SIMULATIONS OF THIN SHELL AND 4-D INVERSION TECHNIQUES FOR MAPPING OF TOTAL ELECTRON CONTENT.

Robert W Meggs(1), **Cathryn N Mitchell**(2) **and Paul S J Spencer**(3)
(1)Department of Electronic and Electrical Engineering, University of Bath, Bath, BA27AY, UK.
r.w.meggs@bath.ac.uk
(2) As (1) above, but email c.n.mitchell@bath.ac.uk
(3) As (1) above, but email eesps@bath.ac.uk

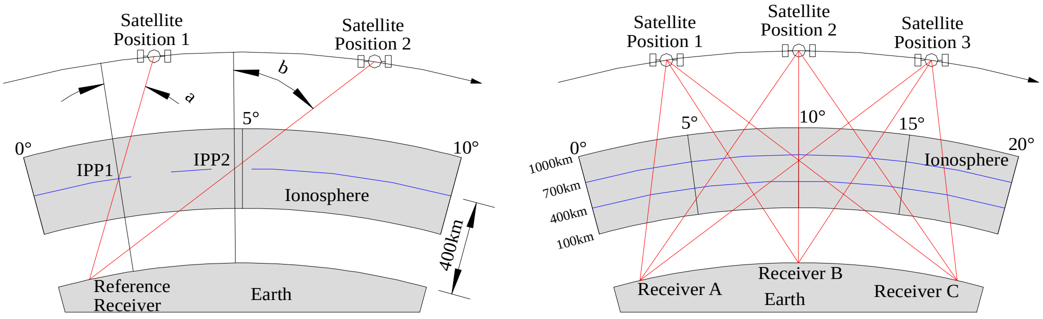
ABSTRACT

A simulation-based comparison between two approaches to Total Electron Content (TEC) mapping is presented. The first method is the thin-shell model, and the second is a full inversion through a 3-D volume. The advantage of the inversion approach is demonstrated quantitatively. In particular, we examine the effect on the calculation of slant TEC from two locations in Europe using the two different approaches to TEC mapping. We also demonstrate the effect of the horizontal grid size on the results.

INTRODUCTION

As satellite-to-ground L-band radio signals travel through the Earth's atmosphere, they experience a delay, the major contribution of which is from the ionosphere. Since the absolute amount of delay is proportional to the integrated value of electron concentration along the satellite-to-receiver ray path, knowledge of the spatial distribution of electron concentration can be used to determine the delay of a signal along a particular path. The ionosphere is a dispersive medium, so dual-frequency signals can be used to calculate and hence remove the ionospheric effect. However, single-frequency users cannot accurately determine the ionospheric delay and hence their position remains uncertain.

A solution to this problem is to provide single-frequency users with an ionospheric correction derived from fixed dual-frequency GPS receivers. Fig. 1 illustrates two different approaches used for mapping TEC over a wide geographical area. In Fig. 1(a), it is assumed that the entire ionosphere, extending from 80 to over 1000 km, can be approximated as a thin shell at a fixed height above the Earth. A problem with this approach is that the ionosphere is highly variable and the assumption of a shell height at one time may not be valid at another. In addition to the diurnal variation of the F-layer altitude, storm conditions can dramatically change the vertical distribution of the electron concentration. Furthermore, the observation of a single structure by more than one satellite or receiver could create several artificial features on a shell approximation map. This indicates that it is not possible to improve the situation merely by making a high horizontal resolution thin shell map. A better choice would be an inversion method, shown in Fig. 1(b).



(a) (b)

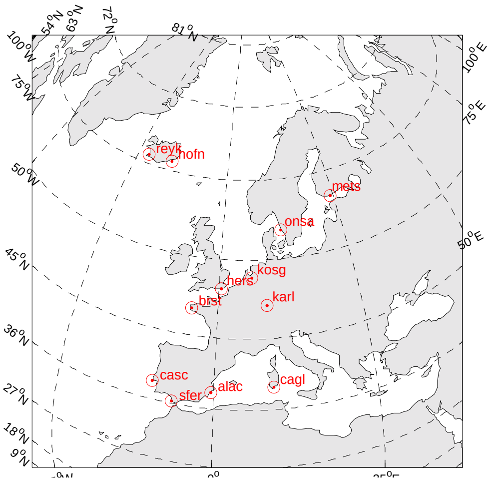
Fig. 1. Diagram showing the two methods of ionospheric mapping: a) the thin-shell and b) the inversion approach.

