which uses roof top basis functions combined with a rigorous Galerkin testing procedure evaluated without introducing any approximations. Consequently the formulation introduces higher order numerical integrations which increases the stability of the algorithm. Results shows that the approach is accurate even for large dielectric constant values with good computational efficiency.

FORMULATION

The volume integral equation is formulated defining equivalent polarization currents in the volume of dielectric bodies. An integral equation is then set up by imposing boundary conditions for the fields, and then it is solved with the Method of Moments algorithm. In the present technique, a mixed potential formulation is used combined with roof top basis functions defined in the volume of the dielectric bodies. In order to take into account for the charge distribution at the surface of the dielectric bodies, half roof top functions are defined on the boundaries of the objects. The scalar potential interactions are consequently evaluated on the divergence of the basis functions, which leads to a constant volume charge in the volume and of a constant surface charge at the boundaries between dielectric objects (including the transition to free space). For the testing procedure the same roof top functions are chosen, therefore obtaining a rigorous Galerkin formulation. In order to reduce the singular behavior of the Green's functions the gradient is transfered to the test functions as a divergent operator. The chosen test functions assures that the remaining surface integrals on each discretization cell vanished inside the volume of the objects, thus avoiding one of the sources of numerical instabilities [4, 5]. However, the discontinuity generated by the half roof top functions at the boundaries produces the required surface integrals for the charge during the testing procedure, therefore resulting in a complete symmetric formulation. It is important to observe that following this approach, the transition between two dielectric regions must be treated with separate half roof top functions defined on either side of the boundary. Therefore, full roof top functions must not be defined for the cells ending into a boundary interface. This definition of half roof top functions at the two sides of the boundaries assures the correct modeling of the discontinuity of the polarization currents across dielectric regions.

In order to avoid the definition of half roof top functions at the boundaries between dielectric objects, the integral equation can also be formulated for the electric flux density as derived in [3]. Following this formulation, cells on the two sides of dielectric boundaries can form a new volume basis and test function. This is due to the fact that electric flux density remains continuous across dielectric regions. Only for the transition to free space half roof top functions must be defined at the outer interface. In this case, however, the half roof top functions are of different nature as compared to the half roof top functions defined in the previous formulation. Now, half roof tops don't take zero value at the outer boundary, and are therefore terminated in a continuous way in order to respect the continuous behavior of the electric flux density. The divergence of the basis functions, then, does not produce any additional surface integrals; not even at the surface representing the transition to free space. However, these surface integrals are generated by the divergence of an additional permittivity constant which appears in the formulation (see, [1]). A different situation is found for the testing functions. The divergence of the test functions does not generate the corresponding surface integrals for the charge. This is true not only for interfaces between dielectric regions but also for the interface to free space due to the continuous nature of the terminated half root top functions. We know, however, that a surface charge density must be present at the outer interface. The associated surface integrals are in fact generated by the process of transferring the gradient of the Green's function to the test function as a divergent operator. We know that in this process additional surface integrals on each cell are generated. These integrals are always zero for the volume and inner interfaces due to the selected test functions. However, the presence of the continuous half roof top functions at the free space boundary surface prevents this integral to be zero for this particular interface, thus resulting in the last sought for term of the formulation. For instance, a general matrix impedance element with all divergence terms transfered to the test functions and full Galerkin procedure leads to:

$$
\begin{aligned}
Z_{m,n} = & \int_{V_m} \frac{\vec{F}_m(r)\cdot\vec{F}_n(r)}{\hat{\varepsilon}}\,dV - \frac{\omega^2\mu_0}{4\pi}\int_{V_m}\vec{F}_m(r)\,dV\int_{V_n}K(r')\,\vec{F}_n(r')\,G(r-r')\,dV' \\
& -\frac{1}{4\pi\varepsilon_0}\int_{S_m}\vec{F}_m(r)\cdot\hat{e}_n\,dS\int_{V_n}\Big[K(r')\,\nabla'\cdot\vec{F}_n(r')+\vec{F}n(r')\cdot\nabla'K(r')\Big]\,G(r-r')\,dV' \\
& +\frac{1}{4\pi\varepsilon_0}\int_{V_m}\nabla\cdot\vec{F}_m(r)\,dV\int_{V_n}\Big[K(r')\,\nabla'\cdot\vec{F}_n(r')+\vec{F}_n(r')\cdot\nabla'K(r')\Big]\,G(r-r')\,dV' \qquad (1)
\end{aligned}
$$

where $K(r) = (\hat{\varepsilon}(r) - \varepsilon_0)/\hat{\varepsilon}(r)$ as in [1].

