MODELING ELECTRIC STRUCTURE OF A MESOSCALE CONVECTIVE SYSTEM

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INTRODUCTION

Thunderstorms are widely considered as generators of electric currents which drive the global electric current system. A number of analytical and numerical models has been suggested for description of the interaction of a thunderstorm with its global electrical environment (see, for example, [1] and [2] for review of earlier work). These models were different in terms of the conductivity profile, magnetic field and source function specification, but all of them used rather simple dipole configuration of the current source with horizontal scale being small as compared to the height of the atmosphere. Recent studies, however, gave evidence for more complicated structure of the current sources in the global circuit.

First of all, an importance of huge atmospheric thundercloud formations called mesoscale convective systems (MCSs) as the global circuit drivers became clear [3]. It is also recognized that MCSs have been found associated with the very intensive lightning activity and occurrence of sprites. The basic feature of MCSs is the presence of vast region of stratiform precipitation with the significant horizontal scale (up to a few hundreds kilometers) which determines the electric structure of entire MCS in many respects [4,5].

Second, some electric field profiles measured vertical soundings through storms have been so complex that it is doubtful that even the gross structure of the charge distribution should be approximated as a dipole or tripole; additional vertically separated charge layers appear to be required for the field profiles formation [3]. To examine the tripole paradigm further, the E-profiles by Simpson and colleagues were reanalyzed by Rust and Marshall [6], who noted than many of the profiles were consistent with a more complex thunderstorm charge distribution. Similarly, in [4] many modern E-profiles were analyzed, it and was suggested that some were caused by more than three vertically separated charges.

At present time a current system and, correspondingly, contribution of MCS to global circuit are open questions. An importance of that problem is determined first of all by a considerable net power of electric source like MCS and evidential interconnection of sprites and MCS, which serve as a positive charge reservoir for the sprite-producing flashes [7]. To take into account more complicated vertical structure of the current source and its significant horizontal scale, we develop below a model of the MCS.

A MODEL CURRENT SOURCE AND BASIC EQUATIONS

The results of sounding and radar measurements of electric field, air motion, and other meteorological parameters in MCS allowed to choose a few regions with a different electric structure: stratiform region, convective (updraft and outside updraft) regions and transition zone [4]. Differences of both charge particle sources and air flow pattern (convective or advective flows) in MCS regions lead to a significant diversity of their electric structure. Nevertheless, the results of in situ measurements make it clear that relatively slow changing layers with predominantly vertical electric field exist in MCS. Lateral expansion of these layers in srtatiform region substantially exceeds a scale height of MCS. In the framework of the model it is assumed that an electrification processes in these regions of MCS can be modeled by an effective vertical external current rising from charge separation and charged particle displacement by convective flows.

First, consider a case when an external current is located in a layer in the atmosphere. We determine stationary distribution of electric potential φ in a plain media with conductivity profile

$$\sigma(r,z) = \begin{cases} \sigma_1 = \text{const}, & z \le 0; \\ \sigma_0 \exp z/H, & z > 0, \end{cases}$$

where a vertical external current $\mathbf{j}_{\rm ex}(r,z)$ is embedded:

$$\mathbf{j}_{\rm ex}(\mathbf{r},z)=\begin{cases}0, & z<z_-,\quad z>z_+\quad (0<z_-<z_+);\\ j_{\rm ex}(\mathbf{r})\mathbf{z}_0, & z_-<z<z_+.\end{cases}$$

Assuming that electric field is potential: $\mathbf{E}=-\nabla\varphi$, and conductivity current $\mathbf{j}=\sigma\mathbf{E}$ one can obtain the following simple equation for the electric potential:

$$\mathrm{div}\left[-\sigma\nabla\varphi+\mathbf{j}_{\rm ex}(\mathbf{r},z)\right]=0.$$

Boundary conditions for the above equation are as following. Electric potential is undisturbed at the great distances:

$$\varphi|_{\left\{\begin{smallmatrix} r\to\infty \\ z\to\infty\end{smallmatrix}\right\}}\to 0,$$

and continuous at the boundaries $z=0; z_\pm$ as well as vertical component j_z of electric current density.

