# SOME FINE EFFECTS IN PROPAGATION PROBLEM FOR TRANSIONOSPHERIC SATELLITE RADIOLINKS

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**ABSTRACT**. A series of effects of the field phase and amplitude (log-amplitude) fluctuations on transionospheric GPS radio links are discussed, which are described by means of a recently extended propagation model previously developed by the authors. The following effects have been considered: the evaluation of the rate of change of phase due to the ionospheric electron density fluctuations; evaluation of the accuracy of precise phase measurements; generation of the random time sequences for the field amplitude (log-amplitude) and phase.

## INTRODUCTION

The paper presents a further extension of the propagation model for satellite transionospheric links developed in [1]. The extended model permits the calculation of some fine effects of propagation for transionospheric radio links, in particular, to assess the rate of change of phase when field scintillations occur, to estimate a series of higher order propagation effects, to generate random time sequences for the field amplitude (log-amplitude) and phase fluctuations. Necessary codes have been developed to enable calculation of the enumerated effects for realistic 3D models of the background ionosphere and anisotropic inverse power law spatial spectrum of fluctuations of the electron density of the ionosphere.

Numerical results related to the enumerated problems that will be presented below have been obtained for a profile of the background ionosphere obtained with the NeQuick model [2], corresponding to the low-latitude ionosphere with a total electron content of 69 TEC units. An inverse power law spatial spectrum of the ionospheric electron density fluctuations which has a spectral index $p = 3.7$ , a cross-field outer scale of $l_{tr} = 10\ km$ and aspect ratio $a = 5$ has been employed. Calculations were performed for the variance of the fractional electron density fluctuations $\sigma_N^2 = 0.01$ (10% fluctuations of the fractional electron density) and a carrier frequency of $1575\,MHz$ (L1-GPS fre-

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quency). Calculations have been performed for a path of propagation with an elevation angle of $45^0$, an azimuth of $45^0$ and a dip angle of the Earth's magnetic field of $45^0$. The effective velocity of the horizontal frozen drift of the random ionospheric inhomogeneities of 300 *m/s* perpendicular to the path of propagation was used.

**RATE OF PHASE CHANGES**

When evaluating the operation of a GPS link, an important quantity characterizing the received signal degradation is the rate of phase changes due to fluctuations of the ionospheric electron density. To estimate this phase change rate when scintillations occur, the root mean square (r.m.s.) of the time derivative of the random phase of a signal may be utilized as a measure, given, for instance, by the square root of the variance of the fluctuations of the phase time derivative. Variance of the time derivative of the phase fluctuations at a given point of observation is defined as the following average over the ensemble of realizations

$$\sigma^2(t) = \left\langle \frac{\partial\varphi(t_1)}{\partial t_1}\frac{\partial\varphi(t_2)}{\partial t_2} \right\rangle_{t=t_1=t_2} . \tag{1}$$

Here $\varphi(t)$ is a random phase. It can be shown that in the quasi-stationary approximation, the latter quantity is expressed through the second order time derivative of the time correlation function of the phase fluctuations $B_\varphi(\tau)$in difference time variable $\tau = t_1 - t_2$

$$\sigma^2 = -\frac{\partial^2}{\partial\tau^2} B_\varphi(\tau)_{\tau=0} . \tag{2}$$

When the second time derivative in (2) is calculated numerically in the scope of the approximation of finite differences, it is found that

$$\sigma^2 = \frac{2}{\tau^2}\left[B_\varphi(0) - B_\varphi(\tau)\right] = \frac{1}{\tau^2} D_\varphi(\tau) . \tag{3}$$

Finally, calculation of the appropriate variance according to (3) is simplified to the calculation of the structure function of phase fluctuations $D_\varphi(\tau)$, so that this function is taken at the moment of time $\tau$ corresponding to a given interval of evaluation of the rate of change of phase (the typical value for a standard receiver is $\tau = 20\,ms$ [3]) and properly normalized over this interval. The time structure function of phase fluctuations (or time correlation function of phase fluctuations) is calculated in the approximation of the complex phase method [1]. The appropriate extension of the propagation model [1] has been performed to enable the calculation of the variance (3), or its r.m.s. As a result, the rate of phase change can be calculated for any given 3D distribution of the background ionosphere electron density along the propagation path (straight, or curved) and for fluctuations of the ionospheric electron density of the turbulent type with an anisotropic inverse power law spatial spectrum. In particular, for the conditions of the background ionosphere, parameters of the anisotropic inverse power law spectrum fluctuations and geometry of propagation described in the Introduction the r.m.s. given by $\frac{1}{\tau}\sqrt{D_\varphi(\tau)}$, is $0.45\ s^{-1}$.