The inversion method models the ionosphere as a spherical volume. Consequently, either slant or vertical TEC can be extracted directly from the electron-concentration grid. This can be carried out using a mathematical inversion program, called the Multi-Instrument Data Analysis System (MIDAS), which has been developed to produce spatial maps of Total Electron Content (TEC) that are directly derived by integration through a 3-D grid of electron concentration [1].

PROCEDURE

The MIDAS program has been developed and tested using simulated and experimental data, but here we present simulations in order to facilitate a quantitative evaluation of the results. The basics of the inversion method have been described in [1]. The 3-D inversion is an underdetermined problem due to the geometrical bias and sparse nature of the GPS satellite to ground measurements, so we stabilise the inversion using an ionospheric model in the manner explained in [1]. While any model can be used for stabilisation, we have taken the simplest (Chapman function) for the purposes of this feasibility study.

For a realistic simulation over Europe, actual GPS receiver locations (see Fig. 2) and satellite orbital parameters from the International GPS Service (IGS) network were used. A geomagnetically disturbed day, 30 September 2000, was selected, and an ionosphere based on the International Reference Ionosphere (IRI) was modelled. Since the IRI model does not account for the detailed short-term variability of the ionosphere, a dynamic representation of the mid-latitude trough was superimposed on the model during the evening/night, using information taken from [2] and [3].



For the inversion and thin shell methods, the reconstructions were performed over a one-hour time interval, sampled every two minutes. The spatial resolution was varied from 1° to 5° in latitude and longitude, and the inversion height range was 80 km to 1200 km in 50 km increments. It was assumed that the signal was affected only by propagation though free space and through the ionosphere, with no account taken of the tropospheric delay, antenna pattern, or multipath effects. Hardware biases and noise were also neglected in the simulation. These effects will be implemented in a future study and their impact on the images will be assessed.

Two test locations, Milan (45.5ºN, 9ºE) and Hamburg (53.5ºN, 10ºE), were selected from which to calculate the slant TEC to each satellite in view. A comparison was then made between the accuracy of the calculated slant TEC (*TECc*) and the true slant TEC, *TECm*, found by integration through the original model. From this comparison, an error measurement was produced relating an average error in TEC for all the observations between each of the two fictitious receivers, and each satellite in view at two-minute intervals for the whole hour.

This gave a quantitative measure of the relative merits of each of the inversions evaluated over one hour. Since the error is not normalized, it retains the information on absolute TEC values that are proportional to position error. It is important to note that a larger error in the daytime will result from the greater absolute values of TEC, even though the overall accuracy of the image (from a physical basis) may be better. Thus the measure of error can only be used to evaluate quantitatively the relative merits of the images for a particular ionosphere and absolute values should not be taken to compare simulations for different times.

RESULTS

Fig. 3a shows the vertical TEC mapped with the thin shell model using GPS RINEX data simulated for 12-13 UT on 30 September 2000. The shell was at a mean ionospheric height of 400 km and the voxel size was 5° in latitude by 5° in longitude. Fig. 3b shows a map of vertical TEC produced by vertical integration through an image that was produced from the MIDAS inversion using a grid of fixed voxel size 50 km altitude by 1° latitude and longitude. It can be seen that the maps both appear to be smooth representations of the mid-latitude ionosphere. In order to make a comparison of the accuracy of the images we compare the TEC error that would be found by a user located at Hamburg and for one at Milan. For a user at Hamburg the TEC error using the thin shell model was 2.2 TECU, compared to the high-resolution inversion where the error was 0.7 TECU. At Milan, the corresponding TEC errors were 3.8 for the thin shell model and 0.7 TECU for the inversion. The greater errors at Milan were due the larger TEC values to the south. In Fig. 4, the path length errors in metres for the inversions are plotted as a function of grid resolution. It can be clearly seen that, in this case, the inversion method has generated smaller errors for a given grid resolution.

