# MICROCELL COVERAGE AND DELAY SPREAD PREDICTION USING WAVEGUIDE THEORY

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## ABSTRACT

Waveguide analysis is used to predict power levels received in a microcellular system in an urban environment. An Urban street with tall buildings on both sides is modeled by a plate parallel waveguide made of rough plates. The distribution of power among the waveguide modes tends to concentrate in the low order modes, at large distances from the source antenna. The waveguide–based model predicts a significant decrease of power levels when turning from a street containing a transmitter into another street. Typical delay profiles are calculated for the steady state distribution of power among the modes.

## 1 INTRODUCTION

Waveguide theory is suggested as means of analysing propagation in microcellular environments. This approach is an alternative to ray tracing theory. It offers a prediction of average power levels and delay proriles, which are not readily available in ray theory. The modal theory provides results in an intuitive manner, based on the average properties of the propagation environment.

The perturbations of building walls from perfect smoothness induce coupling among the propagating modes in a street. This can be analysed in an average way, based on the statistics of the wall perturbations. Power levels at different locations along a street are predicted using the coupled modes theory. The initial conditions at the transmitter are determined by the distribution of power among the propagating modes and the overall transmitter power. An intersection of a main street containing a transmitter with a side street induces initial conditions at the side street, that are used to predict the power levels along it.

The delay profile is analyzed by means of the waveguide dispersion, where the steady state distribution of power among the modes induces a typical delay spread.

## 2 THE MODEL

### 2.1 A Smooth Multi-Moded Waveguide

The simple model we present here consists of a slab waveguide or width $2a$, which represents a street with very tall buildings on both sides. The walls of the waveguide are made of a lossy dielectric material.

We are interested in multi–moded waveguides because the normal width of streets is many times the wavelength in the UHF band. We assume a two dimensional model, with propagation along the $z$ direction, and no variation in the $y$ direction. The field expressions for the TE modes inside the waveguide $|x| <= a$ are:

$$\mathbf{E} = E_y \hat{y} = j\frac{k}{k_x} Z_0 \mathcal{H} \left\{ \begin{array}{c} \cos(k_x x) \\ \sin(k_x x) \end{array} \right\} e^{j\omega t - j\beta z} \hat{y} \tag{1}$$

with a similar expression for the magnetic field of the TM modes. The upper function applies to the symmetric modes and the lower to the antisymmetric modes, where the symmetry/antisymmetry characterizes the field component in the $y$ direction. $\omega$ is the angular frequency, $k = \frac{2\pi}{\lambda}$ is the free space wave number, where $\lambda$ is the free space wavelength. $\beta$ and $k_x = u/a$ represent the $z$ and $x$ components of the k vector for propagation inside the waveguide, where $u$ is the normalized k vector in the $x$ direction. $Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$ is the vacuum impedance, and $\mathcal{H}$ is an arbitrary amplitudes.

Using the boundary conditions, the characteristic equation can be formulated in terms of $u$ and the properties of the waveguide, and solved for the wave vector characteristic of each propagating mode.

### 2.2 A Rough Waveguide

In this section we consider slab waveguides made of uniform material, but the geometry of the walls is no longer smooth. Instead of presenting the coupling theory in full detail, we state the assumptions taken and proceed to the final result, namely the coupled power equations. The analysis followed the approach taken by [3] to the mode coupling caused by the roughness of the waveguide walls. The analysis relies on the assumption of small perturbations of the wall geometry.

The two dimensional model is maintained, where there is no variation in the $y$ direction. We characterize the wall perturbations statistically, where we assume that the perturbations on both walls are independent of each other and wide

sense stationary, i.e., the statistical properties do not change along the street. $s$ is the rms deviation of the wall from perfect straightness and $D$ is the correlation length.
The coupling of power among the propagating modes is described by a set of linear equations [3]:

$$\frac{dP_m}{dz} = -\alpha_m P_m + \sum_{n=1}^{N} \Gamma_{m,n} \left(P_n - P_m\right) \tag{2}$$

where $P_m$ is the power carried by the $\mathrm{m}^{\mathrm{th}}$ mode, $\Gamma_{m,n}$ is the coupling factor from mode $n$ to mode $m$ computed in a similar manner to [3], and $N$ is the number of propagating modes.
The coupling between TE and TM modes is not predicted by the model, bacause of the two dimensional assumption. We include cross polarization coupling in order to compensate for this over–simplification of the model.
$\alpha_m$ are the modal loss factors, which equal the imaginary part of the $z$ component of the wave vector of each mode.
The coupled power equations (2) can be expressed as a simple matrix equation, where the unknown $\mathbf{P}$ is a vector containing the power level of each mode:

$$\frac{\partial \mathbf{P}}{\partial z} = \Gamma \mathbf{P} \tag{3}$$

$\Gamma$ is an $N \times N$ matrix which holds all the power coupling coefficients.
The coupled power equation (3) is easily solved in terms of the eigenvalues and eigenvectors of the coupling matrix $\Gamma$. at large distances from a source we obtain the steady state solution $\mathbf{P} = \mathbf{B}_1 e^{-\lambda_1 z}$ where $\lambda_1$ is the eigenvalue of $\Gamma$ with the smallest real part and $\mathbf{B}_1$ is the corresponding eigenvector. The steady state solution varies with distance along the street, but only via the exponential decay. The distribution of power among the modes does not change along the street. The steady state solution normally tends to concentrate most of the power in the low order modes that are not as lossy as the high order ones. When we used realistic street parameters in simulations, all the power tended to concentrate in steady state in the low order TE modes.
In addition to our interest in the steady state solution, we also looked at the dynamic behavior of the power measured at small distances from a source. We model the source as a distribution of power among the waveguide modes, and then solve (3) numerically. The results we present below are the total power along the waveguide calculated using this method.

### 2.3 Ground Reflection

The ground plane affects each of the propagating modes differently, because the wave vectors of the modes point at different directions.
The coupling between the modes reduces the coherency between the direct propagating wave (of a certain modes) and its reflections off the ground. We consider the effect of incoherent addition of the direct and reflected waves for each mode, so we sum the power of these waves. The effect on the total power is an increase of up to 3 dB (achieved at large distances from the source).
When looking at street corners, we consider the junction as an initialization reference for propagation down the side street. Therefore, we calculate the reflection effects from the junction and not from the location of the actual transmitter antenna.

### 2.4 A Street Corner Model

This section describes the model of street corners, where power flows along one street (the ‘main’ street) into another (‘side’) street. We are interested in the behavior of power levels along the side street. We present here an intuitive explanation of the mode coupling mechanism. For a more thorough analysis see [1].
Each mode can be decomposed into a pair of plane waves propagating at equally oblique angles with the $z$ direction. The lower order modes are decomposed into plane waves that propagate in an almost parallel direction to the $z$ axis. High order modes travel in directions increasingly oblique to the $z$ axis. When considering a perpendicular street corner, the low order modes in the main street couple into high order modes in the side street and vice versa.
We assume steady state distribution of power of the modes in the main street, where most of the power is contained in the low order modes. As a consequence, the power coupled into the side street is mostly contained in the high order modes. The power leaking into the side street is re-distributed among the modes as it propagates along the street.
The expected effect in the side street is a significant decrease of power level as the receiver moves away from the junction. At a certain distance, where the modal distribution of power reaches its steady state, the rate of decrease of power loss along the street resumes its steady state rate.

## 3 COMPARISON TO MEASUREMENTS

We compare our theoretical predictions of power levels in an urban grid with measurements obtained by J. H. Whitteker in Ottawa, Ontario, Canada [5]. Fig. 1 shows a map of the measurement area, where the height of buildings may be judged roughly from the area that the occupy. Buildings that fill a large fraction of the space between streets are many stories

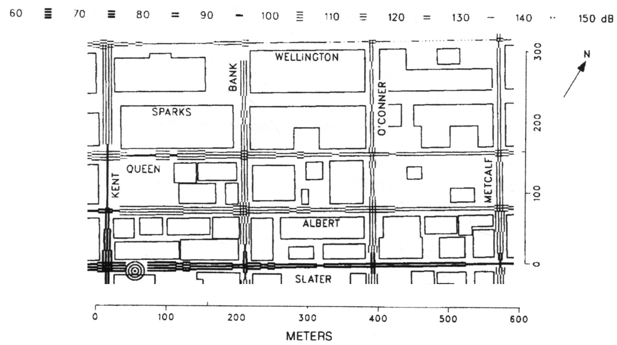


Figure 1: Map of Ottawa, Canada, with a transmitter at 300 Slater St. and path loss levels indicated by bars. Reproduction of fig. 1 from J. H. Whitteker, 'Measurement of Path Loss at 910 MHz for Proposed Microcell Urban Mobile Systems', IEEE Transactions on Vehicular Technology, August 1988, Vol. 37, No. 3, ©1988 IEEE.

high; those that occupy only a small area are typically three stories. All the buildings were higher than the antennas, that were placed 8.5 m above ground (for transmitting) and 3.7 m above ground (for receiving). The location of the transmitter is indicated in fig. 1, as well as the power level measured along the streets.

Fig. 2(a) presents the measured power and the simulation results the east–west streets, and Fig. 2(b) shows the results for the north–south streets. The simulation program calculated the power levels in all the paths that contained the transmitter and up to two turns in intersections.

Significant drops of power level are apparent in the measurements and simulation results as the receiver moves across a junction and looses line of sight with the transmitter (as in fig. 2(b)), or across a junction with a street with high power levels. The latter is apparent in fig. 2(a), for Wellington street. Note the variation in received power level along the side street, and the relaxation of the loss rate to the steady state rate. The agreement between measurement and theory is good is most streets, in particular those where the geometrical assumptions of the model hold. The prediction fails in areas with large openings between buildings, such as parts of Queen Street, O'Conner Street and Metcalf Street.

