

# A Mode Matching Analysis of an Offset Coaxial Iris in a Circular Waveguide and its Applications.

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## ABSTRACT

The resonant behaviour of multiple offset coaxial irises in a circular waveguide is exploited in this paper. As the single iris can be used in the design of band-pass filters, multiple irises on the same diaphragm can be used for the design of multiple pass-bands. The analysis of the irises, as well as the whole structure, is carried out by employing the Mode Matching technique. All the necessary expressions are evaluated analytically, resulting in a fast and accurate analysis method appropriate for effective network synthesis. The coupling between two irises is investigated and it is shown that they are nearly uncoupled. Finally, two dual band filters are designed and the results are compared against those from a commercial finite element software package.

## INTRODUCTION

The introduction of multiple irises at properly selected cross-sections along a waveguide is a well established approach for the design of filters, diplexers, impedance transformers and a variety of other microwave devices, [1]. Traditionally, the irises are used as impedance inverters. However, some specific iris types have a resonant behaviour. The concentric coaxial iris in a circular waveguide has exactly this type of behaviour. This feature was exploited in our previous work [2], for the design of minimal length waveguide filters. A similar resonant behaviour is expected from the offset coaxial iris, but this geometry offers additional degrees of freedom, since multiple irises can be introduced at the same diaphragm. This is exactly the purpose of the present work. An interesting application could be the design of filters with multiple pass-bands.

## MODE MATCHING ANALYSIS OF DUAL OFFSET COAXIAL IRISES

The offset coaxial iris can be described by two junctions between a circular and an offset coaxial waveguide. As, a first step, this junction is analytically described employing the mode matching technique. The concentric case of this junction was studied in our previous work, [3]. In the present work an analytical study of the offset iris is performed. Starting from the step discontinuity between a circular and a coaxial waveguide as shown in Fig. 1. Following the standard mode matching approach, e.g. [4], a discontinuity can be considered as a two-port network characterized by its scattering parameters [S], which are expressed as:

$$\begin{bmatrix} \mathbf{B}^I \\ \mathbf{F}^{II} \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{21} & \mathbf{S}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{F}^I \\ \mathbf{B}^{II} \end{bmatrix} \quad (1)$$

where  $\mathbf{F}^I$ ,  $\mathbf{F}^{II}$  represent the forward and  $\mathbf{B}^I$ ,  $\mathbf{B}^{II}$  the backward travelling wave phasors. Expressions (1) can be written as a function of the coupling integrals [Q] as:

$$[\mathbf{S}] = \begin{bmatrix} [\mathbf{I}] & -[\mathbf{Q}] \\ [\mathbf{Q}]^t & [\mathbf{I}] \end{bmatrix}^{-1} \begin{bmatrix} -[\mathbf{I}] & [\mathbf{Q}] \\ [\mathbf{Q}]^t & [\mathbf{I}] \end{bmatrix} \quad (2)$$

The coupling integrals are defined by:

$$\mathbf{Q}_{pq} = \frac{1}{2} (\mathbf{Y}_p^I)^{1/2} (\mathbf{Y}_q^{II})^{-1/2} \int_{S_0} \bar{\mathbf{e}}_p^I \cdot \bar{\mathbf{e}}_q^{II} ds \quad (3)$$

where  $\mathbf{Y}_p^I$ , and  $\mathbf{Y}_q^{II}$  are the pth and qth mode characteristic admittance of the first and second waveguide respectively, while  $\bar{\mathbf{e}}_p^I$  and  $\bar{\mathbf{e}}_q^{II}$  are the corresponding mode eigenfunctions for the two waveguides. These eigenfunctions are

expressed with respect to a coordinate system having a z-axis coinciding with each waveguide axis of symmetry. When we have a junction of two offset waveguides (Fig. 1), their eigenfunctions must be expressed in a common coordinate system. Since the integration limits are described by the coaxial waveguide aperture, it was found more convenient to transform the eigenfunctions of the circular waveguide to the offset coordinate system of the coaxial one using the Graff's formula, [5, p.363]:

$$J_m \left( \frac{\chi'_{mn}}{a_1} R \right) \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} (m\Phi') = \sum_{k=-\infty}^{k=+\infty} J_k \left( \frac{\chi'_{mn}}{a_1} \rho \right) J_{m+k} \left( \frac{\chi'_{mn}}{a_1} d \right) \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} [k(\pi - \phi')] \quad (4)$$

