Field Radiated by a Gap Fed Infinite Slot Printed Between Two Homogeneous Dieletrics.

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Abstract
This contribution presents the dyadic Green's function of an electric dipole exciting a slot printed between two different homogeneous media and the relevant uniform asymptotic solution valid in any space points. Our formulation is based on the representation of a continuous spectrum of modes which has poles at the solution of the dispersion equation, i.e., at the wavenumbers of the leaky guided modes.

Introduction
In [1], the magnetic current excited by an electric dipole on a slot printed between two different homogeneous media is derived in analytical form under the not restrictive assumption of small slot width in terms of a wavelength. The final formulation in [1] agrees with the general conclusions discussed in [2] and based on the theory presented in a series of papers [3]-[5] which categorizes the asymptotic currents for a quite general class of guiding structures.
We note that an extensive treatment of the continuous spectrum of modes for open ended waveguides can be found in [6], which give emphasis on slot lines (see chapter 7 and 8, and reference therein). In particular, the problem of coupling between dipole sources placed in vicinity of a slot line etched on a rectangular waveguides has been treated in [7], and described in terms of asymptotic leaky-wave ray contributions, with similarity and agreement with the description we will provide here. Another problem which has similar physical contents is that of a semi-infinite transmission line fed by a leaky mode, which is treated in [8].

Formulation
The geometry is presented in Fig. 1 and consists of an infinite x-oriented slot which is printed on an infinite ground plane between two homogeneous dielectric half-spaces of permittivities ϵ_{r2} $(z>0)$, and ϵ_{r1} $(z<0)$, with $\epsilon_{r2}>\epsilon_{r1}$ assumed for convenience. The cross section w_s of the slot is uniform in x and small in terms of a wavelength. The structure is excited by a y-oriented electric dipole of the same length of the slot width w_s, placed at the origin of the reference system.
The closed form approximate expression for the slot magnetic currents $m(x,y)$ is derived in [1], via contour deformation of the relevant spectral integral, and is summarized herein after:

$$m(x,y) = -v(x)\frac{2}{w_s\pi}\left(1-\left(\frac{2y}{w_s}\right)^2\right)^{-1/2} \tag{1}$$

with

$$v(x') \approx \frac{(k_0\zeta_0\pi w_s)\left(k_1 \mathrm{H}_1^{(2)}(k_1|x|)-k_2\mathrm{H}_1^{(2)}(k_2|x|)\right)}{4(k_1^2-k_2^2)|x|} \approx \frac{-j\,k_0\zeta_0 w_s ln(x)}{2(k_1^2-k_2^2)} \qquad \text{for } x \text{ small} \tag{2}$$

$$v(x') \sim \frac{exp(-j\,k_x^{LW}|x'|)}{jD'(k_x^{LW})} + v_{10}\frac{e^{-jk_1|x'|}}{(k_1x)^2} + v_{20}\frac{e^{-jk_2|x'|}}{(k_2x)^2} \qquad \text{for } x \text{ large} \tag{3}$$

where $D'(k_x)$ is the derivative of $D(k_x)$ with $1/D(k_x)$ Fourier trasform of the magnetic current that was derived analytically in [1].
To obtain (3) the original integral had been expressed as the sum of three contributions: the residue of the leaky wave pole at k_x^{LW} plus the integration along the steepest descent paths (SDP's) associated with the two branch points at $k_x=k_1$and $k_x=k_2$. On the other hand, the asymptotic approximation of the spectrum $1/D(k_x)$ for high values of the spectral variable yields the values of the current for small value of x as proportional to $ln(x)$. The coefficients v_{10} and v_{20} are given in eqs. (29)(30) of

[1]. The first term in (3) is a leaky wave that propagates along x with phase constant β_{LW} =Re(k_x^{LW}) and attenuation constant δ_{LW} =*Im* (k_x^{LW}); a well tested approximation for actual slot widths was given in [1].

