

## EFFECT OF HORIZONTAL IONOSPHERIC STRUCTURE ON HF RADIO SYSTEMS

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### ABSTRACT

In the design and operation of many ionospheric radio systems it is assumed for simplicity that the ionosphere is spherically stratified and concentric with the Earth. The errors associated with this approach depend upon the nature of the system and the state of the ionosphere. Ionospheric structure that is anti-symmetric about the mid-point of an oblique path does not affect the time of flight over the path or the maximum useable frequency, whilst structure that is symmetric does. Anti-symmetric structure causes equal and opposite changes to the angles at both end of the ray whilst symmetric structure causes equal changes.

### INTRODUCTION

In the design and operation of many HF radio systems involving ionospheric reflection of radio rays it is frequently assumed that the ionosphere is spherically stratified and concentric with the Earth. This assumption, made for the sake of simplicity, is implicit for example in the common approach based on ionospheric control points.

The errors associated with this approach depend upon both the nature of the HF system and the state of the ionosphere. In the absence of horizontal gradients, the rays are symmetric about the mid point of the path, and this leads to important simplifications. [Here we neglect the influence of the Earth's magnetic field on the ray paths. This is valid except at lower frequencies and on shorter paths.]

The structure of the ionosphere can be represented, in general, in terms of the ionosphere at the mid-point of an oblique path, plus the symmetric and anti-symmetric parts of the deviation of the ionosphere from that at the mid-point. It is possible to draw general conclusions about the effect of these two parts on radio propagation over the path. These conclusions are valid to the first order, and depend upon magneto-ionic effects being negligible over the paths considered. Some HF systems are more severely affected by symmetric ionospheric structure and others by anti-symmetric structure.

### VARIATIONAL FORMULATION

The situation can be studied mathematically using the variational formulation of Bennett [1,2] as follows. The refractive index squared is written

$$\mu^2 = (\mu^2 - \mu_{m,p}^2) m + \mu_{m,p}^2 \quad (1)$$

and

$$\mu_m^2 \delta m = (\mu^2 - \mu_{m,p}^2) m \quad (2)$$

represents the variation of the square of the refractive index. We then examine the changes to the rays that result as  $m$  goes from 0 to 1. In particular, we find the first order changes.

The first order variation of the phase path is given by

$$\delta_m P = \int_A^B \frac{1}{2} \mu_m^2 \delta m \, du \quad (3)$$

where  $u$  is elapsed group path along the unperturbed ray, that is the ray that exists in the spherically symmetric ionosphere. Notice that only the perturbation of the refractive index along the unperturbed ray contributes. The ray is changed by the presence of horizontal gradients, Figs. 3 and 4, but these changes make no first order contributions to the change in the phase path. While it is possible to obtain equations describing the first order changes to the ray path, and various other important quantities by applying the variational method [1,2], many general conclusions may be drawn without such detailed considerations. Because many important can be expressed as variations or derivatives of the phase path, these conclusions may be drawn from (3). Note in particular that the second variation of the phase path, when the other variation involves only a shift in the ray end points, may be written

$$\delta_m \delta_n P = \delta_m \mathbf{p}|_B \cdot \delta_n \mathbf{r}|_B - \delta_m \mathbf{p}|_A \cdot \delta_n \mathbf{r}|_A \quad (4)$$

where  $\mathbf{p}|_A, \mathbf{p}|_B$  are unit vectors lying in the ray direction (the same as the wave-normal direction) at the ends of the ray. The variations  $\delta_n \mathbf{r}|_A, \delta_n \mathbf{r}|_B$  are the variations of the ray endpoint which define the  $n$  variation of the ray. The variation  $\delta_m \delta_n P$  represents the first order change in  $\delta_m P$  due to the changes in the ray end points. We can use this equation to establish facts about the changes in initial and final ray directions due to the introduction of horizontal ionospheric structure.

It is useful to consider the symmetric and anti-symmetric cases separately. Since more can be said about the anti-symmetric case than the symmetric case, and because the anti-symmetric case includes linear changes, whereas quadratic changes are symmetric, the antisymmetric case is considered first.

### ANTI-SYMMETRICAL CASE

Consider the case when the departure of the ionospheric structure from that at the mid-point is symmetric about the mid-point. In order to discuss out of plane changes, we need to be more specific. Let us agree we are considering mirror symmetry.

#### Phase Path

From (3), clearly the phase path  $P$  is unaffected, to the first order, by horizontal ionospheric structure that is anti-symmetric about the mid-point.

#### Group Path

Since the conclusion above applies at every frequency for which a ray exists, and the group path, which represents time of propagation of a pulse along the ray, is given by

$$P' = P + f \partial P / \partial f \quad (5)$$

the group path is also unaffected by horizontal ionospheric structure that is anti-symmetric. [For conciseness the qualifiers “to the first order” and “about the mid-point” are understood from now on.]

