Optimum Criterion for Array Signal Processing, Using Constraints on the Radiation Pattern Amplitude in the Desired Signal Angular Directions

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ABSTRACT

In this communication, we propose an optimal digital beamforming algorithm with the capability of tracking multiple desired signals and nulling deterministic disturbing signals in a Gaussian noise environment. The optimum criterion is accomplished by minimizing the mean square error (MSE) of the array output, with assigned real unitary gain on the radiation pattern amplitude in the desired signal angular directions. Introducing the auxiliary variables z_d (radiation pattern phases in the desired signal directions) and using the Lagrangian theory, the MSE technique leads to the optimization of a non-linear cost functional in the z_d unknowns only. Numerical simulations show that this approach improves the performances in terms of mean square error with respect to a standard algorithm.

INTRODUCTION

Array signal processing has become a classical problem of statistical theory [1] and may be regarded as a technique to discriminate data received from the array. Since several decades, it has found many applications in radar [2] and wireless radio systems [3]. Nowadays, the working conditions of these systems are more and more complex [4]. For radar systems, apart from the desired reflection of direct signals on the target, the array antennas can also receive spurious returns such as multipath and jammers. The former is due to multiple reflections of the desired signal in the environment and jammers are radiated by enemy systems in order to disturb the receiving radar.

In wireless radio systems, one of the most important problems in a multi-user asynchronous environment is the inter-user interference, due to co-channel couplings. Moreover, in practical code-division multiple access systems, the codes can be non-orthogonal because of the varying delays of different users. Smart antennas (adaptive beam-forming) are used in order to increase performances and capacity of the transmission channels and, therefore, to fulfill the requirement of a higher number of users, at the same time.

Array signal processing has the task of maximizing the gain in the direction of the user, and canceling undesired signals, like inter-user interference, multipath and jammers, which highly degrade the system performances that use the hypothesis of uncorrelated and Gaussian noise [5]. In technical literature, the design of digital beam-formers [6] has been accomplished considering different optimal criteria (corresponding to different design requirements), which are generally reduced to the optimization of a nonlinear functional, using standard techniques, like maximum likelihood, signal-to-noise ratio and mean square error.

Unfortunately, these optimal criteria lead to the optimization of a nonlinear functional, which can be onerous from a computational point of view. Differently from standard algorithms, like multiple signal classification (MUSIC) [7] and ESPRIT [8] (Estimation of Signal Parameters via Rotational Invariance Techniques), Neural Networks (NN) [9] reduce remarkably the computation time, thanks to their massive parallelism, fast convergence rates, and very large scale integration (VLSI) implementation. In fact, although NN are time consuming in the learning phase, they are very fast in the array system processing, therefore allowing their use in real time. Moreover, they have the advantage that the parameter estimation is straightforwardly taken into account in the learning procedure, which is carried out only once at the beginning of the algorithm via the Gaussian radial basis function neural network [10].

In [11], the optimum criterion has been generalized to multiple signals, enforcing in the desired signal direction a complex gain on the radiation pattern. Note that the fulfillment of a complex constraint requires the use of a complex unknown (current), and, as a result, the number of variables, available for improving the system performances in terms of the mean square error, decreases of a complex unit.

In this communication, we propose an algorithm for the development of a digital beam-former with constraints on the radiation pattern amplitude only, as it is required in real applications. The basic idea is to satisfy the constraints using only real unknowns. This gives a further degree of freedom to improve the performance in terms of the mean square error. The proposed algorithm is accomplished using the constrained functional minimization. As a first step we introduce a new variable, representing the radiation pattern phase in the angular directions of the desired signals. By using the unconstrained minimization approach, the optimal digital beam-forming is reduced to the optimization of a Lagrangian functional, which depends on the feeding currents, Lagrangian multipliers, and the introduced variable. The stationary point method allows expressing the currents and the multipliers in terms of the phase. Therefore, the original problem is turned into a minimization of a Lagrangian functional, which depends only on the phase, and can be optimized using the proposed iterative technique.

Once the proposed optimum criterion has been applied, the radial basis function neural network is learned using the procedure proposed in [12]. The input of the network is the normalized vector representing the phase of the ratio between the output of the n-th and the first antenna. Finally, some examples (linear array of 10 and 20 elements) have been simulated and the results compared with data available in literature. In particular, the numerical results show that our algorithm improves the performances in terms of mean square error and provides lowers side lobes with respect to the method proposed in [11].