We used the approximate widths of the streets in the calculations, but other parameters were adjusted to give the best match between measurement and theory. The electrical properties of the walls were set at values reasonable for building materials with the relative dielectric constant at 2 or 3 and the conductivity at 0.04 S/m. The geometric perturbation standard deviation was set between 40 cm and 80 cm and the geometric correlation length was 30 m, which corresponds to the dimensions of the buildings. The power at each street intersection and its distribution among the modes at the junction are used as initial conditions for the simulation of each street. The power was assumed equal to the level calculated for the cross street, and its distribution at each junction was uniform. The power distribution at the source simulated a small antennas located at the middle of the street.

## 4 THE DELAY PROFILE

Modal dispersion in the waveguide causes delay spread, a very important parameter in wireless communication systems. Mode coupling decreases the dispersion of waveguides, because it forces some power to switch between fast modes and slow modes [4]. The delay spread calculated for a waveguide without mode coupling is an upper bound on the delay spread in similar waveguides with mode coupling.

The delay profile is calculated by taking the steady state power distribution and considering the variation of group velocity among the modes, where the low order modes are faster than the high order modes. The steady state distribution of power used was calculated for a 10 m wide street, with the relative dielectric constant of the walls set at $\epsilon_r$ =9, the conductivity set at $\sigma$ =0.4 S/m, the variance of the wall perturbation 0.04 $m^2$ and the correlation length of the wall perturbation set at 5 m. The carrier frequency is 2.6 GHz.

The delay profile is very asymmtric because the difference between the group velocities of the low order modes is smaller than the difference between the group velocities of the high order modes. Another reason for the importance of the low delays is the distribution of power among the propagating modes. The steady state distribution of power among the modes contains almost all the power in the low order modes, which propagate with lower delays. This concentration of power, together with the clustering at the low delays, make very asymmetrical delay profile with most of the power in the low delays.

We compare our calculated delay profiles to the empirical model suggested by *Ichitsubo et al.* [2] for line of sight (LOS) in streets at 2.6 GHz. We consider the LOS case the most appropriate for comparison, because our model does not include

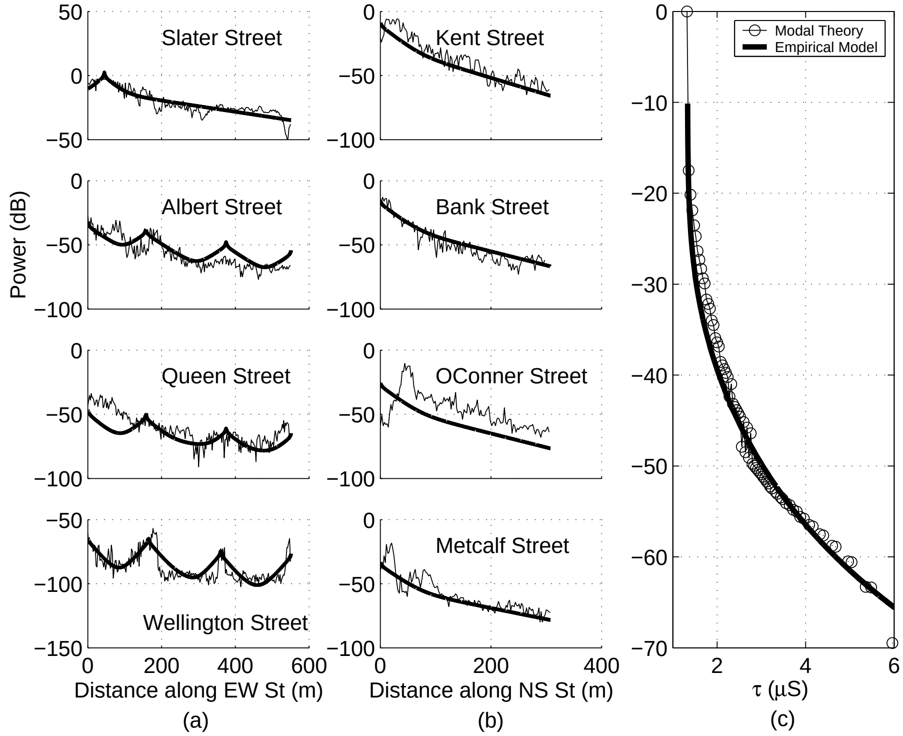


Figure 2: (a) Measured and simulated power levels for east–west streets. (b) Measured and simulated power levels for north–south streets. (c) Delay profile of a multi-moded waveguide with no mode coupling. The graph indicates the (normalized) predicted power at a receiver in the waveguide, 400 m from the transmitter.

propagation paths over the roofs of buildings, and each street intersection where power couples from one street to another effectively resets the shape of the delay profile. Good agreement is seem in fig. 2(c) between the two graphs.

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