Since $\mathrm{div}[\mathbf{j}_{\rm ex}(\mathbf{r},z)]=0$ anywhere except boundaries of the current sheet $z=z_\pm$, it seems necessary to determine general solution of the equation

$$\mathrm{div}(\sigma(r,z)\nabla\varphi)=0.$$

Presenting a solution in the form

$$\varphi=\mathcal{F}(z)G_1(x)G_2(y),$$

and separating variables, one can easily obtain a general expression for the electric potential:

$$\varphi=\phi(k_x,k_y)\exp\left[ik_x+ik_y-\left(1\pm\sqrt{1+4H^2k_\perp^2}\right)\frac{z}{2H}\right]$$

for $z>0$ and

$$\varphi=\phi(k_x,k_y)\exp\left\{ik_x+ik_y+k_\perp z\right\}$$

for $z\le 0$, where $k_\perp=\sqrt{k_x^2+k_y^2}$; $k_{x,y}\in\mathbb{R}$; $H>0$. Using the general solution for the potential and the Fourier transform of the external current in the layer $z_-<z<z_+$,

$$i_{\rm ex}(k_x,k_y)=\frac{1}{2\pi}\iint\limits_{-\infty}^{+\infty} j_{\rm ex}(x,y)\exp(-i\mathbf{k}_\perp\mathbf{r})\mathrm{d}k_x\mathrm{d}k_y,$$

one can obtain an exact solution of the entire problem obeying above boundary conditions. For example, over the region, occupied by external currents, the expression for the potential is

$$\varphi(\mathbf{r},z)=\frac{1}{2\pi}\iint\limits_{-\infty}^{+\infty}\phi(k_x,k_y)\exp(i\mathbf{k}_\perp\mathbf{r}-æ_1(k_\perp)z)\mathrm{d}k_x\mathrm{d}k_y,$$

where

$$\phi(k_x,k_y)=\frac{i_{\rm ex}(k_x,k_y)}{\sigma_0(æ_1-æ_2)}\left[e^{-æ_2z_-}-e^{-æ_2z_+}-\frac{\sigma_1k_\perp+\sigma_0æ_2}{\sigma_1k_\perp+\sigma_0æ_1}\left(e^{-æ_1z_-}-e^{-æ_1z_+}\right)\right],$$

$$æ_{1,2}=\frac{1}{2H}\pm\sqrt{\frac{1}{4H^2}+k_\perp^2}.$$

SAMPLE CALCULATION OF THE ELECTRIC POTENTIAL

In general, using the discussed above exact solution, one can calculate an electric potential and current density arising from an *arbitrary* external current distribution $j_{\rm ex}(x,y)$. As an example, it is relatively easy to consider a case, when the external current has the Gaussian distribution over the horizontal plane:

$$j_{\rm ex}(\mathbf{r})=j_0\exp\left(-r^2/a^2\right),$$

so that

$$i_{\rm ex}(k_x, k_y) = \frac{j_0 a^2}{2} \exp\left(-\frac{k_\perp^2 a^2}{2}\right).$$

Calculations of electric potential φ were performed for different values of the horizontal scale of the current source a and current density magnitude j_0. First calculation was made for $a = 3 \cdot 10^5$ cm and $j_0 = j_0^* = 1\text{A}/(\pi a^2) \approx 3 \cdot 10^{-8}\text{A/m}^2 = 10^{-13}\ \text{SGS}_q/\text{cm}^2$ (that value corresponds to the net external current 1 A for the source of radius 3 km). Second calculation was made for $a = 3 \cdot 10^7$ cm and $j_0 = 0{,}1 j_0^*$, that provides a good agreement of the electric field inside the source with the known data for MCS's stratiform region. The other parameters were $H = 6 \cdot 10^5$ cm; $\sigma_1 = 9 \cdot 10^6\ \text{s}^{-1}$; $\sigma_0 = 4,5 \cdot 10^{-4}\ \text{s}^{-1}$; $z_+ = 9 \cdot 10^5$ cm; $z_- = 3 \cdot 10^5$ cm. Distributions of the electric potential φ for both calculations are presented in Fig.1: right panel corresponds to the first set of the parameters, left panel – to the second one. Distributions of vertical component j_z of the electric current density are presented in Fig.2.