OPTIMUM CRITERIUM FOR MORE DESIRED SIGNALS

Assuming that the radiation impinging on the array of N elements can be expressed as a sum of plane waves, the overall scalar output y is given by:

$$y = \mathbf{w^H r} =< \mathbf{w}|\mathbf{r} > \qquad \mathbf{r} = \mathbf{As} + \mathbf{n} \tag{1}$$

where $\mathbf{w^H}$ is the feeding current vector, $A_{i,k} = h_i(\theta_k)$ is a matrix $N \times K$, being h_i, and K the effective height of the i-th antenna and the number of desired and unwanted signals, respectively. $\mathbf{s} = s_k$ is the complex envelope representation of the time signal, $\mathbf{n}$ is the Gaussian white noise. $(\cdot)^H$ and $< \cdot|\cdot >$ stand for the Hermitian operator and the inner product, respectively.

One typical digital beamforming network is proposed in [11], where a complex gain is assigned to the radiation pattern in all the desired signal directions. The fulfillment of a complex constraint requires that the current vector is onto a hyper-plane, and, as a result, the actual number of unknowns decreases of a complex unit. In most practical applications of interest, the phase of the desired signal does not have any useful information, as a consequence, a positive real gain is assigned on the radiation pattern amplitude. Differently from complex constraints, this approach uses only a real variable to satisfy each condition, but the problem is no longer linear. As a result, for each desired signal, a further real degree of freedom is available in order to improve the performances in terms of mean square error.

As in [11], the optimum criterion is given by:

$$\min_{\mathbf{w}} E[\mathbf{rr^H}] = \min_{\mathbf{w}} \mathbf{w^H R w} \ \text{ subject to } \ |\mathbf{w^H a_d}|^2 = U(D-d) \quad d = 1, \cdots, D \tag{2}$$

where D is the number of desired signals and $\mathbf{R} = E[\mathbf{rr^H}] = \mathbf{A^H S A} + \sigma^2 \mathbf{I}$ is the matrix of correlation, being $\mathbf{S}$, σ and $\mathbf{I}$ the correlation matrix of the signals, the noise standard deviation and the identity operator, respectively. The vector $\mathbf{a_d}$ is given by $\mathbf{a_d} = (h_1(\theta_d), \cdots, h_N(\theta_d))$, where θ_d is the direction of the desired signal.

Introducing the auxiliary variable $z_d \ \ d = 1, \cdots, D$, (2) can be rewritten as:

$$\min_{\mathbf{w}} \mathbf{w^H R w} \quad \text{subject to} \quad \mathbf{w^H a_d} = z_d, \quad |z_d|^2 = 1 \quad d = 1, \cdots, D \tag{3}$$

By using the Lagrangian functional and multipliers, (3) can be turned into an unconstrained condition:

$$\mathcal{L} =< \mathbf{w}|\mathbf{Rw} > - \sum_{d=1}^{D} \alpha_d \left[< \mathbf{w}|\mathbf{a_d} > - z_d\right] - \sum_{d=1}^{D} \beta_r(|z_d|^2 - 1) \tag{4}$$

Evaluating the stationary point of (4) with respect to $\mathbf{w}$ and enforcing the constraints relative to the multipliers α_d, the minimization problem is reduced to the following expression:

$$\min_{z} \sum_{r,s=1}^{D} z_r^* B_{r,s}^{-1} z_s \quad \text{subject to} \quad |z_d|^2 = 1 \quad d = 1, \cdots, D \tag{5}$$

where $B_{r,s} =< \mathbf{a_s} | \mathbf{R^{-1} a_r} >$. As it is shown in [13], it is possible to minimize the functional using the following iteration technique:

$$z_r^{(n+1)} = - \left| \sum_{s=1, r \neq s}^{D} B_{r,s}^{-1} z_s^{(n)} \right|^{-1} \sum_{s=1, r \neq s}^{D} B_{r,s}^{-1} z_s^{(n)} \tag{6}$$

IMPLEMENTATION HINTS AND NUMERICAL SIMULATIONS

Because of the radial basis function neural network (RBFNN) property of uniform convergence to every function, the RBFNN has been used to approximate the inverse mapping between the optimal current vector, found by means of the optimum criterion, and the direction and amplitude of desired signals and jammers. The input of the neural network is the phase of the ratio between the output of the n-th and the first antenna. The activation functions are Gaussian functions. The algorithm consists of two main steps. First, the proposed optimum criterion of the digital beamforming network is applied choosing a suitable set of trial cases. Then, an appropriate RBFNN learning procedure is performed [12].