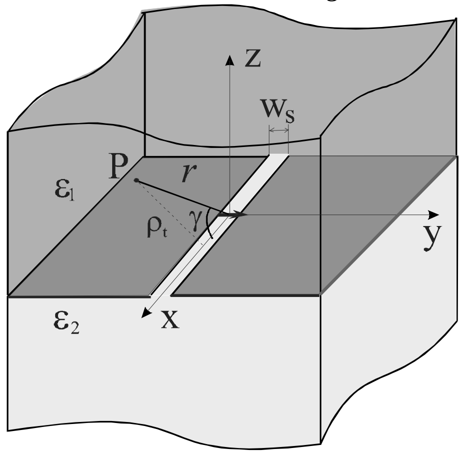

Fig. 1 *Geometry and reference system for an infinite slot excited by an electric dipole. The slot is etched between etched in an infinite ground plane between two different half spaces with dielectric constant ϵ_{r2} (z>0) and ϵ_{r1} (z<0); we assume $\epsilon_{r2}>\epsilon_{r1}$*

Vector potential and dyadic field

The x-oriented electric vector potential, associated to the equivalent magnetic currents defined in (1), in the spatial in medium n=1 *(z<0)* and n=2 *(z>0),* is

$$\mathrm{F}_n(x,y,z)= (-1)^n \int_{-\infty}^{\infty} \int_{-w_s/2}^{w_s/2} \frac{e^{-jk_n \mathrm{R}(x-x',\, y-y',z)}}{4\pi \mathrm{R}(x-x',\, y-y',z)} 2\, m(x,y)\; dx'dy' \tag{4}$$

where $\mathrm{R}(x, y, z)=\sqrt{x^2+y^2+z^2}$ and the factor 2 comes for the application of the image principle (note that for simplicity, we use here a definition of the potential without multiplication for the permittivity). From the potential, it is straightforward to obtain the electric $\mathbf{E}_n(x,y,z)$ and magnetic $\mathbf{H}_n(x,y,z)$ fields as

$$\mathbf{E}_n(x,y,z)= -\nabla \times \mathrm{F}_n \widehat{x} \tag{5a}$$

$$\mathbf{H}_n(x,y,z)= -j\omega\epsilon_n \nabla \times (\nabla \times \mathrm{F}_n \widehat{x})= -j\omega\epsilon_n \left(\mathrm{F}_n \widehat{x} + \frac{1}{k_n^2} \nabla \frac{\partial}{\partial x} \mathrm{F}_n\right) \tag{5b}$$

An asymptotic evaluation of the integral representing the potential provides the basic physical insight for the comprehension of the vector field mechanisms. When performed non uniformly each one of the contributions that arise has a clear interpretation in terms of space wave, leaky wave and lateral wave. However a more rigorous non uniform approximation is required to obtain accurate results at close distance from the slot and the excitinmg dipole.

Uniform asymptotic evaluation

By performing a straigth forward Fourier trasform of eq. (4) the 2-D Fourier representation of the potential and introducing the asymptotic approximation of the Hankel function for large argument, and then transforming the angular spectrum by the change of variable $k_x=k_2 cos\alpha$ with ,

$$\mathrm{F}_2 \sim \int_C e^{-jk_2 r\, cos(\alpha-\gamma)} f(\alpha)\; d\alpha \;\; ; \;\; f(\alpha)= \frac{-\sqrt{2j}\sqrt{k_2 sin\alpha}}{jD(k_2 cos\alpha)\sqrt{r\pi\; sin\gamma}\; 4\pi} \tag{6}$$

where γ is the angle that the observation vector forms with the positive x axis and C is the contour $(-j\infty, 0, \pi, \pi+j\infty)$, which maps into the angular spectral domain the real axis of the k_x plane. The phase of the integrand exhibits a saddle point at $\alpha=\gamma$, a branch point at $\gamma_b=cos^{-1}(k_1/k_2)$ and a pole at $\alpha_{LW}=cos^{-1}(k_x^{LW}/k_2)$. To perform the asymptotic evaluation of the integral on the original contour C, we deform this contour onto the steepest descent path (SDP) through the saddle point $\alpha=\gamma$. In this deformation either the leaky wave pole α_{LW} or the log-type branch cut at γ_b may be captured depending on the position of the observer. Three different situations may be defined, which are represented in Fig. 2a,2b,2c, respectively.