One can see that distributions of the electric potential in the above cases are quite different. First, a magnitude of the potential at the top and bottom boundaries of the external current region are about 36 MV for $a = 3 \cdot 10^7$ cm and 15 MV for $a = 3 \cdot 10^5$ cm. Second, disturbance of the vertical electric field over the source region is significant at the heights up to 25 km for laterally expanded external current region and 12 km in the case of the smaller one. It should be also pointed out that in the first case a region of strong vertical electric field has a significant lateral scale. That can provide a more effective coupling of the thunderstorm to the ionosphere and be closely related with occurrence of sprites.

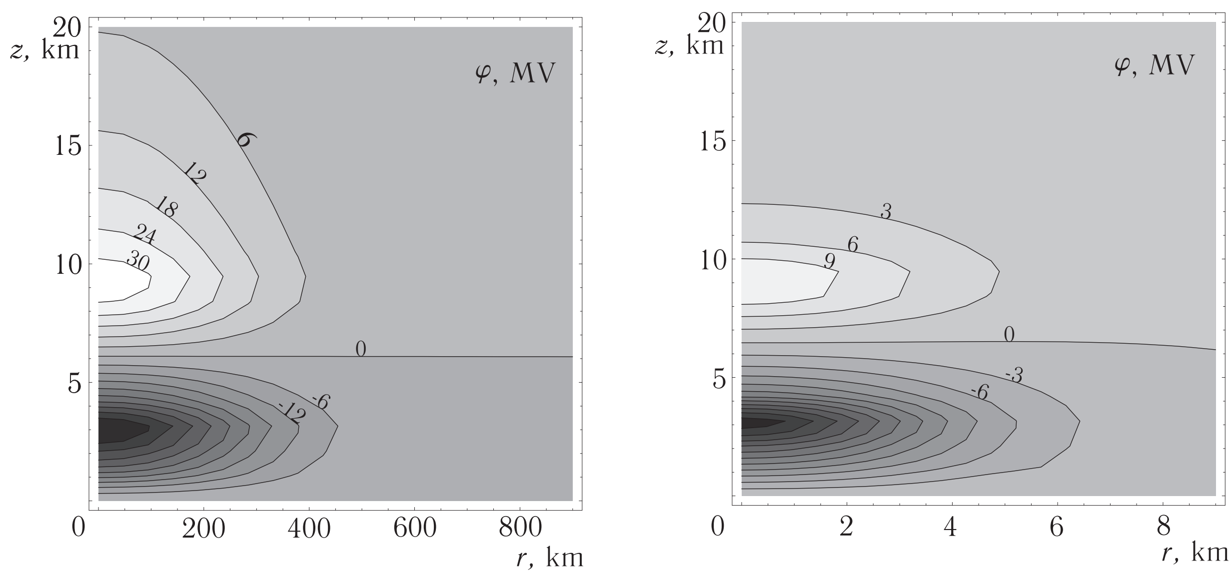

Fig.1. Model distribution of the electric potential φ for the case of one-layer external current

