Investigation of Eigen - Backward –and Leaky – Waves Modes of an Axially Magnetized Lossy Cylidrical Ferrite Substrate

S. G. Diamantis(1), G. A. Kyriacou(1), A. A. Mavrides(1) and J.N. Sahalos(2)

(1) Department of Electrical and Computer Engineering, Demokritos University of Thrace, GR-67100, Xanthi, Greece, (Fax:+30-541-27264, E-mail: gkyriac@ee.cc.duth.gr).
(2) Department of Physics, Aristotle University of Thessaloniki, GR-54006, Thessaloniki, Greece.

ABSTRACT

The characteristic equation of the axially magnetized cylindrical ferrite substrate is first reduced to a single transcendental one. The large and small argument approximations are first obtained and numerically investigated for forward and backward surface wave modes. The numerical results define the limits of the asymptotic approximations validity.

INTRODUCTION

The unique features offered by magnetized ferrites, through the dynamic control of their permeability tensor from an electromagnet DC current, is very well established. Especially, axial magnetization presents non-reciprocal phenomena exploited in a variety of applications (gyrators, isolators, circulators), both in waveguide and printed geometries. Recently an increased effort is put toward the exploitation of the magnetized ferrite features in printed antenna applications, e.g. [1]. These features include electronic tuning over wide band frequency ranges, beam steering and possible surface wave and RCS reduction. Conformal patch antennas and especially those printed on cylindrical surfaces are of particular importance. Surface waves excited in any printed structure have significant effects, particularly in antenna applications. An excellent clarification of surface waves excited on planar ferrite substrate magnetized parallel to the ground plane was given by Yang et al, [2]. However, magnetization in different directions and especially those excited on cylindrical substrate are not investigated yet. The latter subject constitutes the aim of the present research effort. Furthermore it was found out that the axially magnetized cylindrical ferrite substrate supports also "backward waves" and "Leaky waves". These, along with surface waves may enable the development of novel antennas and other microwave circuits with dynamically controllable characteristics.
The characteristic equation of the axially magnetized cylindrical ferrite substrate was obtained in our previous works, e.g. [3], in the form of a 6x6 complex determinant. The analysis is based on the procedure given by Lewin [4] and Baden Fuller [5]. The characteristic equation was investigated numerically in [3] and proper-surface wave modes (normalized propagation constant $\beta/k_0\geq1$) and improper-leaky wave modes ($\beta/k_0<1$) are sought. Serious numerical problems are encountered during the above numerical investigations, especially close to the surface wave cut-off ($\beta/k_0\approx1$) and when the arguments of the involved Bessel (J_n), Neumann (Y_n) and MacDonalt (K_n) functions becomes large. Also, the determinantal form of the characteristic equation does not offer any clear physical insight. In order to deal with this problem, an extensive algebraic manipulation was undertaken in order to formulate the determinantal equation into a single transcendental one. Moreover, in order to gain a physical insight into the problem both large and small argument asymptotic expansions are carried out. The former serves to avoid the large argument numerical problems, but also offers a physical insight of the higher order mode behavior. The small argument expansion clarifies both the surface and leaky wave modes behavior around their cut-off and especially describes the important low-order modes.

ASYMPTOTIC APPROXIMATION OF THE CHARACTERISTIC EQUATION

The geometry of the axially magnetized cylindrical ferrite substrate is shown in Fig.1 As already mentioned, the characteristic equation was obtained in our previous works, e.g.[3] in a determinant form. In order to avoid the numerical problems involved in its arithmetic investigation, the characteristic equation was reduced to a single transcendental form after a long algebraic manipulation. The main advantage of this form is that the Bessel and

Neumann functions appear now only as products p_n, q_n, r_n, and s_n given in [6,p.p.361,§9.1.32], while the MacDonalt function appear only as a ratio (K'_n/K_n, where the prime denotes derivative with respect to its argument). This in turn offers improved numerical accuracy and more convenient asymptotical expansions.

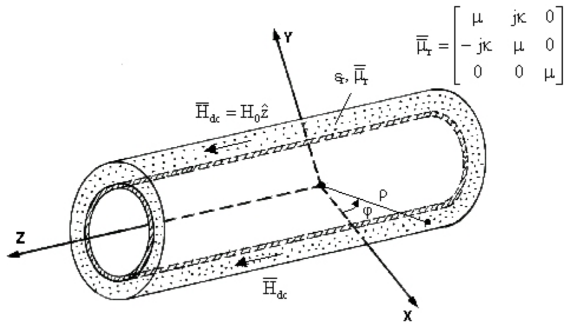

Fig. 1. Geometry of grounded cylindrical ferrite substrate.