**ACCURACY OF PRECISE PHASE MEASUREMENTS**

It is known that measuring over longer time intervals generally increases the accuracy of range finding. To some extent, this procedure is equivalent to an averaging over the interval of ergodicity that, in turn, results in an elimination of the effects of phase fluctuations in the approximation of the first order term in the expansion of the phase fluctuations in powers of the electron density fluctuations. However, the phase fluctuations due to the electron density fluctuations contribute to range error for higher orders. It is of interest to estimate the higher order contributions, which determine the real limit of the accuracy and cannot be eliminated by averaging over long periods of time.

The full phase of a signal measured at a given point of observation can be represented in the following form

$$\varphi(t) = \varphi_0 + \varphi_1(t) + \varphi_2(t) + \ldots \quad (4)$$

Here the deterministic part of phase $\varphi_0$ is specified in the explicit form, whereas the stochastic contribution to the phase is given by its expansion into a perturbation series in the strength of fluctuations of the electron density of the ionosphere, so that $\varphi_1$ represents the linear term, $\varphi_2$ represents the second order contribution, etc. Obviously, averaging (4) over the ensemble of realizations (or averaging in time, if the ergodicity is assumed) results in the following relationship

$$\langle\varphi\rangle = \varphi_0 + \langle\varphi_2\rangle + \ldots \quad (5)$$

The latter equation (5) shows that the estimate of the accuracy of the phase measurements is defined by the mean value of the phase fluctuations of the second order in the magnitude of the electron density fluctuations of the ionosphere, whereas the mean value of the first order phase fluctuation (linear in the strength of fluctuations) is, obviously, equal to zero identically. To calculate the second order average correction to the phase, necessary codes have been developed allowing calculations of $\langle\varphi_2\rangle$ for any given 3D distribution of the electron density of the background ionosphere along the propagation path (straight, or curved) and for fluctuations of the ionospheric electron density of the turbulent type with an anisotropic inverse power law spatial spectrum. In particular, for the conditions of the background ionosphere, parameters of the anisotropic inverse power law spectrum fluctuations and geometry of propagation described in the Introduction the calculated value of $\langle\varphi_2\rangle$ is $0.025$ radians (or 0.76 mm for L1 frequency).

## GENERATING RANDOM TIME SEQUENCIES

The propagation model [1] has been further extended to also enable the calculation of the frequency spectra (power spectra) of the phase and level (log-amplitude) fluctuations in a transionospheric channel of propagation, which forms the basis for generating random time sequences for the field amplitude (log-amplitude) and phase for transionospheric paths of propagation [4]. Relevant codes have been created to first calculate the time correlation functions of the phase and log-amplitude fluctuations for real 3D models of the background ionosphere and the anisotropic inverse power law spatial spectrum of fluctuations of the electron density of the ionosphere. Calculated power spectra of these processes are then employed to produce random time sequences of the log-amplitude and phase of the field. The time series are generated under the assumption that both random processes have the random complex frequency Fourier spectra with the absolute values being the square roots of the appropriate calculated power spectra and the arguments uniformly distributed in the interval $0-2\pi$. When producing these spectra the same series of random values uniformly distributed in the ($0-2\pi$)-interval is employed to obtain both phase and level realisations. This is necessary to provide a correct cross-correlation function between the phase and level fluctuations.

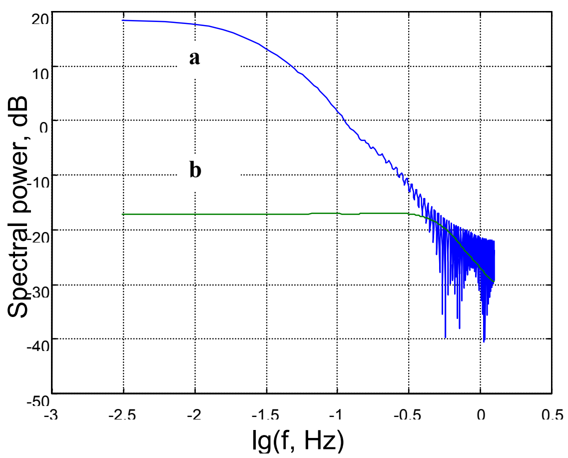


Fig. 1. Frequency spectra of phase (a) and level (b) fluctuations in double logarithmic scale.

Examples of calculations and results, obtained according to the described procedures, are given below in Figs 1, 2. Calculations have been performed for a model of the background ionosphere, parameters of the anisotropic inverse power law spectrum fluctuations and geometry of propagation described in the Introduction. In Fig. 1 the frequency spectra of the field phase and level (log-amplitude) fluctuations (power spectra of both the random processes) are given. The upper curve (a) represents the frequency spectrum of phase fluctuations and the lower one (b) is the spectrum of log-amplitude (level) fluctuations. Plots in Fig. 2 demonstrate the time series for phase (left) and level (right) fluctuations generated utilizing the power spectra in Fig. 1.