When $\gamma > \gamma_b$, (Fig 2a) neither α_{LW} nor γ_b are captured during the contour deformation; in this case the total field is reconstructed by the sole SDP integration. For $\gamma < \gamma_b$ (Fig 2b) the branch point γ_b is captured, thus requiring an additional integration around the branch-cut. For $\gamma < \gamma_{sb}$ (Fig 2c) the pole α_{LW} is captured in the contour deformation, thus leading in addition to the SDP and to the branch cut integrations, the pole residue contribution. The boundary between the situation of Figure 2b and 2c occurs when the SDP crosses the pole, that is when $\mathrm{Re}(\cos(\alpha_{LW} - \gamma_{sb})) = 1$ which, defining $\alpha_{\mathrm{LW}}^{re} = Re(\alpha_{LW})$ and $\alpha_{\mathrm{LW}}^{im} = Im(\alpha_{LW})$, implies $\gamma_{sb} = \alpha_{\mathrm{LW}}^{re} + cos^{-1}(1/cosh(\alpha_{\mathrm{LW}}^{im}))$. Eventually, the potential F_2 can be expressed as

$$F_2 = (I_{sdp} + I_{LW} U(\gamma_{sb} - \gamma) + I_{lat}\, U(\alpha_b - \gamma)) \qquad (8$$

where $U(x)$ is the unit step function ($U(x)$=1 for x>0 $U(x)$=0 for x<0) which accounts for the existence domain of the various contributions as described in Fig.s 2a,b,c. In accordance with the non-uniform ray description in [2], the mathematical definition and the corresponding physical meaning of the various contributions is

i) I_{sdp} = SDP integral $\rightarrow$ space wave contribution ($\rightarrow F^{\mathrm{space}}$)

ii) I_{lat} = integral around the log-branch cut at α_b $\rightarrow$ lateral wave contribution ($\rightarrow F^{\mathrm{lat}}$)

iii) I_{LW} = residue at α_{LW} $\rightarrow$ leaky wave contribution ($\rightarrow F^{\mathrm{LW}}$)

In (3), the unit step functions $U(\gamma_{sb} - \gamma)$ identifies the existence region of the leaky wave inside a shadow boundary cone $\gamma = \gamma_{sb}$. In the same way, the existence function $U(\gamma_b - \gamma)$ defines a shadow boundary cone (SBC) at $\gamma = \gamma_b$ which bounds the existence region of the lateral wave.

Actually, in all the practical configurations we have investigated, the lateral wave contribution has systematically been found negligible due to prevalence of the space-wave contribution.