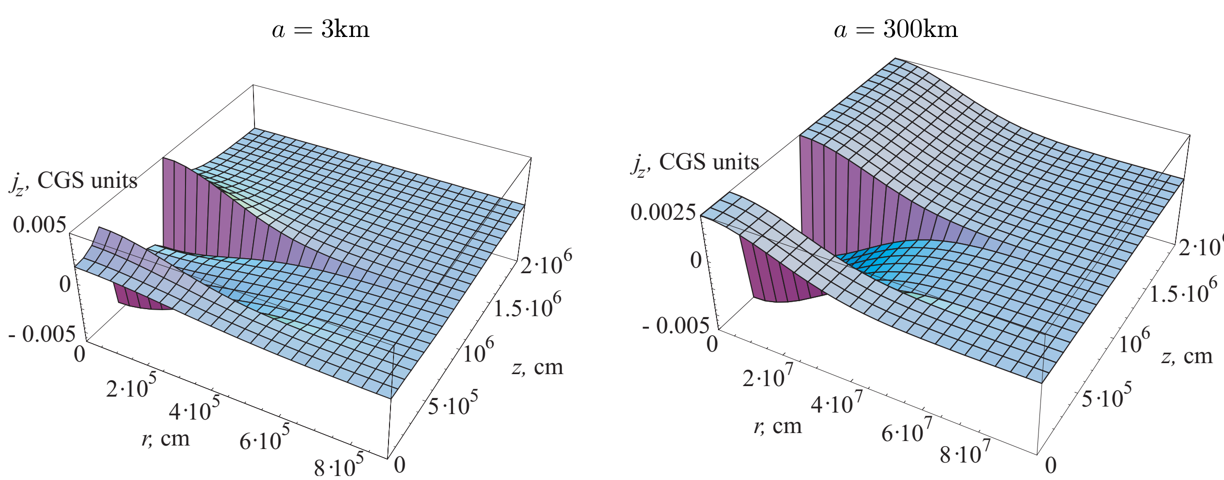

Fig.2. Model distribution of the vertical component j_z of conductivity current

One can apply the above results to investigate the electric potential and current density arising from more complex current systems. In particular, for the illustration of the method we model an electric environment of the MCS stratiform region. Using the known measurements of vertical component of electric field [5], we have to solve an «inverse problem» and restore external current distribution which provides an electric field according to experimental data. The results of such a modeling are presented in Fig.3, where the following multi-layer distribution of external current is accepted:

$$j_{\rm ex}(r,z)=\begin{cases}0, & z<0.2\ \text{km};\\ j_{01}\exp(-r^2/a^2), & 0.2\ \text{km}<z<3\ \text{km};\\ j_{02}\exp(-r^2/a^2), & 3\ \text{km}<z<4\ \text{km};\\ j_{03}\exp(-r^2/a^2), & 4\ \text{km}<z<6\ \text{km};\\ 0, & z>6\ \text{km}.\end{cases}$$

Here, $j_{01}=3.5\cdot10^{-9}$ A/m^2; $j_{02}=-1.8\cdot10^{-9}$ A/m^2; $j_{03}=7.1\cdot10^{-10}$ A/m^2; $a=300$ km.

It should be pointed out that the method presented can be applied to the modeling of the MCS current system on the whole, including the stratiform and updraft regions. Combination of the layers location, size and external current distribution is flexible enough to model entire MCS: any layer can be represented as a sepatrate current source, and using these sources one can assemble complex current systems satisfying experimental data. In general, conductivity distribution also can be modified to fit measured electric field profiles.

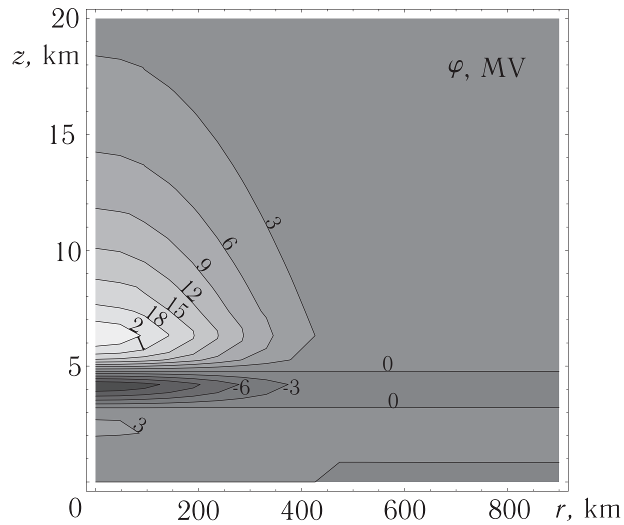

Fig.3. Model distribution of the electric potential φ for the case of multi-layer external current

Finally, the elaborated model gives an ability to calculate an electric field and current in the vicinity of stationary MCS, to determine net vertical current over the thunderclouds that is an important measure of its contribution to global circuit, and make clear a possible role of MCS in sprite events.

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