When a GPS receiver has a high-pass filter with a typical cut-off frequency of 0.1 *Hz* that is used to separate the slow quasi-regular trends from the scintillation effects, then to model numerically the procedure of measuring log-amplitude and phase scintillations by means of this type receiver the theoretically calculated power spectra of both processes should, first, be subject to the numerical process of high-pass filtering. If these processed power spectra are then employed to generate time series, the following random sequences are obtained, which are given in the plots in Fig. 3 (left – phase, right – log-amplitude). Comparing Fig. 2 and Fig. 3 clearly shows the effect that has the cut-offon the resultant phase and amplitude scintillation measurements as was suggested by Forte and Radicella [5]. In particular, when calculated for spectrally processed realisations from Fig. 3, the r.m.s. ($\sqrt{\sigma_S^2}$) of the phase fluctuations and the index $S_4$ are given by the quantities 0.25 and 0.185 respectively, whereas the same variances calculated for the "pure" series from Figure 2 are 0.5 and 0.195.

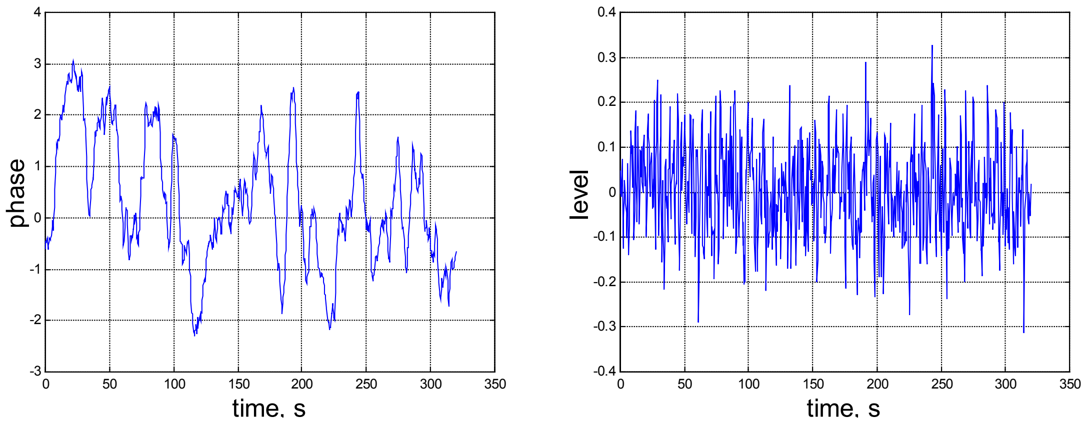


Fig. 2 The time series of the phase (left) and level (right) fluctuations generated utilizing the power spectra in Fig. 1.

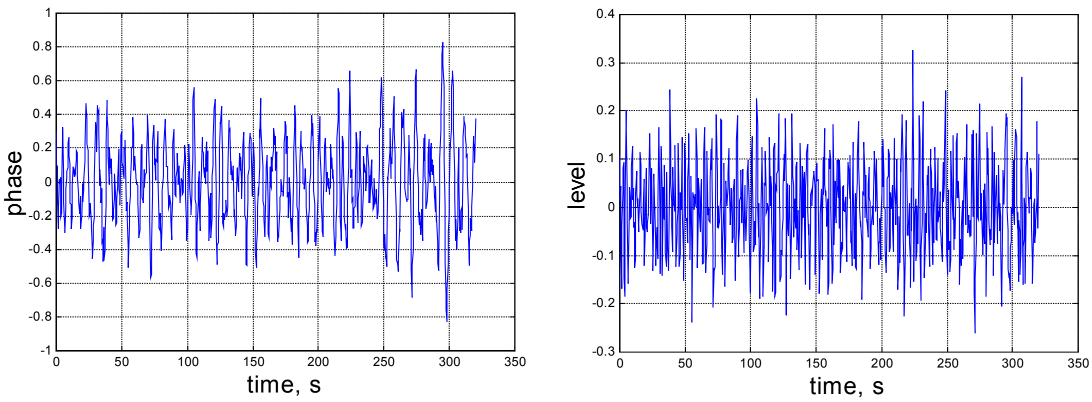


Fig. 3. The time series of the phase (left) and level (right) fluctuations generated utilizing the power spectra in Fig. 1 and cut-off frequency 0.1 *Hz*.

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