Semi-empirical Model of the Polarimetric Radar Responses from Soil Surfaces

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ABSTRACT

A semi-empirical model of the ensemble-averaged differential Mueller matrix for microwave backscattering from bare soil surfaces is presented. Based on existing scattering models and data sets measured by polarimetric scatterometers and the JPL AirSAR, the parameters of the co-polarized phase-difference probability density function, namely the degree of correlation and the co-polarized-phase-difference, in addition to the backscattering coefficients for VV, HH, VH-polarizations, are modeled empirically in terms of the volumetric soil moisture content and the surface roughness parameters such as the rms height and the correlation length. Consequently, the ensemble-averaged differential Mueller matrix is specified completely by the backscattering coefficients, the degree of correlation and the co-polarized-phase-difference.

INTRODUCTION

There is a strong need for a good polarimetric scattering model for backscattering from bare soil surfaces for various reasons. One of the reasons is to use the phase-difference statistics, in addition to the backscattering coefficients, for retrieving the soil moisture and the surface roughness parameters from synthetic aperture radar (SAR) data. The other reason for the need to generate a good model for the ensemble-averaged differential Mueller matrix (or the Stokes scattering operator) for backscatter by bare soil surfaces is that it is a prerequisite for the development of a scattering model for vegetation-covered surfaces. The polarimetric scattering model can also be used to synthesize the polarization response for any possible combination of transmit and receive antenna polarizations.

Experimental data acquired by coherent polarimetric SAR systems and by polarimetric scatterometer systems have shown that the probability density function (PDF) of the co-polarized phase angle $\phi_c = \phi_{hh} - \phi_{vv}$, as well as the backscattering coefficients, are strongly dependent upon the incidence angle, the wavelength, the soil moisture content and surface roughness. In contrast, the cross-polarized phase angle $\phi_x = \phi_{vh} - \phi_{vv} = \phi_{hv} - \phi_{vv}$ is uniformly distributed over $[0, 2\pi]$, and therefore contains no target-specific information . The PDF of ϕ_c is characterized completely by two parameters, namely the degree of correlation α and the co-polarized phase-difference ζ [1].

Unlike the backscattering coefficients of bare soil surfaces, no theoretical models currently exist for the parameters α and ζ, even though many experimental observations have been reported. The goal of this study is to model the empirical models for the parameters α and ζ, as well as σ_{vv}^0, σ_{hh}^0 and σ_{vh}^0, based on an extensive database obtained by the JPL airborne SAR system and ground-based scatterometers, thereby providing a complete model for all of the ensemble-averaged differential Mueller matrix elements.

The PDF of the co-polarized phase angle $\phi_c = \phi_{hh} - \phi_{vv}$ can be specified by two parameters: the parameter α, referred to as the degree of correlation, which is a measure of the width of the PDF, and the parameter ζ, referred to as the mean value of the co-polarized phase-difference, which is the value of ϕ_c at which the PDF has a maximum.

The ensemble-averaged terms of the co-polarized scattering amplitudes in the differential Mueller matrix elements were derived in [2] using the parameters α and ζ as follows:

$$\left\langle \mathrm{Re}\,(S^0_{vv}S^{0*}_{hh})\right\rangle = \alpha \cos\zeta \sqrt{M^0_{11}M^0_{22}} \tag{1}$$

$$\left\langle \mathrm{Im}\,(S^0_{vv}S^{0*}_{hh})\right\rangle = \alpha \sin\zeta \sqrt{M^0_{11}M^0_{22}} \tag{2}$$

The ensemble-averaged differential Mueller matrix elements can be computed from the three backscattering coefficients σ^0_{vv}, σ^0_{hh}, σ^0_{vh} and the two phase-difference parameters α, ζ as follows:

$$M^0_{11} = \sigma^0_{vv}/4\pi\,,\ M^0_{22} = \sigma^0_{hh}/4\pi\,,\ M^0_{12} = M^0_{21} = \sigma^0_{vh}/4\pi\,,\ M^0_{33} = \left(\alpha\cos\zeta\sqrt{\sigma^0_{vv}\,\sigma^0_{hh}} + \sigma^0_{vh}\right)/4\pi\,,$$
$$M^0_{44} = \left(\alpha\cos\zeta\sqrt{\sigma^0_{vv}\,\sigma^0_{hh}} - \sigma^0_{vh}\right)/4\pi\,,\ M^0_{43} = -M^0_{34} = \alpha\sin\zeta\sqrt{\sigma^0_{vv}\,\sigma^0_{hh}}\,/4\pi\,,$$
$$M^0_{13} = M^0_{14} = M^0_{23} = M^0_{24} = M^0_{31} = M^0_{32} = M^0_{41} = M^0_{42} = 0 \tag{3}$$