Both formulations have been implemented and they are equivalent. Only the second formulation is more efficient when there are several dielectric to dielectric interfaces, since then the total number of unknowns in the MoM matrix is reduced due to the elimination of the half roof top functions at the inner interfaces.

Another interesting aspect of the formulation derived is that the Galerkin technique implemented produces singular integrals when source and observer cells coincide. Therefore, care must be exerted while evaluating numerically these singular integrals. For the analysis of 2D dielectric objects both surface and line integrals appear in the formulation. The surface integrals for the singular cases are computed by transforming the basis integral to the polar plane, therefore effectively extracting the singularity of the Green's functions. For the singular line integrals, on the contrary, a transformation to the polar plane is not possible. In this case, however, the asymptotic behavior of the Hankel function for small arguments is used, and the resulting integral is evaluated analytically as shown in [10]. In a similar way, the analysis of 3D dielectric bodies requires the evaluation of both volume and surface integrals as described previously. Again the singular surface integrals are evaluated by transforming the basis integral to the polar plane. Finally, for the evaluation of the volume integrals an extension to the previous technique has been derived. In this case the volume of the basis integral is described using spherical coordinates. The transformation of the integral to the spherical volume produces an additional convergence term which behaves as the square of the distance. With this additional convergence factor on the integrand the function to be integrated behaves very smoothly, and the resulting integral converges very fast even with a limited number of integration points (a total of 27 points were used in the evaluation of each volume integral).

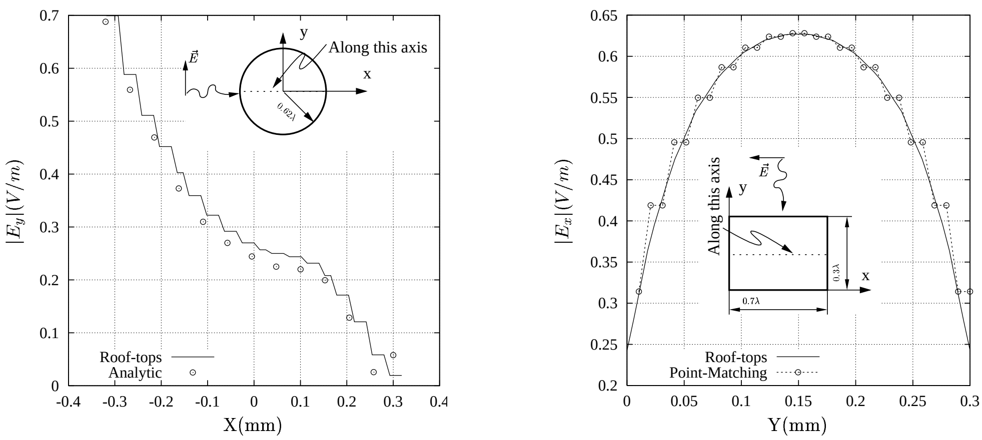
To effectively carry out the spherical transformation, the original brick cell is first divided into six pyramids. The four lateral based pyramids can now be described using spherical coordinates in a straight forward way:

$$I = \int_{\arctan\left(\frac{y_1-y_m}{x_2-x_m}\right)}^{\arctan\left(\frac{y_2-y_m}{x_2-x_m}\right)} d\varphi \int_{\arctan\left(\frac{x_2-x_m}{(z_2-z_m)\cos\varphi}\right)}^{\arctan\left(\frac{x_2-x_m}{(z_1-z_m)\cos\varphi}\right)} \sin\theta \, d\theta \int_0^{\frac{x_2-x_m}{\sin\theta\,\cos\varphi}} G(r-r')\, r^2 \, dr \qquad (2)$$

where x_2, y_1, y_2, z_1, z_2 are the constant equations of the cell sides, and (x_m, y_m, z_m) are the coordinates of the pyramid vertex. For the top and bottom based pyramids, however, this direct procedure can not be applied. In these two cases the basic pyramids are further decomposed into four tetrahedral type regions, which are then described using the spherical coordinate system following the same idea as before, thus completing the procedure.