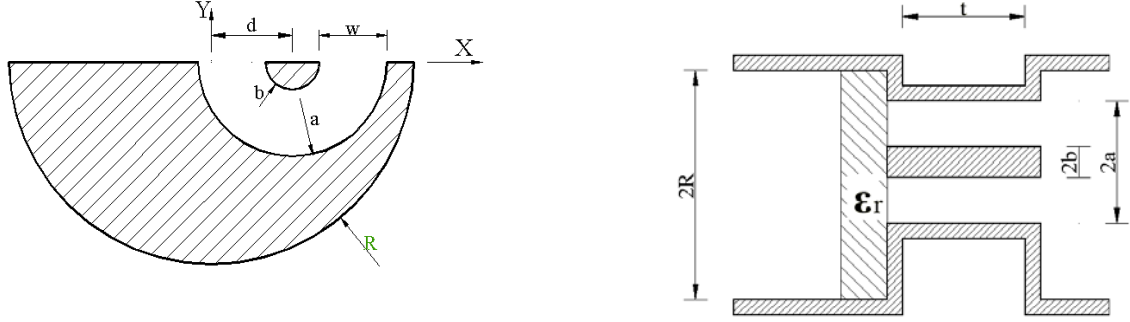


Fig. 1. Geometry of an offset coaxial iris in a circular waveguide.

Then by properly transforming the eigenfunctions of the two waveguides from the cylindrical to rectangular coordinate system, the coupling integrals take the form of a product of Bessel functions with the same order. This expression is known as the Lommel integral and can be evaluated analytically, e.g. Abramowitz and Stegun, [5, p.484]:

$$\mathcal{F}_m(\mathcal{Z}_m, \mathcal{B}_n, a) = \int_0^a \mathcal{Z}_m(x_1 \rho) \mathcal{B}_n(x_2 \rho) \rho d\rho = \frac{a}{x_1^2 - x_2^2} \{x_1 \mathcal{Z}_{m+1}(x_1 a) \mathcal{B}_m(x_2 a) - x_2 \mathcal{Z}_m(x_1 a) \mathcal{B}_{m+1}(x_2 a)\} \quad (5)$$

where,  $\mathcal{Z}_m$  and  $\mathcal{B}_n$  are Bessel and/or Neumann functions.

In this way, the coupling integrals involved in the mode matching technique are evaluated analytically and finally the junction generalized scattering parameters are given in closed form.

We can distinguish five cases for the coupling integrals, depending on the type of mode of each waveguide. These are:

#### 1. TE-TE coupling.

$$\mathbf{Q}_{cx}^{hh} = \frac{1}{2} (Y_c^h)^{1/2} (Y_x^e)^{-1/2} N_c^h N_x^e \pi \frac{\chi'_{mn} \kappa'_{il}}{a_1} \{Y_i'(\kappa'_{il} a_2) [X_{i+1} + X_{i-1}] - J_i'(\kappa'_{il} a_2) [\Psi_{i+1} + \Psi_{i-1}]\} * \left[ (-1)^i J_{m+i} \left( \frac{\chi'_{mn}}{a_1} d \right) \cos[(m+i)\phi_0] + J_{m-i} \left( \frac{\chi'_{mn}}{a_1} d \right) \cos[(m-i)\phi_0] \right] \quad (6)$$

#### 2. TE-TM coupling.

$$\mathbf{Q}_{cx}^{he} = \frac{1}{2} (Y_c^h)^{1/2} (Y_x^e)^{-1/2} N_c^h N_x^e \pi \frac{\chi'_{mn} \kappa_{il}}{a_1} \{Y_i(\kappa_{il} a_2) [X_{i+1} - X_{i-1}] - J_i(\kappa_{il} a_2) [\Psi_{i+1} - \Psi_{i-1}]\} * \left[ (-1)^i J_{m+i} \left( \frac{\chi'_{mn}}{a_1} d \right) \cos[(m+i)\phi_0] + J_{m-i} \left( \frac{\chi'_{mn}}{a_1} d \right) \cos[(m-i)\phi_0] \right] \quad (7)$$