SEMI-EMPIRICAL MODEL

In support of the model development, an extensive database was generated of the *vv*-, *hh*-, and *vh*-polarized backscattering coefficients, the degree of correlation, and the co-polarized phase-difference, obtained by a combination of ground-based scatterometers and the JPL airborne SAR system over a wide variety of bare soil surfaces. This database also includes precise ground truth data such as the surface roughness parameters and the volumetric soil moisture contents for all soil surfaces. The database includes seven polarimetric measurements. A summary of dynamic range for soil parameters m_v, *ks* and *kl* is given in Table 1.

Table 1. Dynamic range of soil parameters.

Range / Parameter	90% range		95% range	
	Min.	Max.	Min.	Max.
m_v (cm^3/cm^3)	0.043	0.283	0.040	0.291
ks (*s*: rms height)	0.14	6.00	0.13	6.98
kl (*l*: cor. length)	1.81	21.65	1.67	22.12
s/l	0.059	0.387	0.048	0.388

The input parameters for the intended polarimetric model include incidence angle θ, the volumetric soil moisture content m_v, and the roughness parameters *ks* and *kl*, where *s* is the rms height, *l* is the correlation length and *k* is the wavenumber. Because the backscatter is only weakly dependent on soil type, in comparison with its response to surface roughness and soil moisture, the soil type has been excluded in this model. The soil moisture content m_v is used in the model instead of the complex dielectric constant for simplicity. Moreover, the soil moisture content of the top 3-cm soil-surface layer is used at all frequencies because it was shown that the top 2~3 cm soil layer exhibits the greatest influence on the radar backscatter response even though the wave may penetrate deeper into the soil for a dry surface.

It was found that the cross-polarized backscattering coefficient of the semi-empirical model described in [3] agrees very well with the measurements, especially with regard to its dependence on θ. The model expresses σ^0_{vh} in terms of the Fresnel reflectivity (or indirectly through the complex dielectric constant), the incidence angle and the roughness parameter *ks*. For a typical agricultural soil, such as silt loam or sandy loam, the Fresnel reflectivity exhibits an approximately linear dependence on the volumetric soil moisture content. At nadir the relationship assumes the approximate form $\Gamma_0 = m_v^{0.7}$, and is valid over the soil moisture range of $0.03<m_v<0.35$.

After examining the angular patterns of the measured data, we selected $(\cos\theta)^c$ as a candidate function for characterizing the angular dependence, and $1-\exp[-a(ks)^b]$ function is used to account for the response to surface roughness. This roughness function satisfies the conditions that the cross-polarized backscattering coefficient approaches zero for a smooth (near flat) surface ($ks \to 0$) and that on the other extreme it becomes independent of *ks* for a very rough surface ($ks \rightarrow \infty$). Hence, an overall functional form for the cross-polarized backscattering coefficient is proposed as follows:

$$\sigma_{vh}^{0} = a\, m_v^{b}\, (\cos\theta)^{c}\, \left[1 - \exp\left(-d\,(ks)^{e}\right)\right] \tag{4}$$

The magnitudes of constants *a*, *b*, *c*, *d* and *e* were determined through data fitting, using the database, by applying the minimum mean square error (MMSE) technique. The process led to the following values: *a*=0.11, *b*=0.7, *c*=2.2, *d*=0.32 and *e*=1.8. Table 2 shows the root-mean-square errors obtained from the data-fit process.

Table 2. Root-mean-square errors for the data-fitting process.

	rms error	Data range (95%)		Number of points
		Min.	Max.	
σ_{vh}^{0} (dB)	2.35	-44.35	-14.94	651
q (dB)	2.06	-22.03	-8.49	651
p (dB)	0.82	-4.90	+0.93	651
α	0.116	0.445	1.010	637
ζ (deg.)	9.4	1.2	49.4	623

