ANALYSIS OF PLASMA WAVE SCATTERING BY USING TECHNIQUE OF WAVE DIGITAL FILTERS

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ABSTRACT

There is a fluid theory describing the plasma state as one of the systems of plasma equations, which consists of a system of Maxwell's equations and that of Euler's equations. A system of partial differential equations is transformed into an equivalent Kirchhoff circuit by considering all physical quantities as currents. Using technique of wave digital filters (WDF), Kirchhoff circuits are transformed into WDF algorithm, leading to a numerical one. This numerical method is applied to an analysis of plasma wave scattering in a magnetized plasma.

INTRODUCTION

There is a fluid theory describing the plasma state as one of the systems of plasma equations, which consists of a system of Maxwell's equations and that of Euler's equations. Euler's equation system is nonlinear, while Maxwell's one is linear. Therefore, it is difficult for both equation systems to be treated unitedly in general, when we get a numerical solution of the system of plasma equations. Usually, Euler's equation has been linearized and solved on the assumption that disturbance in plasma is very small. Therefore, this equation system cannot apply to the large amplitude phenomena in numerical calculation. However, a new method for numerically solving partial differential equations has been proposed, which is based on the technique of wave digital filters (WDF). It has been applied to linear partial differential equations such as a system of Maxwell's equations [1,2]. However, introducing the power waves, it is applicable to the nonlinear equations such as a system of Euler's equations [3].
At first, a system of partial differential equations is transformed into an equivalent Kirchhoff circuit by considering all physical quantities as currents. Then, both Maxwell's and Euler's equation systems are transformed into equivalent Kirchhoff circuits, which are named by considering that each equation represents the Kirchhoff's second law [1-3].
In plasmas, the electric field, which is caused by the moving charged particles, acts on the moving particles again. The Lorentz force due to the magnetic field brings more complicated interaction than the electric force. Therefore, the transformed Kirchhoff circuits for both equation systems are no longer independent of each other. Thus, the Kirchhoff circuits corresponding to both equation systems have to be connected each other [4]. Connected Kirchhoff circuits are transformed into the well-known WDF algorithm, leading to a numerical solution of the plasma equations.
As an example of numerical calculations, plasma wave scattering phenomena from a cylindrical conductor immersed in a magnetized plasma are analyzed.

BASIC PLASMA EQUATIONS

First, Maxwell's equations are shown as

$$\nabla \times H = j + \frac{\partial (\varepsilon_0 E)}{\partial t}, \tag{1}$$

$$\nabla \times E = -\frac{\partial (\mu_0 H)}{\partial t}, \tag{2}$$

where j is given by

$$j = -n_e q_e v_e, \tag{3}$$

and n_e, v_e and $-q_e$ stand for density, velocity and charge of electron, respectively.
Second, as for a system of Euler's equations, ion is considered to be static as background for simplicity. From this, we have only to consider the electron fluid alone for Euler's equation, continuity equation and equation of state, which are shown as

$$m_e n_e\left[\frac{\partial v_e}{\partial t}+(v_e\cdot\nabla)v_e\right]+\nabla p_e = f, \tag{4}$$

$$\frac{\partial n_e}{\partial t}+\nabla\cdot(n_e v_e) = 0, \tag{5}$$

$$p_e = C(m_e n_e)^{\kappa_e}, \tag{6}$$

where m_e, p_e and κ_e stand for mass, pressure and the ratio of specific heats of electron fluid, respectively. f is an external force against electron particle, which expresses the electric force and the Lorentz force shown as,

$$f = -n_e q_e\left[E+v_e\times\mu_0(H_0+H)\right], \tag{7}$$

where H_0 denotes a static magnetic field.

EQUIVALENT KIRCHHOFF CIRCUITS

Problems are analyzed from a viewpoint of multidimensional signal processing. First, we introduce usually four-dimensional manifold by adding time axis to three dimensional space. Then, introducing a coordinate transformation and multidimensional operators, we have transformed a system of Maxwell's equations as follows:

$$D''_7(\varepsilon''\hat{E}_1)+(D''_3-D''_6)\hat{H}_2+(D''_5-D''_2)\hat{H}_3+\hat{j}_1=0, \tag{8}$$

$$D''_7(\varepsilon''\hat{E}_2)+(D''_1-D''_4)\hat{H}_3+(D''_6-D''_3)\hat{H}_1+\hat{j}_2=0, \tag{9}$$

$$D''_7(\varepsilon''\hat{E}_3)+(D''_2-D''_5)\hat{H}_1+(D''_4-D''_1)\hat{H}_2+\hat{j}_3=0, \tag{10}$$

$$D''_7(\mu''\hat{H}_1)+(D''_6-D''_3)\hat{E}_2+(D''_2-D''_5)\hat{E}_3=0, \tag{11}$$

$$D''_7(\mu''\hat{H}_2)+(D''_4-D''_1)\hat{E}_3+(D''_3-D''_6)\hat{E}_1=0, \tag{12}$$

$$D''_7(\mu''\hat{H}_3)+(D''_5-D''_2)\hat{E}_1+(D''_1-D''_4)\hat{E}_2=0, \tag{13}$$