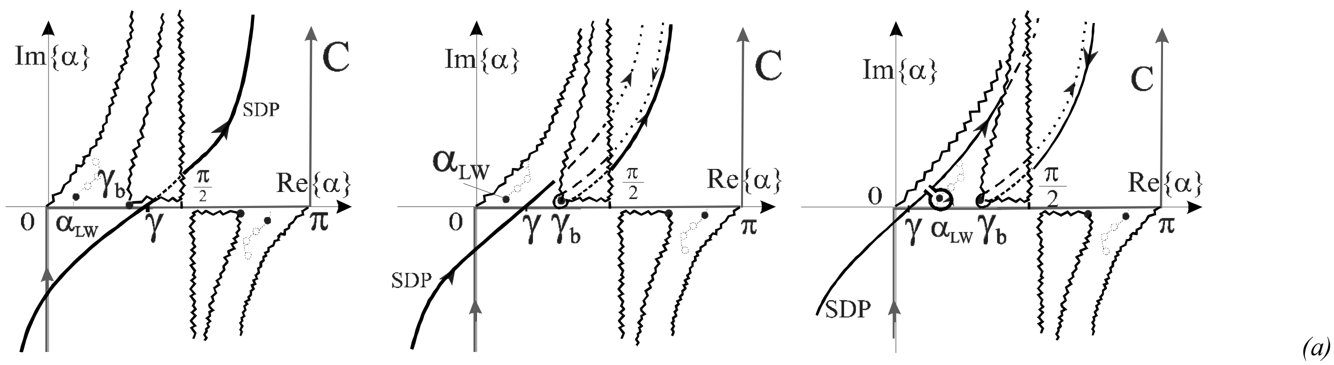

Fig. 2. *Angular spectral plane. The original contour C=($-j\infty$, 0, π, $\pi+j\infty$) is deformed into the SDP through the saddle point γ. Branch points (fig. b and c) and leaky wave pole (fig. c) are captured in the contour deformation. L-type branch point at γ_b maps the square-root branch cuts at k_1 in the k_x-plane. Other log-type branch cuts at γ_b and 0 occur, that correspond to the log-type branch cuts at k_1 and k_2 in the k_x-plane, respectively. The continuous line paths are on the top Rieman sheet (RS) associated with every branch cuts. The long dashed line paths are on top RS of the square-root type branch cut at γ_b and on bottom RS of the log-type branch cut at γ_b. Short dashed line paths are on bottom RS of the square-root type branch cut at γ_b and on top RS of the log-type branch cut at γ_b. Dotted line are on bottom RS of both type of branch cuts at γ_b.*

Numerical example and conclusion

The first set of curves in Fig. (3) check the accuracy of the asymptotic with respect to a reference solution obtained by numerical space domain integration of (6) in [2] (dots). In this figure, the normalized field potential is presented at $r=3\lambda_0$ versus the scan angle $\theta = 90^o - \gamma$, for slots width $w_s = 3\ 10^{-4}\lambda_0$ and dielectrics ϵ_{r1}=1 and ϵ_{r2}=11.7. The curve relevant to the leaky wave contribution alone is also presented (dash-dotted line), which is truncated at the shadow boundary. The continuity of the total potential across the shadow boundary is guaranteed by the space wave (short dashed line). The sum of the LW and the SW uniformly recovers the reference solution. In the zone where the leaky wave exists one can observe that the total field presents amplitude oscillations due to the interference between leaky and space wave. A maximum of the total potential occurs at the first in-phase summation of this two contributions beyond the shadow boundary.

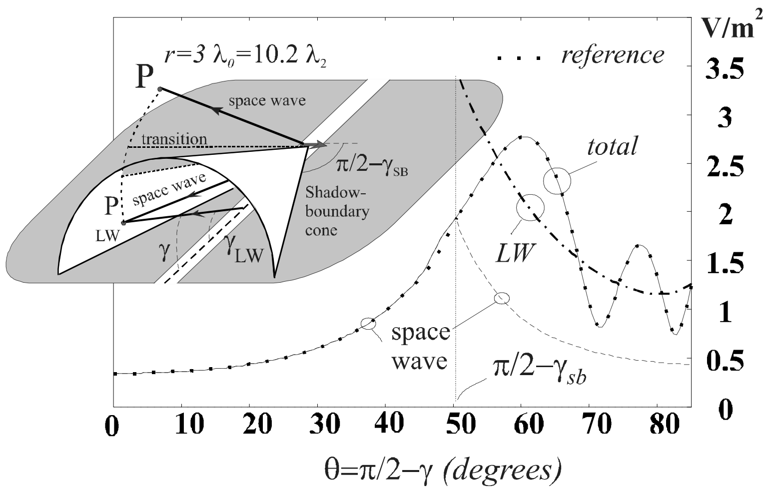

Fig. 3 *Electric potential F_2 in medium 2 at a distance $r = 3\lambda_0$ as a function of the scan angle for a slot with $w_s=3\ 10^{-4}\lambda_0$ and for permitivities $\epsilon_{r1}=1$ and $\epsilon_{r2}=11.7$. The reference curve (dots) is almost superimposed to the uniform asymptotics (solid line) obtained by implementation of (3). Leaky wave (LW, dash-dotted line) and space wave (dashed line) are also presented.*

To conclude a rigorous pole saddle point uniform asymptotics has been carried out starting from the continuous spectrum representation for the slot's Green's function solution. It is found that the conical leaky-wave is exponentially attenuated along radial scans in its existence region, and therefore disappears in the far zone. The directive pattern is then constructed by the sole space wave contribution, whose directive property are explained by the vicinity of the leaky wave pole to the visible portion of the space-wave spectrum. The validity and accuracy of the asymptotics has been verified by comparison with independent numerical integration, for observation points up to distance 0.5 λ_n, where λ_n (n=1,2) is the wavelength pertinent to the medium n=1,2.

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