In Fig. 5, vertical TEC maps are presented for 21-22 UT on 30 September 2000, both produced using the same parameters as the 12-13 UT simulation described above. The ionospheric conditions were disturbed during this evening, indicated by the Kp value of 6. The reconstruction of the trough can be seen clearly in both of the maps. However, the contours show it to be deeper and smoother in the full inversion than in the thin shell model. For a receiver at Hamburg the TEC error using the thin shell model was 1.3 TECU, compared to the high-resolution inversion where the error was 0.3 TECU. For a receiver at Milan the corresponding TEC errors were 2.1 for the thin shell model and 0.5 TECU for the inversion. The TEC mapping for the receiver at Hamburg would have been affected by the accuracy of the TEC on the equatorward wall of the trough. The path length errors in metres are plotted in Fig. 6, and the inversion method has again generated smaller errors.

CONCLUSIONS AND DISCUSSION

Since TEC is the line-integration of electron concentration the production of a 4-D map of electron concentration should be used to calculate arbitrary slant TEC information. The work here confirms that this is not just conceptually feasible but also can be successfully implemented using realistic geometrical configuration of GPS satellites and receivers.

The work demonstrates a new technique for the mapping of TEC. This technique could lead to an improved ionospheric algorithm for Space-Based Augmentation Systems (SBAS) such as the Wide Area Augmentation System (WAAS) currently under development by the Federal Aviation Administration (FAA). An introduction to the WAAS can be found in [4]. Similar systems are under development in Europe and in Asia.

The tomographic approach used here, extended into four-dimensions, makes use of the line-of-sight propagation delay information from GNSS signals in matrix inversion. A comparison has been made between a full inversion and a thin-shell model. The results have shown that for a given horizontal resolution greater accuracy in the determination of slant TEC can be achieved using a full inversion to map the electron concentration, then to integrate along the required paths. The importance of a time-dependent inversion is stressed, since the sparse nature of GPS data and the temporal changes in the ionosphere necessitates this approach. In separate studies, we have found that it is not valid to assume that the medium is stationary over a period long enough to make an image.

The choice of the IRI model (to represent the real ionosphere) and the Chapman function (to assist the inversion) were arbitrary and our choice does not imply that any particular model is an accurate representation of the ionosphere. While the addition of the trough aids the simulation, the variability in the location and shape of the trough is large and current knowledge does not permit its accurate specification in models. The horizontal structure in the image was represented with the use of spherical harmonics. This approach can be deficient in data sparse regions and we intended to try wavelet representations in future studies. It is intended that the simulation work here will provide a useful reference from which to interpret experimental results.

REFERENCES

[1] Spencer, P.S.J. and Mitchell, C.N., “Multi-Instrument Inversion Technique for Ionospheric Imaging,” *Proceedings of the International Beacon Satellite Symposium, Boston, MA, USA, 2001.*

[2] Mitchell, C.N., Kersley, L. and Cannon, P.S., “The Latitudinal Position of the Mid-Latitude Trough – Seasonal Variations and Model Comparisons,” *Proceedings of the 4th COST 251 Workshop, Funchal, 1999*.

[3] Collis, P.N. and Haggstrom, I., “Plasma convection and auroral precipitation processes associated with the main ionospheric trough at high-latitudes,” *J. Atmos. Terr. Phys., 50, 389 - 404, 1988*.

[4] Bakry El-Arini, M., Poor, W., Lejeune, R., Conker, R., Fernow, J., Markin, K., “An Introduction to Wide Area Augmentation System and its predicted performance,” *Radio Science 36, 5, 1233-1240, 2001.*

ACKNOWLEDGEMENTS

RWM acknowledges sponsorship from BAE SYSTEMS. We are grateful to the IGS for the use of GPS data. We also acknowledge the use of the IRI model.