#### 3. TM-TE coupling.

$$\mathbf{Q}_{cx}^{eh} = \frac{1}{2} (Y_c^e)^{1/2} (Y_x^h)^{-1/2} N_c^e N_x^h \pi \frac{\chi_{mn} \kappa'_{il}}{a_1} \{Y_i'(\kappa'_{il} a_2) [X_{i-1} - X_{i+1}] - J_i'(\kappa'_{il} a_2) [\Psi_{i-1} - \Psi_{i+1}]\} * \left[ -(-1)^i J_{m+i} \left( \frac{\chi'_{mn}}{a_1} d \right) \cos[(m+i)\phi_0] + J_{m-i} \left( \frac{\chi'_{mn}}{a_1} d \right) \cos[(m-i)\phi_0] \right] \quad (8)$$

#### 4. TM-TM coupling.

$$\mathbf{Q}_{cx}^{ee} = -\left(Y_c^e\right)^{1/2}\left(Y_x^e\right)^{-1/2} N_c^e N_x^e \pi \frac{\chi_{mn} K_{il}}{2a_1} \left\{ Y_i(\kappa_{il} a_2) [X_{i-1} + X_{i+1}] - J_i(\kappa_{il} a_2) [\Psi_{i-1} + \Psi_{i+1}] \right\} * \left[ (-1)^i J_{m+i} \left( \frac{\chi_{mn} d}{a_1} \right) \cos[(m+i)\varphi_0] - J_{m-i} \left( \frac{\chi_{mn} d}{a_1} \right) \cos[(m-i)\varphi_0] \right] \quad (9)$$

### 5. TM-TEM coupling.

$$\mathbf{Q}_{cx}^{et} = \sqrt{\frac{\gamma_p}{k_0}} N_c^e N_x^{tem} \frac{\pi \chi_{mn}}{a_1} \cos(m\varphi_0) J_m \left( \frac{\chi_{mn} d}{a_1} \right) \frac{J_0(\chi_{mn} a / a_1) - J_0(\chi_{mn} b / a_1)}{\chi_{mn} / a_1} \quad (10)$$

where we used the following notations:

$$X_k = \int_b^a J_k \left( \frac{\chi_{mn}}{R_1} \rho \right) J_k(\kappa_{il} \rho) \rho d\rho = \mathfrak{G}_k(J_k, J_k, a, b) \quad \Psi_k = \int_b^a J_k \left( \frac{\chi_{mn}}{R_1} \rho \right) Y_k(\kappa_{il} \rho) \rho d\rho = \mathfrak{G}_k(J_k, Y_k, a, b) \quad (11)$$

$$\mathfrak{G}_m(Z_m, \mathfrak{B}_n, a, b) = \mathcal{F}_m(Z_m, \mathfrak{B}_n, a) - \mathcal{F}_m(Z_m, \mathfrak{B}_n, b) \quad (12)$$

The above closed forms enable the efficient design of complicated structures involving multiple irises/diaphragms. When two or more coaxial irises are introduced on the same diaphragm the above procedure is repeated, but the coupling integrals are evaluated separately for each iris.

## NUMERICAL RESULTS

The single offset coaxial iris behaves as a shunt parallel resonant circuit, while multiple irises at the same diaphragm offer multiple resonances. Moreover, in between the two-shunt parallel resonances, a shunt series resonance occurs. The equivalent circuit of the dual offset coaxial irises in a circular waveguide is shown in Fig. 2a. It consists of two parallel LC networks connected in series. The equivalent impedance of this geometry is given in Fig. 2b, where the three resonances are presented.

A parametric investigation of closely positioned irises on the same diaphragm was performed, in order to exploit the coupling effects of dual irises. It was found that in most cases the coupling between the two coaxial irises was less than -35dB.

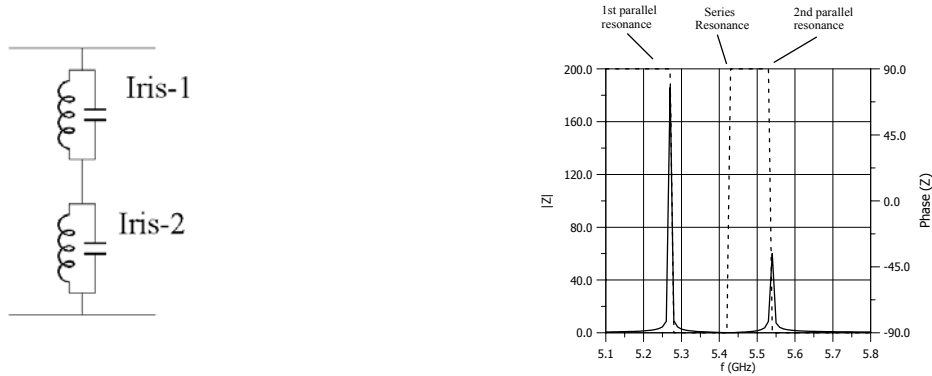


Fig. 2. a) Equivalent circuit of dual offset coaxial irises in a circular waveguide.  
b) Equivalent impedance of two coaxial irises connected in parallel.