However, even though this reduced form behaved better than the determinant form, there still major numerical difficulties in the calculation of J_n, Y_n and K_n for large arguments, especially when the argument becomes complex with large imaginary part. In order to improve the accuracy as well to reduce the computation time, but mostly to gain physical insight into the problem, the large and small argument asymptotic approximations were undertaken. For the large argument expansion a first order approximation, [6,§9.2,§9.7], is employed. Similarly, for the small argument expansion we used the small argument approximations again from [6]. Unfortunately, due to space limitation we can not quote here the resulting final expressions of these asymptotic approximations.

However, these expressions are investigated numerically, for all type of modes: surface waves, leaky waves and backward wave. For the latter case the sign of the DC magnetizing field (H_{DC}) is changed so that the wave propagation and H_{DC} are in opposite directions. It was then observed that the dispersion curves, especially for the low order modes are quite different. But, the more interesting observation is that there are regions where the group velocity is negative, while the phase velocity remains positive. This means that even though the wave propagates toward the positive z-direction there is an electromagnetic energy flow toward the negative z-direction, in other words the so-called "backward wave" is supported by the structure. This phenomenon is already known in ferrite filled waveguides where it is exploited in remanent phase-shifter applications. Likewise, it can be used for the development of remanent phase-shifters printed on cylindrical ferrite substrate. For this purpose one actually drives the magnetizing electromagnet by a bipolar current pulse, which provides the switching between positive and negative directed H_{DC}, which in turn enables the forward and backward waves excitation.

Moreover, the mode spectrum is discriminated into three frequency ranges simply identified by the sign of the effective permeability (μ_{reff}). These are defined using the frequencies where μ_{reff} or μ_r vanish, as:

$$\mu_r=0 \quad \leftrightarrow \quad \mu_{reff}\rightarrow\infty: \quad f=f_L=[f_o(f_o+f_m)]^{1/2} \tag{1}$$

$$\mu_{reff}=0 \quad \leftrightarrow \quad \mu_r+k_r=0: \quad f=f_u=f_o+f_m \tag{2}$$

In this manner the three frequency regions are defined as:

a) $0<f<f_L$: region where an infinite number of modes are excited.
b) $f_L<f<f_u$: region where the modes are typically turned off ($\mu_{reff}<0$), but a strong non-reciprocal complex mode is possible.
c) $f>f_u$: Quasi-isotropic region, with very weak ferrite effects. The ferrite behaves almost like a dielectric material.

NUMERICAL RESULTS

The numerical code is programmed in Fortran employing Bessel functions with complex arguments. The real part of each root is estimated first using a bisection technique. This is in turn used as an initial estimate for the calculation of the complex root using the IMSL subroutine ZXSSQ which is based on the Levenberg-Marquard minimization technique.

A magnetically saturated ferrite material is considered in all the cases with $\varepsilon_r = 12.6$, $4\pi M_s = 2750 Gauss$, f_m=7.7GHz and linewidth $\Delta\mathrm{H} = 225Oe$. Also, the DC-bias level is $\omega_o / \omega_m = 1$, where $\omega_o = \gamma\mu_o \mathrm{H}_{DC}$ and $\omega_m = \gamma\mu_o \mathrm{M}_s$ with γ, the gyromagnetic ratio. For a given inner and outer radius (a,b) the frequency was varied and for each value of frequency the normalized propagation constant (β/Ko) was scanned seeking for roots. The normalized propagation constant β/K_0 versus the frequency for a grounded cylindrical ferrite with inner and outer radius a=25mm and b=32mm for forward waves ($\omega_o / \omega_m = 1$) is shown in Fig.2, while (β/Ko) for the backward waves ($\omega_o / \omega_m = -1$) is shown in Fig.3. In both cases the red symbols represents the results from the exact characteristic equation. Blue symbols denote results when the asymptotic approximation is applied only for the J_n, Y_n functions, while the exact values of the K_n function are considered. The excellent agreement in this case is obvious. The black symbols in Fig.2 correspond to the case when the large argument limit $K_n'(z)/K_n(z) \to -1$ as $z \to \infty$ is also introduced. This approximation behaves well, only for $\beta/Ko>2$ for the low order modes, while its accuracy for the second and higher order modes is acceptable even for relatively small arguments. This behavior was expected since when $\beta / K_o \to 1^+$ the argument of K_n and K'_n tends to zero ($K_{co} \to 0$).