#### Maximum Useable Frequency

The above remark about group path is also valid for all frequencies. Hence the oblique ionogram and the maximum useable frequency associated with each ionospheric layer is also unaffected. However, changes of ray elevation angles may lead to low angle, and even high angle rays, being lost due to obscuration by the Earth. These effects over-ride the previous point. Such changes do affect the oblique ionogram, and may even lead to the disappearance of nose associated with the maximum useable frequency.

#### Elevation angles

We now make use of an  $n$  variation and (4). Consider the  $n$  varied ray as shown in Fig. 1(a). The end points of this varied ray have been moved equal distance away from the actual ray end points. The variation of the phase path due to anti-symmetric ionospheric structure will be zero for a ray with these new end points, since it is still symmetric about the mid-point. Hence the second variation is zero, and from (4)

$$\delta_m p_h|_B = - \delta_m p_h|_A. \quad (6)$$

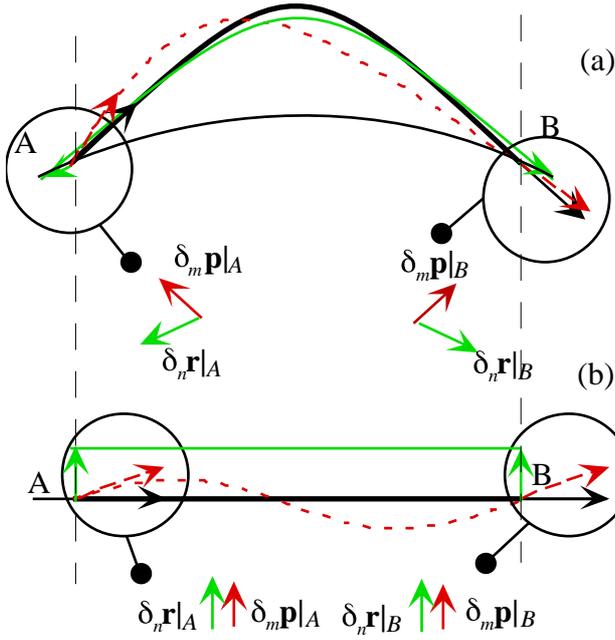


Fig. 1. Unperturbed ray and ray variations in anti-symmetrical case. (a) elevation, (b) plan view.

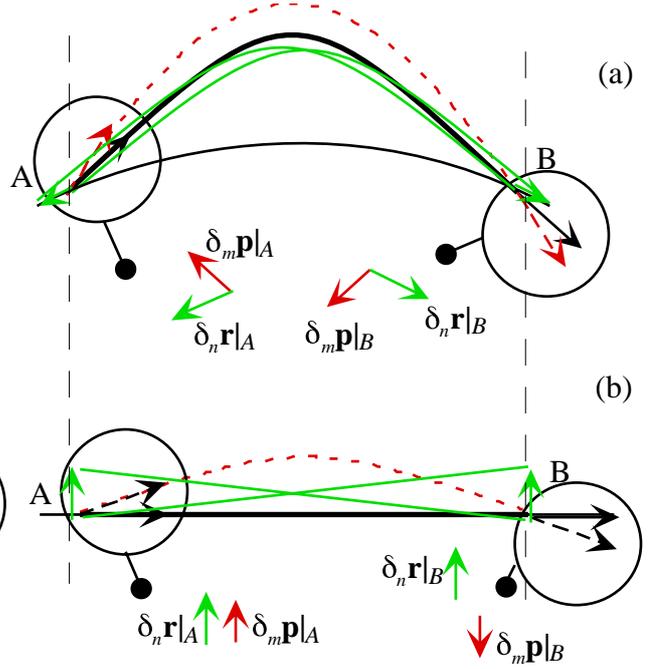


Fig. 2. Unperturbed ray and ray variations in symmetrical case. (a) elevation, (b) plan view.

Here the subscript  $h$  indicates the horizontal component. Thus (6) shows that in this case the changes to the cosine of the elevation angles at the two ends of the ray, and hence to the elevation angles themselves, are equal and opposite.

### Azimuth angles

Now consider the  $n$  varied ray as shown in Fig. 1(b). On the assumption of mirror anti-symmetry, again the second variation given by (4) is zero. In this case

$$\delta_m p_{\perp|B} = \delta_m p_{\perp|A}. \quad (7)$$

Whether this is regarded as equal or opposite changes of azimuth is a matter of definition, see Fig. 3(b).

## SYMMETRICAL CASE

### Phase Path, Group Path, Maximum Useable Frequency

These all suffer changes due to ionospheric structure which is symmetric about the ray mid-point.