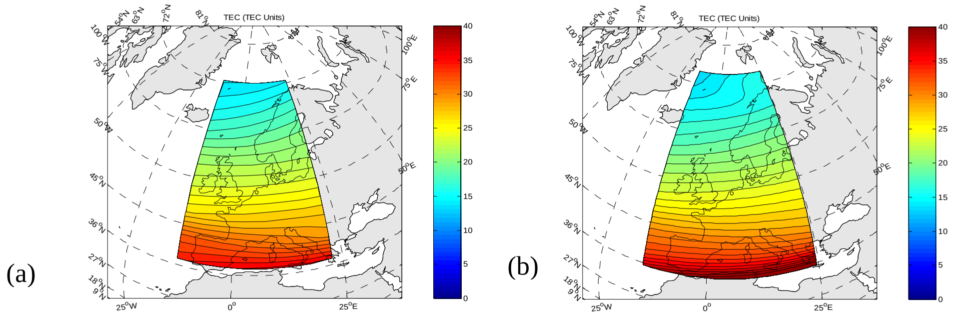


Fig. 3. Vertical TEC (in TEC Units) for a simulation for 12-13 UT (13 UT shown) on 30 September 2000 produced using a) the thin shell and b) the inversion approach.

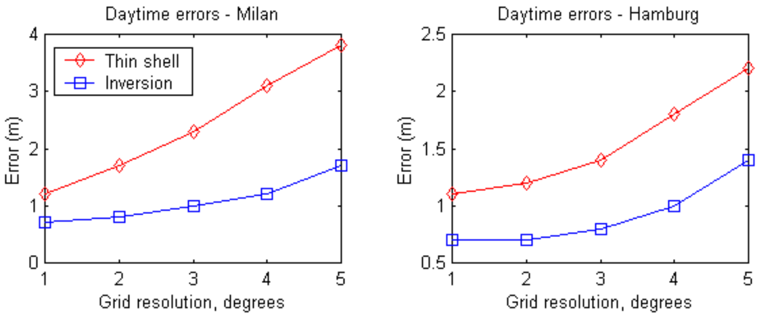


Fig. 4. Comparison of path length errors from the 12-13 UT simulations.

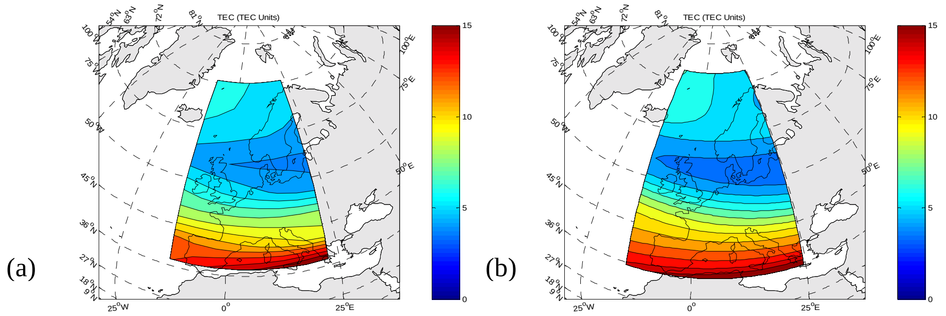


Fig. 5. Vertical TEC (in TEC Units) for a simulation for 21-22 UT (22 UT shown) on 30 September 2000 produced using a) the thin shell and b) the full inversion approach.

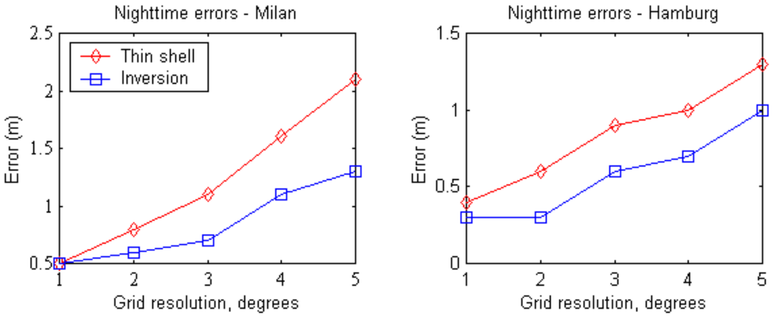


Fig. 6. Comparison of path length errors from the 21-22 UT simulations