RESULTS AND DISCUSSIONS

The technique presented has been implemented for the analysis of 2D and 3D dielectric objects. The roof top functions have been defined on both rectangular and triangular cells for the 2D case and on tetrahedral and bricks type cells for the 3D case. In its 2D version the volume formulation with roof top functions defined on rectangular cells has been compared for an infinite cylinder with analytical data (Fig. 1(a)). Also the technique has been compared for a rectangular dielectric area with standard

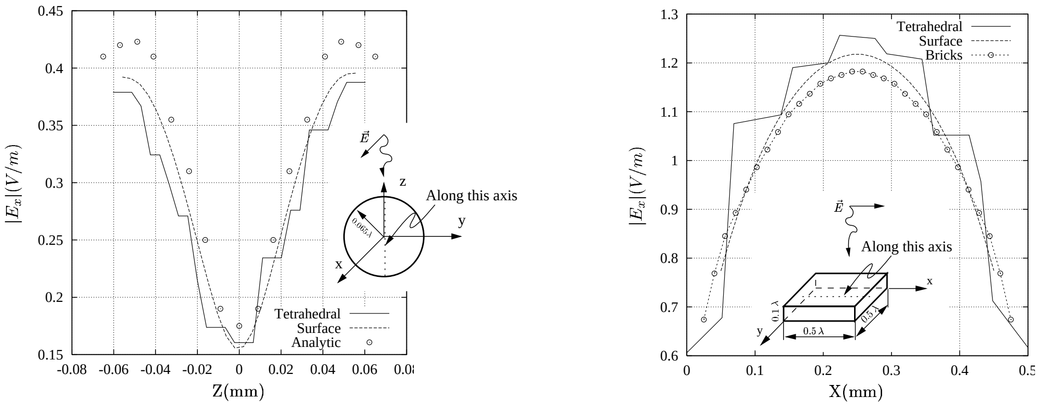


(a) Infinite cylinder $\varepsilon_r = 2.56 - j\,2.56$. **(b)** Infinite rectangle $\varepsilon_r = 4.0$.

Figure 1: Absolute value of electric field along shown axis.

pulse-point matching technique (Fig. 1(b)). The agreement is good, but the gain in the accuracy for the representation of the currents along the longitudinal direction can be clearly seen in the figure.

Finally, the 3D version of the integral equation has been used to study 3D dielectric bodies. In Fig. 2(a) we present results obtained for a dielectric sphere with a surface approach and with a volume approach using tetrahedral basis functions. Also analytical results are presented for this case, showing good agreement. In Fig. 2(b) we further present results obtained for a substrate type dielectric object analyzed again with a surface and with a volume approach, using tetrahedral and brick type basis functions. As can be seen the agreement in all cases is good, therefore validating the theoretical and



(a) Sphere $\varepsilon_r = 36$. **(b)** Substrate type $\varepsilon_r = 4$.

Figure 2: Absolute value of electric field E_x component along shown axis.

numerical techniques derived in this contribution.

CONCLUSIONS

This contribution has presented the analysis of 2D and 3D arbitrary dielectric bodies using the volume equivalent formulation of the integral equation. In order to avoid the numerical instabilities traditionally associated with this technique, roof top functions have been used as basis and test functions in the method of moments algorithm. The testing procedure has also been rigorously formulated for both the polarization currents and the electric flux density, and it has been evaluated without introducing any approximations. Finally, the singular terms of the integral equation kernel have been treated with special numerical techniques derived in this work. Results show that the proposed approach leads to accurate results even for dielectric objects of high relative dielectric constant values.

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