The multiple irises diaphragm configuration can be used for the design of multiple pass-bands filters. Specifically, two dual band pass filters are designed, the first one for the 5.15-5.35GHz and 5.50-5.75GHz frequency ranges having a response shown in Fig. 3. The validity of the proposed analysis method is proved by comparing the frequency response of the designed filter with the one obtained from a simulation using Hewlett-Packard general purpose Finite Element software, HFSS [6]. It is important to note the steepest response of the dual filter (Fig. 3) which isolates the two pass bands, as compared to the responses of two single filters of the same order (3<sup>rd</sup>). The second filter was designed for the 12.00-12.75GHz and 14.25-14.75GHz frequency ranges, and its response is shown in Fig. 4.

Since the irises can be nearly uncoupled, the design procedure does not involve any coupling effects between the irises, which would disturb the filter response. This feature enables the independent design of each filter, in a dual band configuration. The overall dimensions of the dual band proposed filters are significantly smaller compared with the alternative approach, which consists of two single band filters joined with power dividers and impedance matching sections.

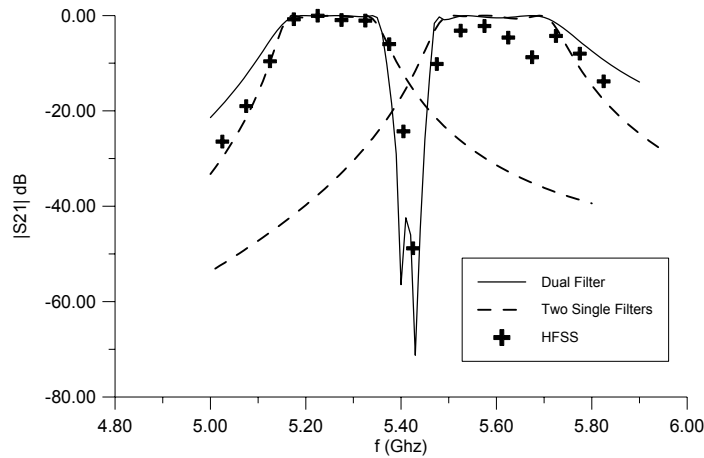


Fig. 3. A 5GHz third order dual band filter consisting of double coaxial irises in circular waveguide.

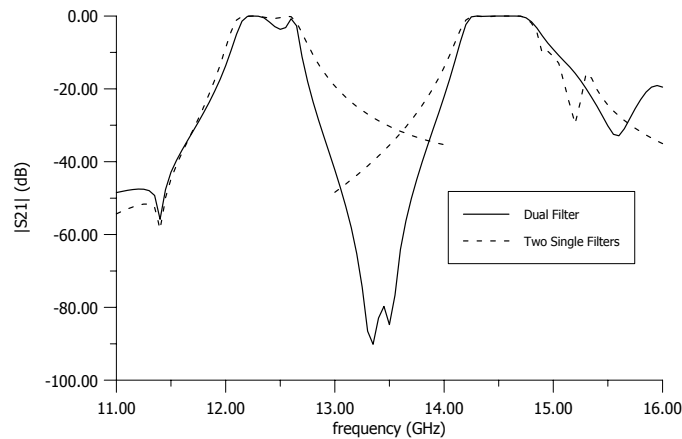


Fig. 4. A 12/14GHz third order dual band filter consisting of double coaxial irises in circular waveguide.

## CONCLUSIONS

A mode matching analysis of the offset coaxial iris is proposed. The involved coupling integrals are evaluated analytically, resulting in a fast and accurate algorithm, which can be included in microwave network synthesis software. The proposed method is validated against commercially available electromagnetic simulators. Finally, the employment of dual irises diaphragm in the design of dual band pass filters is demonstrated.

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