It can be concluded that for the surface waves ($\beta/Ko>1$) the J_n and Y_n Bessel functions can be always substituted by their large argument approximation. Furthermore, Fig.2 and Fig.3 refer to the lower frequency ($0<f<f_L$) mode spectrum, but equally good agreement is obtained for region-3 ($f>f_u$). The investigation of region-2 is under consideration, since even through it appears as a prohibited one ($\mu_{reff}<0$), interesting complex modes are found in the corresponding planar geometry,[8]. Also leaky waves are under investigation.

Finally the results of Figs.2 and 3 are compared in Fig.4. It can be easily observed that the change of the DC-bias sign results in a switch from a forward to a backward wave with different phase constants. This could be exploited in a lot of applications including phase shifters.

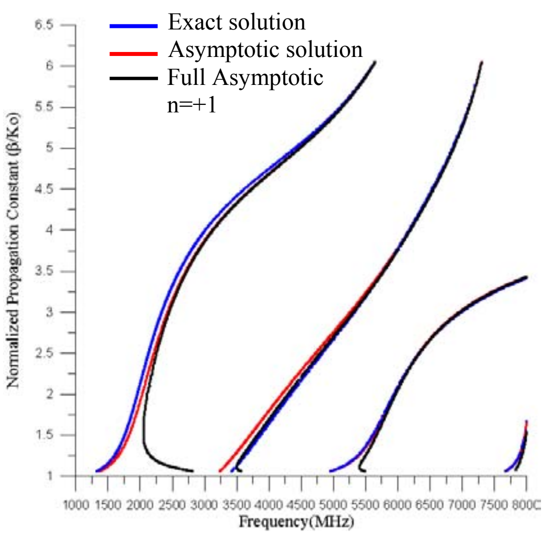

Fig.2. Comparison of the exact and large argument asymptotic expressions for forward waves.

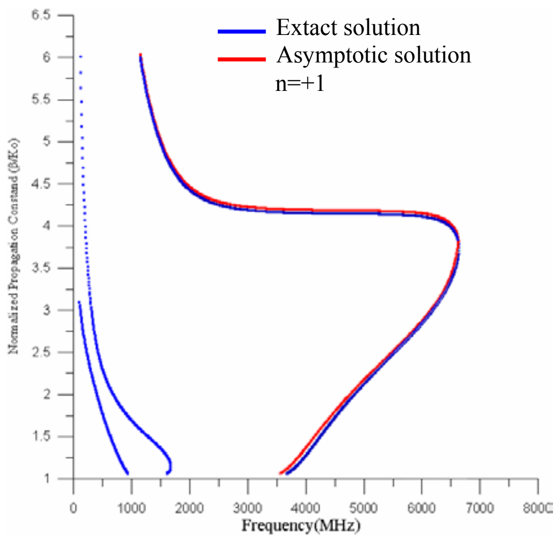

Fig.3. Comparison of the exact and large argument asymptotic expressions for backward waves.

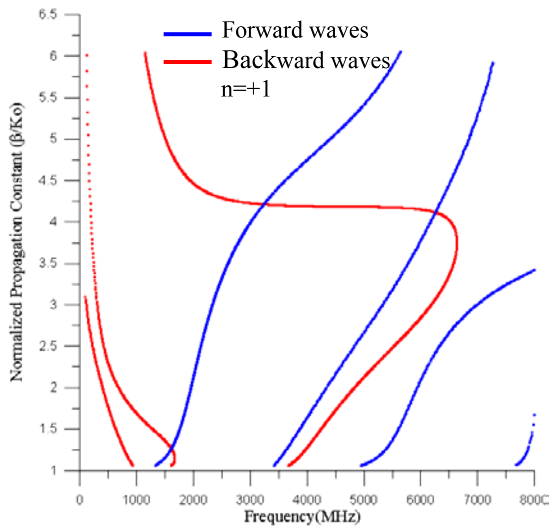

Fig.4. Normalised Propagation constant for forward and backward waves.

CONCLUSIONS

The characteristic equation of the axially magnetized cylindrical ferrite substrate is first reduced to a single transcendental equation, starting from its original 6x6 determinantal form. The large argument approximation is obtained and studied. It was found that for the surface waves the J_n ,Y_n functions can be always substituted by their large argument approximation provided that the exact value of $K_n'(z)/K_n(z)$ is preserved. In contrary when the ratio $K_n'(z)/K_n(z)$ is approximated to -1 (complete large argument expansion), the expression is accurate for higher order modes and for the dominant mode only when $\beta/Ko>2$.

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