Similarly, the cross-polarized ratio q, the co-polarized ratio p, the degree of correlation α and the co-polarized phase-difference ζ are modeled empirically using the extensive database, as follows:

$$\sigma_{vh}^{0} = 0.11\, m_v^{0.7}\, (\cos\theta)^{2.2}\, \{1 - \exp[-0.32 (ks)^{1.8}]\}\,, \tag{5}$$

$$\sigma_{vv}^{0} = \sigma_{vh}^{0} / q, \qquad q = 0.10\, [ks/kl + \sin(1.3\theta)]^{1.2} \{1 - \exp[-0.9 (ks)^{0.8}]\}\,, \tag{6}$$

$$\sigma_{hh}^{0} = p\, \sigma_{vv}^{0}, \qquad p = 1 - \left(\theta / 90^{0}\right)^{0.35\, m_v^{-0.65}} \cdot e^{-0.4\,(ks)^{1.4}}\,, \tag{7}$$

$$\alpha = 1 - \left(0.17 + 0.01\, kl + 0.5\, m_v\right) \cdot \left(\sin\theta\right)^{1.1 (ks)^{-0.4}}\text{, and} \tag{8}$$

$$\zeta = \left(0.44 + 0.95\, m_v - ks/kl\right)\theta \tag{9}$$

Table 3 shows maximum sensitivities of the models for σ_{vv}^{0}, p, q, α and ζ over the 95% ranges of the soil parameters ks, m_v, kl, s/l (Table 1), as well as over the range of θ ($10^{0} \le \theta \le 70^{0}$).

We should note that the data measured by the AirSAR show lower values of α and ζ than those measured by the polarimetric scatterometers. The phase-difference parameters measured by the polarimetric scatterometers are accurate because the data were calibrated by the differential Mueller matrix technique using the polarimetric response of a calibration target over the entire mainlobe of the scatterometer. When a traditional calibration technique for a distributed target is used, *i.e.* the differential Mueller matrix is approximated by the Mueller matrix divided simply by an illuminated area, the α and the ζ are inaccurate. For the scatterometer data, the α values obtained by the old illumination-integral calibration technique were about 0.8 times the α values determined by the new accurate differential Muller matrix calibration technique.

CONCLUDING REMARKS

A semi-empirical polarimetric backscattering model was developed for random bare soil surfaces using a combination of truck-mounted scatterometer measurements and airborne SAR observations, both supported by extensive ground observation of the soil surface statistics and moisture content. The functional form of the model was constrained to insure that its predictions are consistent with known theoretical values, such as $\sigma_{vv}^{o} = \sigma_{hh}^{o}$ at normal incidence, $\sigma_{vv}^o = \sigma_{hh}^o$ for an electromagnetically very rough surface, and $\sigma_{vh}^o / \sigma_{vv}^o$ approaches a constant as surface roughness exceeds *ks*=3. The two distinguishing features of the model is that it not only agrees with experimental observations over a wide range of soil surface conditions, but it also agrees with the IEM and geometrical optics model over their individual regions of validity, thereby encompassing the full range of surface roughness encountered under natural conditions.

REFERENCES

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Table 3. Maximum sensitivity of the model on each parameter in its 95% range.

Model	Para-meter	Maximum Sensitivity
σ_{vh}^0	ks	20.9 dB
	θ	10.3 dB
	m_v	6.0 dB
p	ks	7.1 dB (θ =70^0, m_v =0.29) 0 dB (θ =10^0, m_v =0.04)
	θ	6.2 dB (ks =0.13, m_v =0.29) 0 dB (ks =6.98, m_v =0.04)
	m_v	4.1 dB (ks =0.13, θ =70^0) 0 dB (ks =6.98, θ =10^0)
q	ks	7.9 dB
	θ	7.0 dB (s/l =0.048) 4.3 dB (s/l =0.388)
	s/l	4.2 dB (θ =70^0) 1.5 dB (θ =10^0)
α	θ	0.453 (ks =0.13, kl =22.1, m_v =0.29) 0.115 (ks =6.98, kl =1.67, m_v =0.04)
	ks	0.285 (θ =27^0, kl =22.1, m_v =0.29) 0.023 (θ =70^0, kl =1.67, m_v =0.04)
	kl	0.198 (θ =70, ks =6.98) 0.003 (θ =10, ks =0.13)
	m_v	0.122 (θ =70^0, ks =6.98) 0.020 (θ =10^0, ks =0.13)
ζ	θ	40.1^0 (m_v =0.29, s/l =0.048) 5.4^0 (m_v =0.04, s/l =0.388)
	s/l	23.8^0 (θ =70^0), 3.4^0 (θ =10^0),
	m_v	16.7^0 (θ =70^0), 2.4^0 (θ =10^0)