where hatted physical quantities stand for normalized ones.
On the other hand, a system of Euler's equations are also transformed as follows:

$$\sum_{k=1}^{6}\eta_{0k}D''_k(\eta_{0k}\hat{p}_e)+\sum_{k=1}^{3}[D''_k(\hat{p}_e+\hat{v}_{ek})+D''_{k+3}(\hat{p}_e-\hat{v}_{ek})]=0, \tag{14}$$

$$\sum_{k=1}^{6}\eta_k D''_k(\eta_k\hat{v}_{el})+D''_l(\hat{v}_{el}+\hat{p}_e)+D''_{l+3}(\hat{v}_{el}-\hat{p}_e)=\hat{f}_{el} \qquad (l=1,2,3), \tag{15}$$

Considering all physical quantities as currents and also replacing the differential operators with inductances, a system of partial differential equations is transformed into what is called an equivalent Kirchhoff circuit, which is named by considering that each equation of the system mentioned above represents the Kirchhoff's second law.
Kirchhoff circuit of Maxwell's equations is shown in Fig. 1. On the other hand, Kirchhoff circuit of Euler's equations is shown in Fig. 2. Fig. 3 shows a connecting network consisting of ideal gyrators and ideal transformers between Kirchhoff circuit of Maxwell's equations and that of Euler's equations [4, 5].

DERIVATION OF WDF ALGORITHM

The desired WDF algorithm is obtained by applying the theory of WDF [2, 3]. Obtained WDF signal-flow diagrams corresponding to both Kirchhoff circuits and the connecting network are shown in Figs. 4 ~ 6, respectively. The algorithm is easily derived from these diagrams.

NUMERICAL CALCULATIONS AND DISCUSSION

As an example, analyzed are plasma wave scattering phenomena from a cylindrical conductor immersed in a plasma which is magnetized perpendicularly to the paper plane in Fig. 7. When a plane electromagnetic wave with stimulated longitudinal waves is incident from lefthand side on a cylindrical conductor whose radius a is given by $ka = 1$, calculations are done in the area $3\lambda \times 3\lambda$, where λ is wavelength of the incident electromagnetic wave at initial time. In Fig. 8, the change of total intensity of electric field is shown by line elements, according as the scattering direction changes. The situation is drawn visually with time, because sampling is done along time axis and spatial and physical variables are calculated every sampling time. Therefore, it is found that this method is suitable for the time-dependent phenomena. However, even in steady state, this method is applicable if the time is sufficiently elapsed. The scattering pattern is also actually obtained by this manner.

CONCLUSION

Introducing power waves and the technique of wave digital filters, nonlinear equation systems in physics are treated in a same manner as linear ones. Applying this method to the plasma equation system, plasma wave scattering phenomena are analyzed here. Some interesting results are easily obtained.
And since numerical calculations are done after constructing a digital filter, stability and accuracy are guaranteed. Furthermore, since sampling is done along time axis and spatial and physical variables are calculated every sampling time, it is needless to calculate huge matrices and required time in calculation is short. Therefore, it is found that this method is suitable for computer experiments such as nonlinear interaction among various plasma waves.

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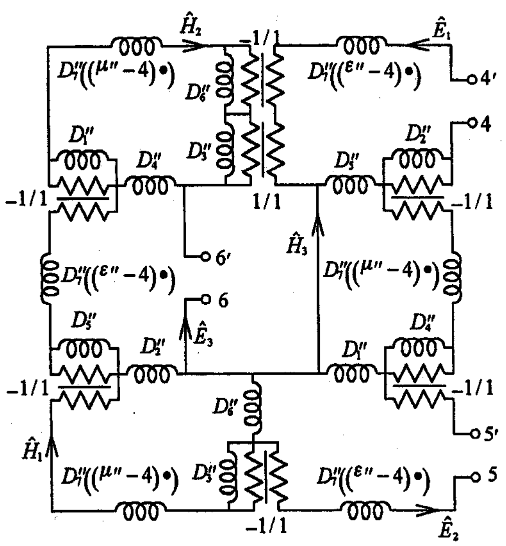

Fig. 1. Kirchhoff circuit for a system of Maxwell's equations.

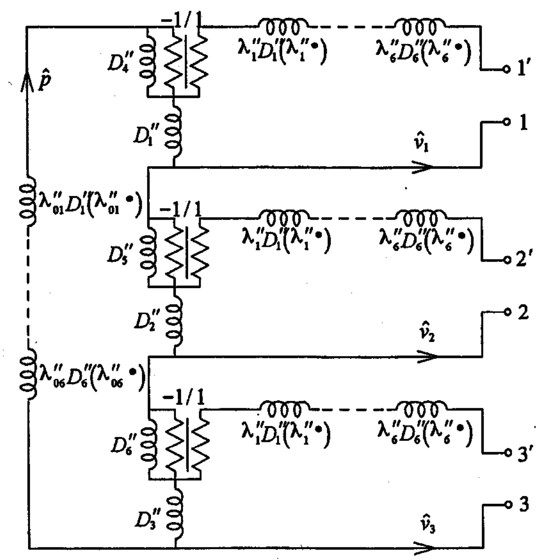

Fig. 2. Kirchhoff circuit for a system of Euler's equations

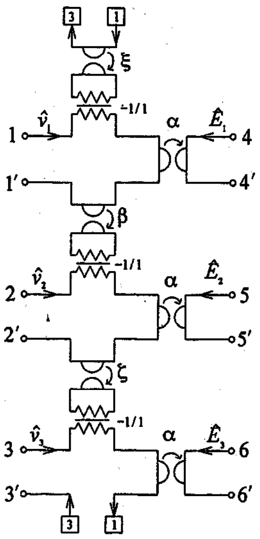

Fig. 3 Connecting network.