In order to assess the proposed algorithm, we tested our technique on some examples presented in [11]. In particular we considered an isotropic linear array of $N = 10$ antennas, uniformly equispaced by half wavelength($\lambda/2$). Fig.s 1.a) and 1.b) show the output (y) of the array in presence of two desired signals and two jammers for 10 and 20 radiators, respectively. In our approach, the mean square error, given from noise and jammers, is reduced of 1.9dB (for 10 radiators) and 2.48dB (for 20 radiators) with respect to the algorithm in [11]. In Fig.s 2.a) 2.b), the noise and interfering signals are decreased of 0.92dB and 5.25dB, for 10 and 20 radiators, respectively.

CONCLUSIONS

In this communication, we have proposed a technique to obtain an optimal digital beamforming criterion, with constraints on the radiation pattern amplitude, in presence of multiple desired signals and jammers with Gaussian noise. The basic idea is to satisfy the constraints using only real unknowns. This gives a further degree of freedom to improve the performance in terms of the mean square error. Numerical simulations are provided to prove the better performances of the proposed algorithm (in terms of MSE) in comparison with other theoretical approaches available in literature.

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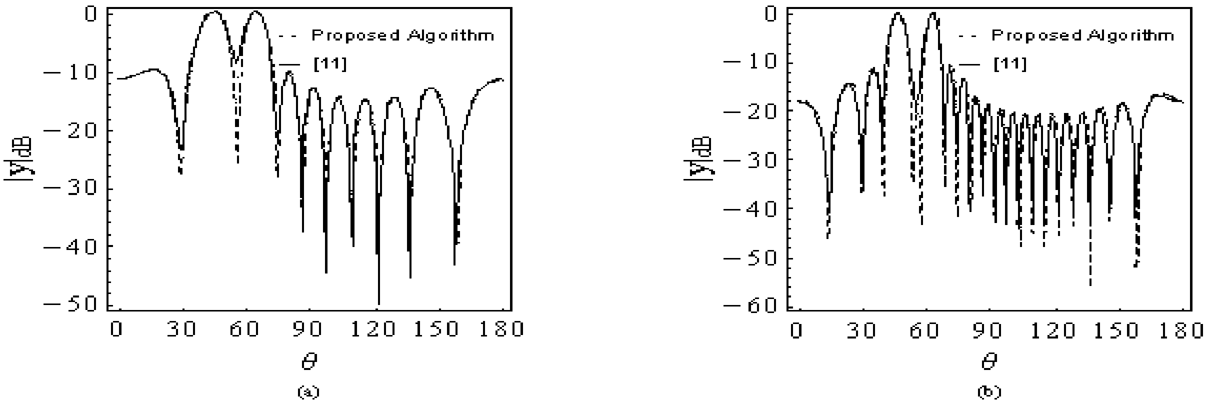

Figure 1: Uniform linear array of 10 (a) and (b) 20 isotropic radiators in presence of two desired signals $(47.5°, 62.5°)$ and two jammers $(32.5°, 77.5°)$.

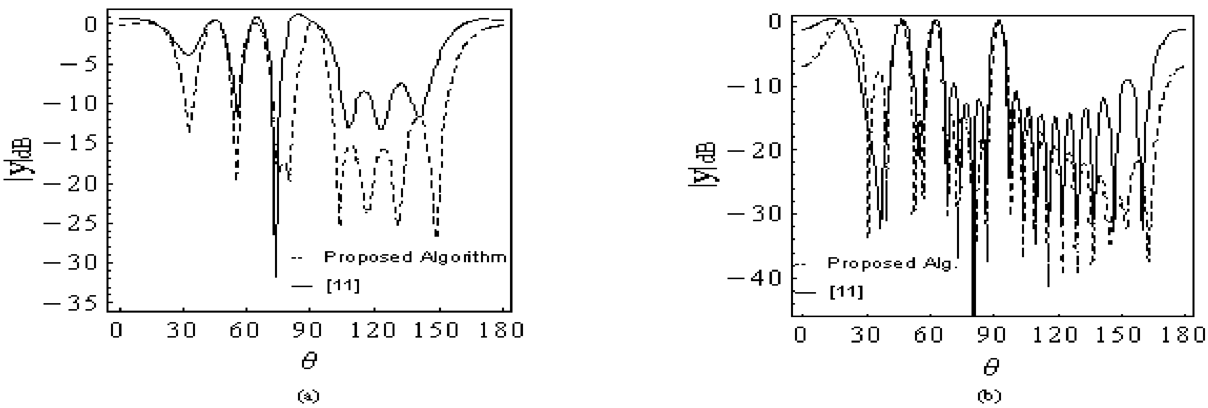

Figure 2: Uniform linear array of 10 (a) and (b) 20 isotropic radiators in presence of four desired signals $(17.5°, 47.5°, 62.5°, 92.5°)$ and two jammers $(32.5°, 77.5°)$.