### Elevation angles

We now make use of two distinct  $n$  variations as shown in Fig. 2(a). Each involves a variation at only one end of the ray. From symmetry, the two second variations will be equal. It follows that in this case elevation changes are equal,

$$\delta_m p_{h|B} = \delta_m p_{h|A}. \quad (8)$$

### Azimuth angles

Now by a similar argument using the two  $n$  variations as shown in Fig. 2(b), it follows that

$$\delta_m p_{\perp|B} = -\delta_m p_{\perp|A}. \quad (9)$$

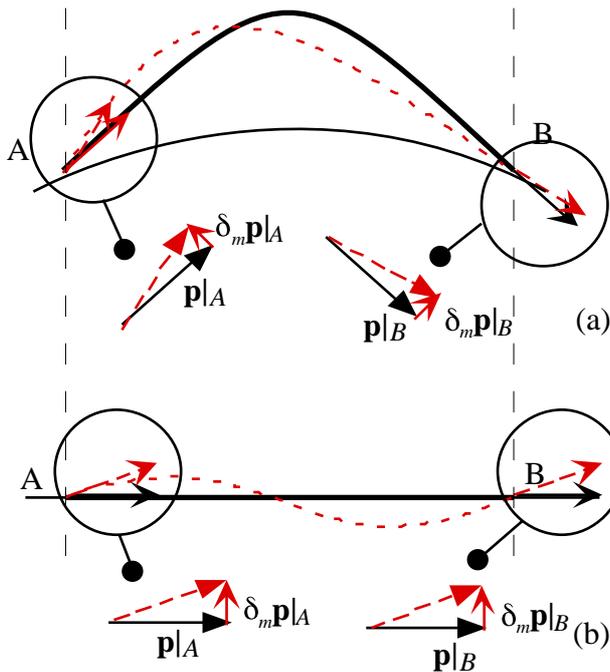


Fig. 3. Rays in anti-symmetrical case showing (a) changes in elevation, (b) changes in azimuth

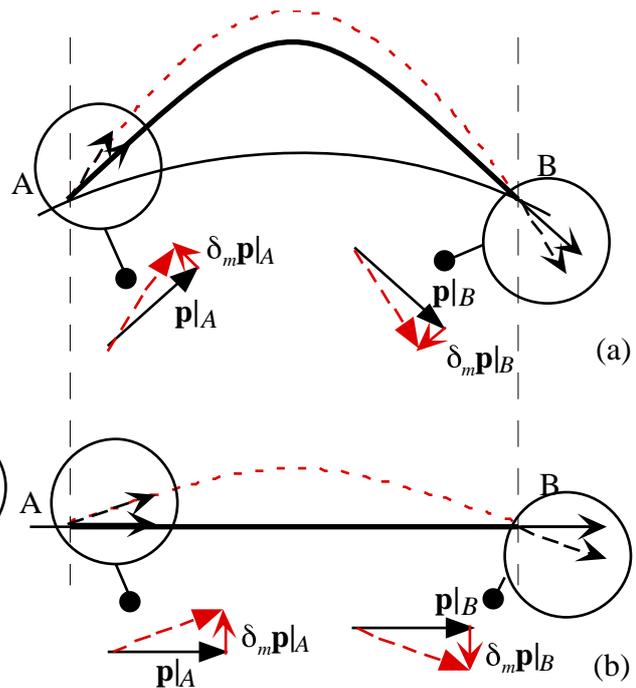


Fig. 4. Rays in symmetrical case showing (a) changes in elevation, (b) changes in azimuth

## DISCUSSION

Structure which is anti-symmetric about the ray mid-point affects ionospheric systems only through changes in initial and final ray directions. Thus, provided the correct control point is chosen, such gradients do not affect the operation of over-the-horizon-radars which make use of time of flight to a target. Ray paths may be lost because of obscuration by the Earth. Point-to-point communication systems are affected by such structure, if the vertical patterns of the antennas have deep nulls, or if the usually more reliable low angle rays are lost by obscuration. Single station location systems which use elevation measurements and direction finding systems are clearly affected. An example of structure which is anti-symmetric is an ionospheric tilt about the mid-point.

Structure that is symmetric generally does affect the group path and maximum useable frequency. There are also symmetric changes in the elevation angles. Over-the-horizon-radars, point-to-point communication systems and single station location and direction finding systems are all affected by such structure. If the horizontal structure is strong, the effects can be large. For example the ionosphere is approximately symmetric about the magnetic equator, and significant increases in maximum useable frequency are often observed in trans-equatorial propagation.

In summary, in the case where anti-symmetric horizontal structure dominates, the common practice of predicting the performance of oblique ionospheric radio paths using the mid-point ionosphere is justified. For systems where angle of arrival is important, changes in elevation and bearing can be simply estimated. However, when symmetric structure is significant the common approach will lead to errors.

## References

- [1] J. A. Bennett, "On the Application of Variation of Variation Techniques to the Ray Theory of Radio Propagation," *Radio Sci.* vol. 4, pp. 667-678, 1969
- [2] J. A. Bennett, "Variations of the Ray Path and Phase Path: A Hamiltonian Formulation," *Radio Sci.* vol. 8, pp. 737-744, 1973.