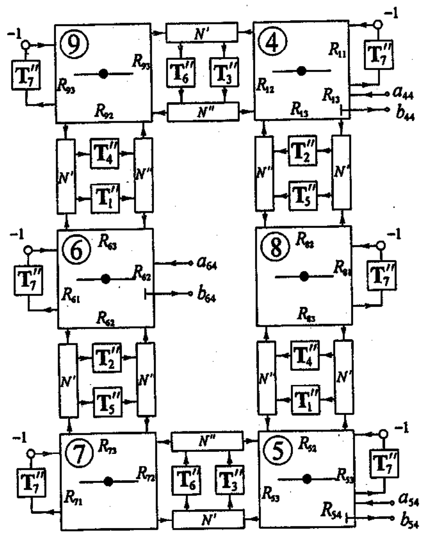

Fig. 4 Signal-flow diagram for Maxwell's equations.

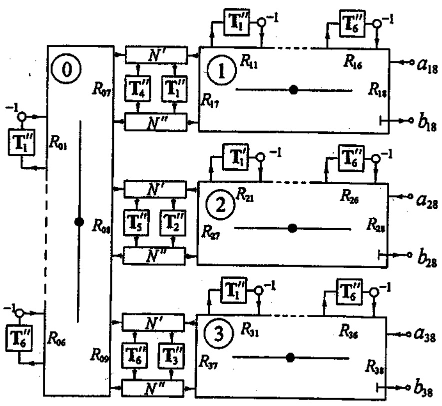

Fig. 5 Signal-flow diagram for Euler's equations.

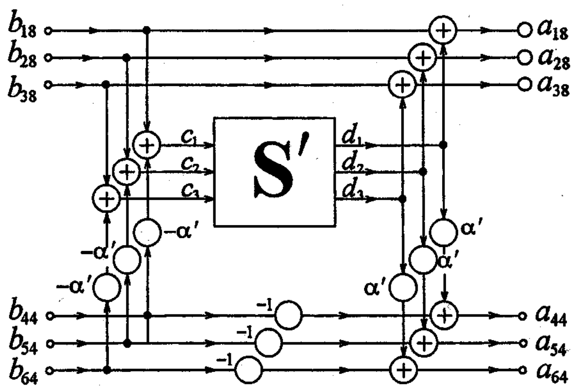

Fig. 6 Signal-flow diagram for connecting network.

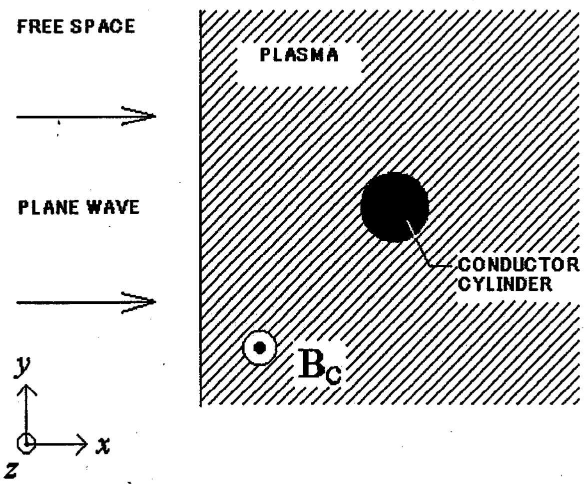

Fig. 7 Setup for the numerical calculations.

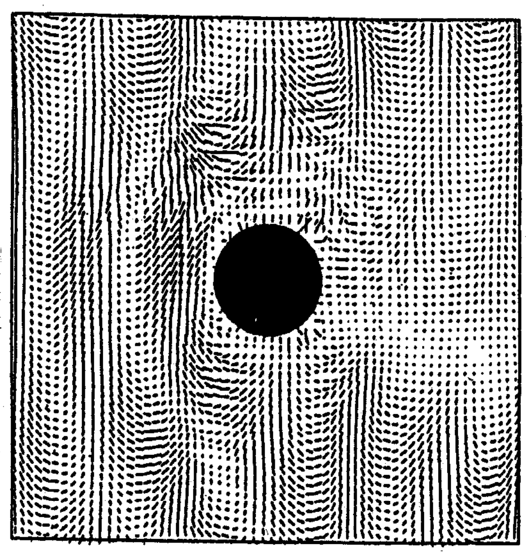

Fig. 8 Plasma wave scattering against cylindrical conductor.